Example 5The Terminal Voltage Of A 2-H Inductor Isv = 10(t-1) VFind The Current Flowing Through It At

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Understanding the behavior of inductors in electrical circuits is fundamental for students, engineers, and enthusiasts working with transient responses and energy storage components. In this detailed article, we will analyze the problem where a 2-H (Henry) inductor has a terminal voltage defined by the function v = 10(t - 1) volts. Our goal is to determine the current flowing through the inductor at a specific instant, leveraging the principles of inductance, differential equations, and circuit analysis.

Introduction to Inductors and Their Voltage-Current Relationship

An inductor is a passive electrical component that stores energy in a magnetic field when current flows through it. Its fundamental property is described by the relation:

$$v(t) = L\, di(t)/dt$$

where:
- v(t) is the voltage across the inductor at time t,
- L is the inductance (Henry),
- di(t)/dt is the rate of change of current with respect to time.

Key Characteristics of Inductors:
- Oppose sudden changes in current.
- The voltage across an inductor is proportional to the rate at which current changes.
- When the voltage is known as a function of time, the current can be found by integrating.

Problem Statement Breakdown

Given:
- Inductance, L = 2 H
- Terminal voltage, v(t) = 10(t - 1) V

Find:
- The current i(t) flowing through the inductor at a specific time, often at t = some value, or generally as a function of time.

Note: Since the problem states "Find The Current Flowing Through It At" without specifying a time, we often interpret this as at $t = 0$ or at a specific point in time. For completeness, the analysis will include general expressions and particular solutions at $t = 0$ and $t > 1$.

Analyzing the Voltage Function $v(t) = 10(t - 1)$ V

The voltage function is linear in time:
- For $t < 1$, $v(t) = 10(t - 1) < 0$, indicating the voltage is negative.
- For $t > 1$, $v(t) = 10(t - 1) > 0$, indicating the voltage is positive.
- At $t = 1$, $v(t) = 0$ V.

This suggests a change in the direction or influence of the voltage at $t = 1$, which impacts how current evolves.

Formulating the Differential Equation

Using the fundamental inductor relation:

$$v(t) = L\, di(t)/dt$$

Substituting the given values:

$$10(t - 1) = 2\, di(t)/dt$$

Rearranged:

$$di(t)/dt = (10/2)\ (t - 1) = 5(t - 1)$$

This is a first-order ordinary differential equation (ODE):

1. **$di(t)/dt = 5(t - 1)$**

Solving the Differential Equation for Current $i(t)$

To find $i(t)$, integrate the differential equation:

1. Rewrite:
   $di(t) = 5(t - 1)\, dt$

2. Integrate both sides:
   $i(t) = \int 5(t - 1)\, dt + C$

Performing the integration:
- $\int 5(t - 1)\, dt = 5 \int (t - 1)\, dt = 5 \left[(t^2)/2 - t \right] + C$

Therefore:

$$i(t) = (5/2)\, t^2 - 5\, t + C$$

where C is the constant of integration determined by initial conditions.

Determining the Integration Constant C

To fully specify $i(t)$, initial conditions are necessary. Commonly, in transient analysis:

- If the inductor is initially uncharged (no current at $t=0$), then $i(0) = 0$.
- Alternatively, if the initial current is known, it can be used to find C.

Assumption: For simplicity, assume the inductor is initially uncharged:

$i(0) = 0$

Substitute $t = 0$:
$0 = (5/2)(0)^2 - 5(0) + C$

$\Rightarrow C = 0$

Thus, the current as a function of time (for $t \geq 0$) is:

$$i(t) = (5/2)\, t^2 - 5\, t$$

Analyzing the Current Behavior Over Time

The expression:

$$i(t) = (5/2)\, t^2 - 5\, t$$

has important implications:

- At $t = 0$: $i(0) = 0$
- At $t = 1$:
$i(1) = (5/2)1 - 51 = 2.5 - 5 = -2.5$ A
- At $t > 1$, the quadratic term dominates, and current becomes positive again after a certain point.

Find when current crosses zero:

Set $i(t) = 0$:

$$(5/2)\, t^2 - 5\, t = 0$$

$$t \left[(5/2)\, t - 5 \right] = 0$$

$$t = 0 \text{ or } t = (5) / (5/2) = 2$$

- $t = 0$: initial condition
- $t = 2$: current returns to zero

Interpretation:
- The current starts at zero at $t=0$.
- It decreases to −2.5 A at $t=1$.
- It reaches its minimum at $t=1$, then increases back to zero at $t=2$.

Physical Interpretation of Results

The behavior of current through the inductor is governed by the voltage function. Since $v(t)$ varies linearly with time, the current exhibits a quadratic variation, reflecting the energy stored and released in the magnetic field.

- For $t < 1$, the inductor sees a negative voltage, causing the current to decrease.
- At $t = 1$, the voltage is zero, and the current reaches its minimum.
- For $t > 1$, the voltage becomes positive, causing the current to increase.

This analysis is crucial for designing circuits that involve inductors subjected to transient voltages, such as in pulse power applications, filters, and energy storage systems.

Special Cases and Additional Considerations

Initial Conditions:
- If the inductor initially carried a non-zero current, C would need to be adjusted accordingly.
- The solution applies for $t \geq 0$; for $t < 0$, initial conditions depend on circuit pre-conditions.

Steady-State Considerations:
- Since the voltage is time-dependent, the inductor's current will vary indefinitely unless controlled or damped.

Practical Applications:
- Transient analysis like this is essential for understanding switch operations, relay actions, and pulse response in circuits.

Summary and Key Takeaways

- The terminal voltage of an inductor directly relates to the rate of change of current.
- For $v(t) = 10(t - 1)$ V and $L = 2$ H, the current is obtained by integrating the differential equation $di/dt = 5(t - 1)$.
- Assuming zero initial current, the current flowing through the inductor at time t is:

$$i(t) = (5/2) t^2 - 5 t$$

- The current behavior reflects the influence of the voltage function, with a minimum at t=1 and crossing zero at t=0 and t=2.

Conclusion

Understanding the transient response of inductors is vital in electrical engineering. By analyzing how the terminal voltage impacts the current, engineers can predict circuit behavior under various conditions. The example explored here demonstrates the process of translating voltage functions into current solutions, emphasizing the importance of initial conditions, mathematical integration, and physical interpretation in circuit analysis.

Keywords for SEO Optimization:
- Inductor voltage and current relationship
- Transient analysis of inductor circuits
- Differential equations in circuit analysis
- Step-by-step solution for inductor current
- Magnetic energy storage in inductors
- Circuit analysis with time-varying voltage
- Energy transfer in inductive circuits
- Practical applications of inductors in electronics

Frequently Asked Questions

What is the significance of terminal voltage in an inductor circuit?

The terminal voltage in an inductor indicates the potential difference across it, which is directly related to the rate of change of current flowing through the inductor, according to the formula $V = L\, di/dt$.

How do you determine the current flowing through a 2-H inductor given its terminal voltage V = 10(t – 1) V?

Using the relation V = L di/dt, rearranged as di/dt = V / L, substitute V = 10(t – 1) V and L = 2 H to find di/dt = (10(t – 1))/2 = 5(t – 1). Then, integrate di/dt with respect to t to find the current i(t).

What is the expression for the current flowing through the inductor at time t?

Integrating di/dt = 5(t – 1), the current i(t) = (5/2)(t – 1)^2 + C, where C is the initial current at t = 0. Additional information about initial conditions is needed to find C.

At what time does the terminal voltage of the inductor become zero?

Since V = 10(t – 1), the voltage becomes zero at t = 1 second.

Why is the inductor's current dependent on the integral of the terminal voltage over time?

Because an inductor opposes changes in current, its current is related to the integral of voltage over time, as per the relation i(t) = (1/L) ∫ V dt + C, indicating that the current depends on the accumulated voltage effect.

Additional Resources

Example 5: The Terminal Voltage Of A 2-H Inductor Is v = 10(t-1) V. Find The Current Flowing Through It At a Given Time

Understanding the behavior of inductors in electrical circuits is fundamental for students, engineers, and hobbyists alike. In this detailed guide, we will analyze a specific problem involving an inductor with a given terminal voltage function: "The terminal voltage of a 2-H inductor is v = 10(t-1) V". Our goal is to determine the current flowing through the inductor at any given time. This problem combines concepts of inductance, differential equations, and initial conditions, providing a comprehensive example of how to approach such circuit analysis challenges.

Introduction to Inductors and Their Behavior

Before diving into the problem, it’s essential to understand the fundamental properties of inductors:

- An inductor (denoted by L) resists changes in current.
- The voltage across an inductor is proportional to the rate of change of current:

\[
v(t) = L \frac{di(t)}{dt}

\]

- The inductance L (measured in henrys, H) indicates how much voltage is generated per unit rate of change of current.

In our case, the inductor has an inductance $L = 2\,\text{H}$.

Problem Statement Recap

Given:

$$
v(t) = 10(t-1) \quad \text{V}
$$

Find:

$$
i(t) \quad \text{at a specified time}
$$

The key points are:

- The terminal voltage varies linearly with time.
- The inductor's inductance is 2 H.
- To find current $i(t)$, we need to relate voltage and current using the inductor's fundamental voltage-current relation.

Step 1: Establish the Differential Equation

From the fundamental inductor relation:

$$
v(t) = L \frac{di(t)}{dt}
$$

Substitute the known values:

$$
10(t-1) = 2 \frac{di(t)}{dt}
$$

Rearranged:

$$
\frac{di(t)}{dt} = \frac{10(t-1)}{2} = 5(t-1)
$$

This is a first-order linear differential equation for the current $i(t)$.

Step 2: Solve the Differential Equation

The differential equation:

$$
\frac{di(t)}{dt} = 5(t-1)
$$

can be integrated directly because it's separable:

$$
di(t) = 5(t-1) dt
$$

Integrate both sides:

$$
i(t) = \int 5(t-1) dt + C
$$

Calculating the integral:

$$
i(t) = 5 \int (t-1) dt + C
$$

$$
i(t) = 5 \left(\frac{t^2}{2} - t \right) + C
$$

$$
i(t) = \frac{5}{2} t^2 - 5 t + C
$$

where C is the constant of integration, determined by initial conditions.

Step 3: Determine Initial Conditions

To fully specify $i(t)$, we need initial current $i(0)$.

Common assumptions:

- If not explicitly given, often the inductor is assumed to have zero initial current at $t=0$:

$$
i(0) = 0
$$

Apply this initial condition:

$$
i(0) = \frac{5}{2} \times 0^2 - 5 \times 0 + C = C
$$

Thus:

$$
C = 0
$$

Final expression for current:

$$
i(t) = \frac{5}{2} t^2 - 5 t
$$

Summary of the Solution

Parameter	Expression
Current $i(t)$	$i(t) = \frac{5}{2} t^2 - 5 t$
Initial current $i(0)$	0 A (assuming zero initial current)

Analyzing the Results

Now, with the explicit formula, you can compute the current at any specific time t. For example:

- At $t=1\,s$:

$$
i(1) = \frac{5}{2} \times 1^2 - 5 \times 1 = \frac{5}{2} - 5 = 2.5 - 5 = -2.5\,\text{A}
$$

- At $t=2\,s$:

$$
i(2) = \frac{5}{2} \times 4 - 5 \times 2 = 10 - 10 = 0\,\text{A}
$$

The negative current indicates the direction of current flow relative to the assumed polarity.

Special Considerations

1. Significance of the Voltage Function

- The voltage $v(t) = 10(t-1)$ V increases linearly from negative to positive as time progresses.
- At $t=1$ second, voltage is zero:

$$
v(1) = 10(1-1) = 0\,V
$$

- Before $t=1\,s$:

$$
v(t) < 0 \Rightarrow \text{Voltage opposes the current flow}
$$

- After $t=1\,s$:

$v(t) > 0 \Rightarrow$ voltage supports the current flow in the assumed direction.

2. Behavior of the Current

- The quadratic term indicates that current will change non-linearly over time.
- The initial condition ensures that at $t=0$, the inductor current is zero.
- The current reaches zero again at $t=2\,s$, with the negative value at $t=1\,s$ indicating a reversal in current direction.

Extending the Analysis

Suppose we are asked to find the current at a specific time, say $t=3\,s$:

$$
i(3) = \frac{5}{2} \times 9 - 5 \times 3 = 22.5 - 15 = 7.5\,A
$$

This positive current suggests that after a certain point, the current flows in the original assumed direction as the voltage continues to increase.

Additional Considerations

- Initial Conditions: If initial current $i(0)$ is different, the constant C would change accordingly.
- Steady-State Behavior: Since the voltage varies linearly, the current will continue to grow quadratically unless some other circuit element (like a resistor or a source) limits it.
- Physical Constraints: Real inductors have parasitic resistances; including resistance would complicate the differential equation to a second-order one.

Summary and Key Takeaways

- The terminal voltage of an inductor directly relates to the rate of change of current: $v(t) = L \frac{di(t)}{dt}$.
- Given a voltage function, the current can be found by integrating the differential equation, considering initial conditions.
- In this example, with $v = 10(t-1)$ V and $L=2\,H$, the current:

$$
i(t) = \frac{5}{2} t^2 - 5 t + C
$$

with C determined by the initial current (assumed zero here).

- The behavior of current over time reflects the linear variation of voltage and the inductor's resistance to sudden changes in current.

Conclusion

Analyzing the terminal voltage of a 2-H inductor given by $v = 10(t-1)$ V reveals a rich interplay between voltage, current, and inductance. By translating the voltage into a differential equation and integrating, we derive the current as a quadratic function of time. This process exemplifies fundamental circuit analysis techniques and highlights the importance of initial conditions, especially in transient phenomena involving inductors. Whether designing filters, energy storage elements, or transient response models, understanding how to solve such problems is crucial for electrical engineers and enthusiasts alike.

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