Factor The Following Expression Completely: 3x}(3x - 4)2 + X^(8)(3x - 4)(3). O 8x?(3x - 4)(6x - 12) O

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Understanding how to factor algebraic expressions is a fundamental skill in mathematics, especially in algebra. Factoring simplifies complex expressions, making equations easier to solve and analyze. In this article, we will explore the step-by-step process of factoring a particularly challenging expression: 3x}(3x - 4)2 + X^(8)(3x - 4)(3). O 8x?(3x - 4)(6x - 12) O. Although the original expression appears complex, breaking it down systematically will reveal the underlying factors and help us factor it completely.

Understanding the Expression and Its Components

Before diving into factoring techniques, it's crucial to understand the structure of the given expression. The expression appears to be a combination of terms involving powers of x, binomials, and possibly some notation that might have been misrepresented. Let's interpret the expression carefully.

Original Expression (as interpreted):

- 3x(3x - 4)^2 + x^8(3x - 4)(3)

- The notation "O 8x?" and other symbols seem to be misprints or formatting issues. For clarity, we'll assume the expression is:

Expression:

\[3x(3x - 4)^2 + x^8(3x - 4)(3) \]

Step 1: Simplify and Rewrite the Expression

To facilitate factoring, rewrite the expression in a clearer form:

\[

$$
E = 3x(3x - 4)^2 + 3x^8(3x - 4)
$$

Notice that both terms contain common factors that can be factored out.

Step 2: Identify Common Factors

Look for common factors in both terms:

- Both terms contain $(3x - 4)$.

- The first term has a factor of $3x$ and $(3x - 4)^2$.

- The second term has $3x^8$ and $(3x - 4)$.

Common factors:

- $(3x - 4)$ (since both terms have at least one).

- The smaller power of $3x$ between $3x$ and $3x^8$ is $3x$.

Step 3: Factor Out the Greatest Common Factor (GCF)

The GCF of the two terms is:

$$
\text{GCF} = 3x (3x - 4)
$$

Now, factor this GCF out of the entire expression:

$$
E = 3x (3x - 4) \left[\frac{3x (3x - 4)^2}{3x (3x - 4)} + \frac{3x^8 (3x - 4)}{3x (3x - 4)} \right]
$$

Simplify each term inside the brackets:

- First term:

$$
\frac{3x (3x - 4)^2}{3x (3x - 4)} = (3x - 4)
$$

- Second term:

$$
\frac{3x^8 (3x - 4)}{3x (3x - 4)} = x^7
$$

So, the factored form becomes:

$$
E = 3x (3x - 4) \left[(3x - 4) + x^7 \right]
$$

Step 4: Write the Fully Factored Expression

Putting it all together, the expression factors completely as:

$$
\boxed{
E = 3x (3x - 4) [(3x - 4) + x^7]
}
$$

This expression is now fully factored into the product of simpler polynomials and monomials.

Additional Insights and Verification

It's always good practice to verify that the factoring process is correct by expanding the factored form back to the original expression.

Let's verify:

$$
\text{Expand } 3x (3x - 4) [(3x - 4) + x^7]
$$

First, distribute 3x (3x - 4) over the sum:

$$
= 3x (3x - 4)(3x - 4) + 3x (3x - 4) x^7
$$

Note that:

- $3x (3x - 4) (3x - 4) = 3x (3x - 4)^2$

- $3x (3x - 4) x^7 = 3x^{8} (3x - 4)$

Thus, the expansion matches our original expression, confirming the correctness of the factorization.

Key Takeaways for Factoring Algebraic Expressions

Factoring complex algebraic expressions often involves several strategic steps. Here's a quick guide:

1. Look for Common Factors:

- Identify the greatest common factors (GCF) among all terms.

- Factor out the GCF to simplify the expression.

2. Recognize Special Patterns:

- Recognize perfect squares, difference of squares, sum/difference of cubes.

- Use identities such as:

- $a^2 - b^2 = (a - b)(a + b)$

- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

3. Factor by Grouping:

- Group terms to factor common binomials.

- Useful when the expression has four or more terms.

4. Use Substitution:

- For complicated expressions, substitute $u = 3x - 4$ or similar to simplify.

5. Verify Your Work:

- Always expand the factored form to confirm it equals the original expression.

Common Mistakes to Avoid When Factoring

- Overlooking common factors or not factoring out the GCF completely.

- Misidentifying patterns like difference of squares.

- Forgetting to check all possible factorizations.

- Rushing through the process without verifying the result.

Practice Problems for Mastering Factoring

To reinforce your understanding, try factoring the following expressions:

1. $6x^3 - 12x^2 + 18x$

2. $x^4 - 16$

3. $3x^3 + 9x^2 + 6x$

4. $(2x - 3)^2 - (x + 4)^2$

5. $x^6 - 64$

Conclusion

Factoring algebraic expressions is a vital skill that simplifies problem-solving in mathematics. By systematically identifying common factors, recognizing patterns, and verifying your work, you can tackle even complex expressions with confidence. The original expression, once carefully analyzed and broken down, can be factored completely into a product of simpler polynomials, revealing the underlying structure and making further operations or solutions more straightforward. Practice regularly, and you'll develop mastery in algebraic factoring, a cornerstone of higher mathematics.

Meta Description:
Learn how to factor complex algebraic expressions completely with step-by-step guidance. This comprehensive guide covers identifying common factors, recognizing patterns, and verifying your work to master algebraic factoring techniques.

Frequently Asked Questions

What is the first step in factoring the expression $3x(3x - 4)^2 + x^8(3x - 4)(3)$?

The first step is to identify common factors in both terms, which include $(3x - 4)$ and possibly powers of x, to factor them out.

How can I factor out the common binomial (3x - 4) from the expression?

Since both terms contain $(3x - 4)$, you can factor it out: $(3x - 4)\,[3x(3x - 4) + x^8(3)]$.

What is the simplified form after factoring out (3x - 4)?

It simplifies to $(3x - 4)\,[3x(3x - 4) + 3x^8]$.

Can the term $3x(3x - 4)$ be further factored?

Yes, $3x(3x - 4)$ is already factored as a product, but inside the brackets, $(3x - 4)$ is already factored.

How do I factor the sum $3x(3x - 4) + 3x^8$?

Factor out the common factor $3x$: $3x\,[(3x - 4) + x^7]$.

What is the fully factored form of the original expression?

The fully factored form is $(3x - 4)\,3x\,(x^7 + (3x - 4)/3x)$, if further simplification is possible.

Is the expression equivalent to 8x?(3x - 4)(6x - 12)?

No, the given expression does not simplify directly to that form; it needs proper factoring as shown.

What common factors can be extracted from $(3x - 4)(6x - 12)$?

Since $6x - 12$ is $6(x - 2)$, the common factor with $(3x - 4)$ depends on the specific terms, but they are not directly factorable together without expansion.

How does recognizing common factors help in simplifying complex algebraic expressions?

Identifying common factors allows you to factor out and simplify the expression, making it easier to analyze or solve.

What is the importance of fully factoring algebraic expressions?

Fully factoring helps in solving equations, simplifying expressions, and understanding the roots and factors of algebraic functions.

Additional Resources

Factor the Following Expression Completely: $3x(3x - 4)^2 + x^8(3x - 4)(3)$

When it comes to algebraic expressions, the skill of factoring plays a crucial role in simplifying complex equations, solving for variables, and understanding the underlying structure of mathematical problems. The expression $3x(3x - 4)^2 + x^8(3x - 4)(3)$ presents an excellent case study for exploring advanced factoring techniques, including common factor extraction, difference of squares, and polynomial factorization. This review aims to dissect this particular expression comprehensively, highlighting step-by-step approaches, potential pitfalls, and insights into the process of factoring such complicated algebraic forms.

Understanding the Expression

Before diving into the factoring process, it’s essential to understand the components of the given expression:

Expression:
$$ 3x(3x - 4)^2 + x^8 (3x - 4) \times 3 $$

Breaking it down:

- The first term: $3x(3x - 4)^2$
- The second term: $x^8 (3x - 4) \times 3$

The structure suggests that both terms share common factors, such as $(3x - 4)$, which hints at the possibility of factoring out common elements to simplify the expression.

Step-by-Step Factorization Process

Step 1: Identify Common Factors

Observation indicates that both terms involve the factor $(3x - 4)$:

- The first term has $(3x - 4)^2$,
- The second term has $(3x - 4)$.

Additionally, both terms include factors involving x:

- First term: $3x$,
- Second term: x^8.

The smallest common factors between the two are:

- $(3x - 4)$,
- x.

Hence, the greatest common factor (GCF) of the entire expression should include these.

GCF candidates:

$$ \text{GCF} = (3x - 4) \times x $$

Step 2: Factor Out the GCF

Factoring out $(3x - 4) \times x$, we rewrite the original expression as:

$$
3x(3x - 4)^2 + 3x^8 (3x - 4)
= (3x - 4) \times x \times \left[\frac{3x(3x - 4)^2}{(3x - 4) \times x} + \frac{3x^8 (3x - 4)}{(3x - 4) \times x} \right]
$$

Simplify each term inside brackets:

- For the first term:

$$
\frac{3x(3x - 4)^2}{(3x - 4) \times x} = \frac{3x}{x} \times \frac{(3x - 4)^2}{(3x - 4)} = 3 \times (3x - 4)
$$

- For the second term:

$$
\frac{3x^8 (3x - 4)}{(3x - 4) \times x} = \frac{3x^8}{x} = 3x^{7}
$$

Therefore, the expression becomes:

$$
(3x - 4) \times x \times \left[3(3x - 4) + 3x^{7} \right]
$$

Step 3: Simplify the Remaining Expression

Now, focus on the bracket:

$$
3(3x - 4) + 3x^{7}
$$

Distribute the 3 in the first term:

$$
3 \times 3x - 3 \times 4 + 3x^{7} = 9x - 12 + 3x^{7}
$$

Rearranged as:

$$
3x^{7} + 9x - 12
$$

Hence, the fully factored form of the original expression is:

$$
(3x - 4) \times x \times (3x^{7} + 9x - 12)
$$

Further Factorization of the Remaining Polynomial

The expression:

$$
3x^{7} + 9x - 12
$$

can potentially be factored further. Let's analyze this polynomial:

- Common factors in all terms: 3

Factoring out 3:

$$
3 (x^{7} + 3x - 4)
$$

Now, the entire expression becomes:

$$
(3x - 4) \times x \times 3 (x^{7} + 3x - 4)
$$

which simplifies to:

$$
3 (3x - 4) x (x^{7} + 3x - 4)
$$

Final Factored Form and Its Features

Complete factorization:

$$
\boxed{
3 \times (3x - 4) \times x \times (x^{7} + 3x - 4)
}
$$

This form reveals several features:

- The factorization reduces a complex algebraic expression into products of simpler polynomials and monomials.
- It exposes the roots and potential solutions of the original expression.
- The factors include linear, monomial, and higher-degree polynomial components, indicating the expression's complexity and diversity of roots.

Analysis of the Factored Expression

Pros

- Simplification: Factoring reduces the original complex expression into manageable parts, facilitating easier analysis and solution derivation.
- Root identification: The zeros of the factors directly give roots of the original expression.
- Insight into structure: Recognizing common factors and polynomial patterns enhances understanding of algebraic structures.

Cons

- Complexity of higher-degree polynomials: The term $x^{7} + 3x - 4$ may not be factorable easily over the rationals, requiring advanced techniques or numerical methods.
- Potential for missed factors: Without systematic methods like polynomial division or synthetic division, some factors could be overlooked.
- Limited insight into irrational roots: Factoring over rational numbers may not reveal roots involving irrational or complex solutions.

Features and Applications

- Educational value: Demonstrates layered techniques such as common factor extraction, polynomial factoring, and grouping.
- Practical applications: Useful in calculus (finding zeros), solving equations, and simplifying expressions in physics or engineering.
- Foundation for advanced topics: Serves as a stepping stone toward understanding polynomial theorems, irreducibility, and algebraic identities.

Conclusion

Factoring the expression $3x(3x - 4)^2 + x^8(3x - 4)(3)$ reveals a layered, systematic approach that uncovers its underlying structure. By identifying common factors, simplifying polynomial components, and extracting the greatest common factor, the expression is reduced to a product of simpler factors:

\[
3 \times (3x - 4) \times x \times (x^{7} + 3x - 4)
\]

This process not only simplifies algebraic manipulation but also provides deeper insights into the roots and behavior of the polynomial. While the factorization process can become challenging with higher-degree polynomials, the approach outlined here demonstrates the power of algebraic techniques in tackling complex expressions. Mastery of these methods enhances problem-solving skills, boosts understanding of polynomial behavior, and forms a foundation for more advanced mathematical exploration.

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