

Fig. 11.43 A Vertical Pole, XY, Is Erected On A Piece Of Level Ground. A Student Whose Eye, E, Is 1.5

Introduction

Fig. 11.43 A Vertical Pole, XY, Is Erected On A Piece Of Level Ground. A Student Whose Eye, E, Is 1.5 meters Above The Ground, Observes The Pole.

This diagram illustrates a fundamental concept in geometry and trigonometry—how to determine heights and distances using angles of elevation and depression. The setup involves a vertical pole, XY, firmly erected on a level ground, with an observer positioned some distance away from the pole, measuring angles to establish the pole's height or the observer's distance from the pole. Such problems are common in surveying, navigation, and engineering, where accurate measurements are crucial.

In this article, we will explore the concepts underlying this setup, including the principles of angles of elevation and depression, how to apply trigonometry to solve practical problems, and the significance of these methods in real-world applications. We will analyze the scenario in detail, explain the mathematical steps involved, and provide examples to help you grasp the concepts thoroughly.

Understanding the Setup

Components of the Diagram

- Vertical Pole XY: Erected on a level ground, standing upright.
- Level Ground: Flat surface where the pole is installed.
- Observer's Eye E: Located 1.5 meters above the ground, representing the height of the student's eye.
- Line of Sight: The line connecting the observer's eye to the top and bottom of the pole, forming angles of elevation and depression.
- Angles of Elevation and Depression: The angles measured between the horizontal line from the observer's eye and the line of sight to the top or bottom of the pole.

Understanding these components is essential for setting up the problem properly and applying trigonometric principles accurately.

Fundamental Concepts in Angles of Elevation and Depression

Definitions

- Angle of Elevation: The angle between the horizontal line from the observer's eye and the line of sight to an object above the horizontal plane.
- Angle of Depression: The angle between the horizontal line and the line of sight to an object below the horizontal plane.

Properties

- When observing objects at the same height as the observer, the angles of elevation and depression are equal if the objects are at the same horizontal distance.
- These angles are measured using instruments like a theodolite, clinometer, or even simplified tools like a protractor in basic problems.

Importance in Real-World Applications

- Surveying: Measuring distances and heights of terrains and structures.
- Navigation: Determining positions based on angles of celestial bodies.
- Engineering: Designing structures and assessing site elevations.
- Meteorology: Estimating cloud heights.

Applying Trigonometry to the Scenario

Setting Up the Problem

Suppose the student at point E measures two angles:

- Angle of elevation to the top of the pole (θ_1)
- Angle of depression to the bottom of the pole (θ_2)

Let:

- h: Height of the pole XY.
- d: Horizontal distance from the student to the base of the pole.
- E: The eye level of the student, 1.5 meters above the ground.

The goal might be to find:

- The height of the pole (XY).
- The distance from the student to the pole.

Assumptions for Simplification

- The ground is perfectly level.
- The angles are measured accurately.
- The pole is perfectly vertical.
- The observer's eye level is known and constant.

Mathematical Formulation

Using basic trigonometry:

- For the top of the pole:

$$\tan \theta_1 = \frac{h - E}{d}$$

- For the bottom of the pole:

$$\tan \theta_2 = \frac{E}{d}$$

From these two equations, the height of the pole can be expressed as:

$$h = d \tan \theta_1 + E$$

and the distance as:

$$d = \frac{E}{\tan \theta_2}$$

Note: If the angles are known, these formulas allow the calculation of both the pole's height and the distance.

Step-by-Step Solution Approach

1. Measure the Angles

Using a theodolite or a simple angle-measuring instrument, the student measures:

- θ_1 : The angle of elevation to the top of the pole.
- θ_2 : The angle of depression to the bottom of the pole (or vice versa depending on

observation).

2. Calculate the Horizontal Distance (d)

Using the angle of depression:

$$d = \frac{E}{\tan \theta_2}$$

This gives the horizontal distance from the student to the base of the pole.

3. Determine the Height of the Pole (h)

Using the angle of elevation:

$$h = d \tan \theta_1 + E$$

This accounts for the height of the pole above the observer's eye level, plus the observer's eye level itself.

4. Final Calculation

Substitute the known values into the formulas to compute the pole's height:

$$h = \left(\frac{E}{\tan \theta_2} \right) \tan \theta_1 + E$$

or, if the angles are directly known, plug in the values to find a numerical answer.

Practical Examples and Applications

Example 1: Calculating the Height of a Pole

Suppose:

- The student measures an angle of elevation to the top of the pole as 45° .
- The angle of depression to the bottom of the pole as 30° .
- The student's eye level, $E = 1.5$ meters.

Solution:

1. Calculate d:

$$\begin{aligned} d &= \frac{1.5}{\tan 30^\circ} = \frac{1.5}{\frac{\sqrt{3}}{3}} = 1.5 \times \frac{3}{\sqrt{3}} = 1.5 \times \sqrt{3} \approx 1.5 \times 1.732 = 2.598 \text{ meters} \end{aligned}$$

2. Calculate h:

$$h = d \tan 45^\circ + 1.5 = 2.598 \times 1 + 1.5 = 2.598 + 1.5 = 4.098 \text{ meters}$$

Result: The height of the pole XY is approximately 4.10 meters.

Example 2: Determining Distance to the Pole

If the height of the pole XY is known to be 6 meters, and the angles of elevation and depression are known, you can reverse the calculations to find the distance.

Significance of the Method in Various Fields

Surveying and Civil Engineering

- Accurate measurements of land elevation and structure heights.
- Planning construction projects based on terrain data.
- Establishing boundary lines and property extents.

Navigation and Maritime Applications

- Using angles of celestial bodies to determine longitude and latitude.
- Navigating ships by measuring angles to landmarks.

Environmental and Geological Studies

- Measuring tree heights or mountain elevations.
- Assessing terrain features for ecological studies.

Educational Importance

- Enhances understanding of basic trigonometry.
- Develops practical problem-solving skills.
- Demonstrates real-life applications of mathematical concepts.

Common Challenges and Considerations

Measurement Accuracy

- Precise measurement of angles is crucial; errors can lead to significant inaccuracies.
- Use of proper instruments and calibration is recommended.

Assumptions and Simplifications

- The level ground assumption may not always hold true in real terrains.
- Adjustments are necessary for sloped grounds.

Visibility and Obstructions

- Obstructions may hinder the line of sight.
- Alternative methods or instruments may be needed.

Environmental Factors

- Atmospheric conditions can affect measurements.
- Using correction factors may be necessary.

Conclusion

Understanding the principles demonstrated in **Fig. 11.43** involves grasping the fundamentals of angles of elevation and depression, applying trigonometric ratios, and recognizing their practical applications. This setup is a classic example used in surveying, navigation, and engineering to measure heights and distances indirectly, especially when direct measurement is challenging.

By carefully measuring angles and applying the right formulas, students and professionals can accurately determine the height of structures like poles, trees, or buildings, and measure distances across terrains. Mastery of these concepts not only enhances

mathematical skills but also provides valuable tools for real-world problem-solving across various disciplines.

Whether in academic exercises or practical fieldwork, the ability to analyze such diagrams and perform these calculations is essential. As technology advances, more sophisticated instruments continue to improve measurement accuracy, but the fundamental principles of trigonometry remain at the core of these

Frequently Asked Questions

What is the main purpose of Fig. 11.43 involving a vertical pole XY on level ground?

The figure is used to illustrate the principles of perspective projection, particularly how the eye's position affects the perception of the pole's height and the angles involved.

How does the height of the eye (E) influence the perception of the pole's length in the diagram?

The height of the eye determines the angles of sight to the top and bottom of the pole, affecting how the pole appears from the observer's point of view and influencing measurements like the apparent height.

What is meant by the term 'level ground' in the context of Fig. 11.43?

Level ground refers to a flat, horizontal surface on which the pole is erected, ensuring no incline or decline affects the projection or perception of the pole's height.

Why is the student's eye position (E) significant in the analysis of the figure?

The eye position is significant because it serves as the station point from which the angles and projections are measured, affecting how the pole's height and position are perceived and calculated.

How can this diagram be used to determine the actual height of the pole XY?

By measuring the angles of elevation from the eye point E to the top and bottom of the pole and using trigonometric relations, the actual height of the pole can be calculated.

What assumptions are typically made about the ground

and the pole in Fig. 11.43 for accurate measurements?

It is assumed that the ground is perfectly level, the pole is perfectly vertical, and the eye point remains stationary to ensure accurate projection and measurement calculations.

In practical surveying, how does understanding the perspective shown in Fig. 11.43 assist in field measurements?

Understanding this perspective helps surveyors accurately determine heights and distances by measuring angles from a known eye point, applying proper trigonometry, and accounting for elevation differences.

What role does the concept of similar triangles play in the analysis of the figure?

Similar triangles formed by the sight lines and the pole allow for proportional reasoning, enabling the calculation of the pole's height based on known distances and angles from the eye point.

Could this diagram be used to explain the principle of parallax? If so, how?

Yes, the diagram illustrates how the apparent position of the top of the pole changes with the observer's eye level, demonstrating parallax and how it can be used to measure heights or distances indirectly.

Additional Resources

Fig. 11.43 A Vertical Pole, XY, Is Erected On A Piece Of Level Ground. A Student Whose Eye, E, Is 1.5

Introduction

Fig. 11.43 depicts a classic scenario frequently encountered in the study of practical geometry and surveying: a vertical pole, labeled XY, erected on a level ground surface, with a student observing the pole from a fixed point on the ground, at a height of 1.5 meters above the ground. This simple yet fundamental setup forms the basis for understanding various principles of angular measurement, perspective, and triangulation—concepts that are essential in fields ranging from civil engineering to navigation. In this article, we will explore the geometrical and practical aspects of such an arrangement, analyze the methods used to determine heights and distances, and discuss the significance of the observations in real-world applications.

Understanding the Setup: The Geometry of the Vertical Pole and the Observer

The Basic Components

The diagram features several key elements:

- Vertical Pole XY: Erected upright on the ground, presumably of known or unknown height.
- Level Ground: A flat, horizontal surface on which the pole stands, ensuring that measurements relate to a consistent baseline.
- Observer's Eye, E: The point from which the student views the pole, located at a height of 1.5 meters above ground level.

The Significance of the Setup

This configuration is a textbook example of a common surveying problem. The primary challenge is to determine the height of the pole, or the distance from the observer to the pole, using only angular measurements taken from the observer's position. Such problems often involve the application of basic trigonometry, particularly the tangent function, which relates angles to lengths in right-angled triangles.

Analyzing the Observation: Angles and Measurements

The Role of Angles of Elevation

When the student observes the top and bottom of the pole from point E, they measure the angles of elevation—let's denote:

- θ_1 : The angle of elevation to the top of the pole.
- θ_2 : The angle of elevation to the bottom of the pole (which is on the ground level).

These measurements are crucial because they allow us to establish relationships between the known and unknown quantities:

- The horizontal distance from the observer to the foot of the pole (say, point A).
- The height of the pole (from ground level to point Y).

Establishing the Triangles

From the observations, two right-angled triangles can be constructed:

1. Triangle 1 (to the top of the pole):

- Vertices: Observer's eye (E), ground point directly beneath the top of the pole (say, point C), and the top of the pole (Y).
- Known: The angle θ_1 , the observer's eye height (1.5 meters).
- Unknowns: Horizontal distance (A E), and the height of the pole (Y).

2. Triangle 2 (to the bottom of the pole):

- Vertices: E, the foot of the pole (A), and the ground level (assumed to be at zero height).
- Known: The angle θ_2 , the observer's eye height.
- Unknown: Horizontal distance (which should be the same as in the first triangle).

Because the ground is level, the horizontal distance from the observer to the pole's foot remains constant in both triangles. Using these triangles, the height of the pole can be calculated if the angles are measured accurately.

Mathematical Approach: Applying Trigonometry

Calculating the Distance from the Observer to the Pole

Suppose:

- The height of the observer's eye above ground: $(h_e = 1.5\text{m})$.
- The angle of elevation to the top of the pole: (θ_1) .
- The angle of elevation to the foot of the pole: (θ_2) .

Then, the horizontal distance (D) from the observer to the pole's foot is given by:

$$D = \frac{h_e}{\tan \theta_2}$$

This follows because:

$$\tan \theta_2 = \frac{\text{height of eye above ground}}{\text{distance to pole}}$$

Determining the Total Height of the Pole

The height of the pole above ground, (H) , is then:

$$H = h_e + (D) \times \tan \theta_1$$

Here:

- $(D) \times \tan \theta_1$ gives the height of the top of the pole above the eye level.
- Adding (h_e) accounts for the observer's eye height, yielding the total pole height.

Summary of the Calculation

- Measure (θ_1) and (θ_2) .
- Calculate (D) using the known (h_e) and (θ_2) .
- Calculate (H) using the calculated (D) and (θ_1) .

Practical Considerations

- Accurate measurement of angles is crucial; tools such as a clinometer or a theodolite are often employed.

- It's essential to ensure that the ground is level; any slope can introduce errors.
- Multiple measurements can improve accuracy, averaging out errors.

Real-World Applications and Significance

Civil and Structural Engineering

Understanding how to measure heights and distances accurately is fundamental in construction and civil engineering. For example, when erecting tall structures or installing communication towers, engineers often rely on such geometric principles to verify heights without direct measurement.

Navigation and Topographical Surveys

Mariners and land surveyors use similar methods to determine the elevation of distant landmarks or features when direct measurement is impractical, especially over rugged terrain.

Environmental and Ecological Studies

Scientists studying landscape features or monitoring changes over time also employ triangulation methods to measure heights and distances between natural features.

Advanced Techniques and Modern Tools

While traditional methods rely on basic trigonometry and manual measurements, modern technology has enhanced accuracy and efficiency:

- Theodolites and Total Stations: Advanced surveying instruments that automate angular measurements.
- Laser Rangefinders: Devices that measure distances directly with laser beams, reducing reliance on angular measurement alone.
- GPS and GIS Technologies: Provide geospatial data with high precision, supplementing traditional methods.

Despite these advancements, the fundamental principles exemplified in Fig. 11.43 remain relevant, especially in educational contexts and scenarios where high-tech equipment is unavailable.

Limitations and Sources of Error

While the geometric approach is straightforward, several factors can affect accuracy:

- Measurement Errors: Small inaccuracies in angle measurement can lead to significant errors in height estimation.

- Refraction and Atmospheric Conditions: Light bending can distort observations, especially over long distances.
- Uneven Ground or Slope: Slope can introduce errors if not properly accounted for.
- Instrument Calibration: Poorly calibrated tools can produce inaccurate readings.

Proper training, repeated measurements, and calibration are essential to mitigate these issues.

Conclusion

Fig. 11.43 offers a window into the practical application of geometry and trigonometry in real-world measurement tasks. By understanding the relationships between angles, distances, and heights, students and professionals can accurately determine the dimensions of objects that are otherwise difficult or impossible to measure directly. Whether in the classroom, on a construction site, or in the field of surveying, these principles form the backbone of many essential tasks. As technology continues to evolve, the fundamental concepts illustrated by this setup remain vital, reminding us that a simple setup—a vertical pole and a student's eye—can unlock complex insights into our environment.

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