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Introduction

Determinants are fundamental concepts in linear algebra, playing a crucial role in understanding the properties of matrices. When working with matrices, especially those involving scalar multiples, it's essential to understand how the determinant behaves under such operations. One common question that arises is: How does the determinant change when a matrix is scaled by a scalar?

In particular, consider an \(n \times n \) matrix \(A \) and a scalar \(r \). What is the value of the determinant of the matrix \(rA \)? The answer to this question not only deepens our comprehension of matrix properties but also has applications spanning across various fields such as physics, engineering, computer science, and mathematics.

This article aims to explore the formula for the determinant of a scalar multiple of a matrix, analyze the correct expression, and understand the reasoning behind it. We will also clarify common misconceptions, provide detailed explanations, and present relevant formulas to ensure a comprehensive understanding.

Understanding the Determinant of a Scalar Multiple of a Matrix

Before delving into the formula, let’s review what a determinant signifies and how it relates to matrix scaling.

What Is a Determinant?

The determinant of an $n \times n$ matrix A, denoted as $\det(A)$, is a scalar value that provides important information about the matrix, such as:

- Whether the matrix is invertible (determinant is non-zero).
- The scaling factor of volume when the matrix is viewed as a linear transformation.
- The orientation-preserving or reversing nature of the transformation.

Scalar Multiplication in Matrices

When we multiply a matrix A by a scalar r, each entry of the matrix is multiplied by r. This operation scales the entire matrix uniformly.

The Key Formula: How Does the Determinant Scale?

The central question is: What is the relationship between $\det(rA)$ and $\det(A)$?

The well-established formula in linear algebra states that:

$$
\boxed{
\det(rA) = r^n \det(A)
}
$$

\]

where:

- \(r \) is a scalar,
- \(A \) is an \(n \times n \) matrix,
- \(n \) is the size of the matrix (number of rows or columns).

Explanation of the Formula

This formula indicates that when a matrix \(A \) is scaled by a scalar \(r \), its determinant is scaled by \(r^n \).

Why?

- Each row (or column) of the matrix gets multiplied by \(r \), and because the determinant is multilinear and alternating, scaling one row multiplies the determinant by \(r \).
- Since the matrix has \(n \) rows, scaling all rows simultaneously results in multiplying the determinant by \(r \) each time, leading to \(r^n \).

Step-by-Step Derivation

1. Start with matrix \(A \):

\[
A =
\begin{bmatrix}
a_{11} & a_{12} & \dots & a_{1n} \\
a_{21} & a_{22} & \dots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \dots & a_{nn}

\end{bmatrix}

\]

2. Form the scaled matrix \(rA \):

\[

rA =

\begin{bmatrix}

r a_{11} & r a_{12} & \dots & r a_{1n} \\

r a_{21} & r a_{22} & \dots & r a_{2n} \\

\vdots & \vdots & \ddots & \vdots \\

r a_{n1} & r a_{n2} & \dots & r a_{nn}

\end{bmatrix}

\]

3. Determinant scaling property:

- When multiplying a single row by \(r \), the determinant of the matrix gets multiplied by \(r \).
- Multiplying all \(n \) rows by \(r \), the determinant becomes multiplied by \(r^n \).

4. Final formula:

\[

\det(rA) = r^n \det(A)

\]

Correct Answer and Validation

Given the above explanation and derivation, the correct formula for the determinant of \(rA \) when \(

A \) is an \(n \times n \) matrix is:

\[
\boxed{
\det(rA) = r^n \det(A)
}
\]

This confirms that the statement "A. Det(rA) = R^n" is correct, assuming the context implies that the determinant scales by \(r^n \).

However, note that the full, precise statement should include the original determinant:

\[
\det(rA) = r^n \det(A)
\]

Applications and Examples

Example 1: 2x2 Matrix

Let \(A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \), and \(r = 5 \).

Calculate \(\det(rA) \):

- First, compute \(\det(A) \):

\[
\det(A) = (2)(4) - (3)(1) = 8 - 3 = 5

\]

- Then, apply the formula:

\[
\det(5A) = 5^2 \times 5 = 25 \times 5 = 125
\]

- Alternatively, directly compute \(5A \):

\[
5A = \begin{bmatrix} 10 & 15 \\ 5 & 20 \end{bmatrix}
\]

and check the determinant:

\[
\det(5A) = (10)(20) - (15)(5) = 200 - 75 = 125
\]

Both approaches agree, confirming the formula.

Example 2: 3x3 Matrix

Let \(A \) be:

\[
A = \begin{bmatrix}
1 & 2 & 3 \\
0 & 4 & 5 \\
1 & 0 & 6

\end{bmatrix}

\]

Calculate \(\det(A) \):

\[

\det(A) = 1 \times (4 \times 6 - 5 \times 0) - 2 \times (0 \times 6 - 5 \times 1) + 3 \times (0 \times 0 - 4 \times 1)

\]

\[

= 1 \times (24 - 0) - 2 \times (0 - 5) + 3 \times (0 - 4)

\]

\[

= 24 - 2 \times (-5) + 3 \times (-4) = 24 + 10 - 12 = 22

\]

Now, for \(r = 3 \):

\[

\det(3A) = 3^3 \times 22 = 27 \times 22 = 594

\]

Alternatively, directly compute \(3A \):

\[

3A = \begin{bmatrix}

3 & 6 & 9 \\

0 & 12 & 15 \\

3 & 0 & 18

\end{bmatrix}

\]

and verify the determinant:

$$
\det(3A) = 3 \times (12 \times 18 - 15 \times 0) - 6 \times (0 \times 18 - 15 \times 3) + 9 \times (0 \times 0 - 12 \times 3)
$$

$$
= 3 \times (216 - 0) - 6 \times (0 - 45) + 9 \times (0 - 36)
$$

$$
= 3 \times 216 - 6 \times (-45) + 9 \times (-36) = 648 + 270 - 324 = 594
$$

Again, the calculation confirms the formula.

Common Misconceptions and Clarifications

Misconception 1: The determinant scales by r regardless of size

Correction: The determinant scales by r^n, not just r, where n is the order of the square matrix.

Misconception 2: The formula applies only to diagonal matrices

Correction: The formula applies to all square matrices, not just diagonal ones, because the scaling property is a general attribute of determinants.

Misconception 3: The determinant of rA equals $r \times \det(A)$

Correction: This is incorrect. The determinant scales by r^n, not just r.

Summary and Key Takeaways

- The determinant of an $n \times n$ matrix A when scaled by a scalar r is given by:

$$
\boxed{
\det(rA) = r^n \det(A)
}
$$

- This property arises because scaling each row (or column)

Frequently Asked Questions

What is the formula for the determinant of a scalar multiple of an N×N matrix A?

Det(rA) = r^n Det(A), where r is a scalar and A is an N×N matrix.

How does scaling an N×N matrix A by a scalar r affect its determinant?

Scaling A by r multiplies its determinant by r^n, so Det(rA) = r^n Det(A).

If A is an N×N matrix and r is a scalar, what is the determinant of r times A?

It is r raised to the power of n times the determinant of A, i.e., Det(rA) = r^n Det(A).

Which of the following is the correct formula for Det(rA) when A is an N×N matrix?

A. Det(rA) = r^n Det(A).

Is the formula Det(rA) = r^n Det(A) valid for any scalar r and matrix A?

Yes, the formula holds for any scalar r and N×N matrix A.

What is the importance of the exponent n in the formula Det(rA) = r^n Det(A)?

The exponent n corresponds to the size of the matrix, indicating how the scalar r influences the determinant.

When A is a 3×3 matrix, what is Det(2A) in terms of Det(A)?

Det(2A) = 2^3 Det(A) = 8 Det(A).

Additional Resources

Determinant: Unlocking the Key to Matrix Scaling and Transformations

When delving into the world of linear algebra, the determinant often emerges as a fundamental concept, serving as a measure of a matrix's invertibility, volume scaling, and transformation properties. For students, researchers, and professionals alike, understanding how the determinant behaves under various operations is crucial. Among these operations, scaling a matrix by a scalar r is one of the most fundamental, especially when analyzing how such a change impacts the determinant.

In this article, we explore the question: "Find a formula for $\det(rA)$ when A is an $n \times n$ matrix." We will dissect the mathematical underpinnings, clarify the key concepts, and provide a comprehensive understanding of the relationship between scalar multiplication and determinants.

Understanding the Determinant of an $n \times n$ Matrix

Before diving into the effects of scalar multiplication, it’s essential to revisit what the determinant signifies for an $n \times n$ matrix.

What is the Determinant?

The determinant of a square matrix A, denoted as $\det(A)$, is a scalar value that encapsulates several important properties:

- Invertibility: A is invertible if and only if $\det(A) \neq 0$.
- Volume Scaling: When A is viewed as a linear transformation, the absolute value of $\det(A)$ indicates how much the transformation scales volume in $\mathbb{R}^n$.

- Orientation: The sign of $\det(A)$ indicates whether the transformation preserves or reverses orientation.

Mathematically, the determinant can be computed via various methods, including cofactor expansion, LU decomposition, or leveraging properties of matrices.

Scalar Multiplication of Matrices and Its Effect on the Determinant

One of the most straightforward yet profound operations involving matrices is scalar multiplication: multiplying every element of A by a scalar r, resulting in rA.

Formulating $\det(rA)$

The core property that guides us is:

$$
\det(rA) = r^n \det(A)
$$

where:

- r is a scalar,
- A is an $n \times n$ matrix,
- n is the dimension of the matrix (number of rows and columns).

Why does this hold? Let's explore the reasoning behind this formula.

Deriving the Formula: $\det(rA) = r^n \det(A)$

The derivation hinges on understanding how scalar multiplication impacts volume scaling, as well as properties of determinants under row operations and matrix decompositions.

Intuitive Explanation

- When you multiply a matrix A by a scalar r, each row (or column) of A is scaled by r.
- The determinant, being related to volume, scales accordingly.
- Since an $n \times n$ matrix has n rows and columns, scaling each row by r multiplies the volume (and thus the determinant) by r for each row.

Thus, the combined effect on the determinant when all rows are scaled is:

$$
\det(rA) = r \times r \times \ldots \times r \times \det(A) = r^n \det(A)
$$

This intuitive approach aligns with the formal properties of determinants.

Formal Proof Outline

1. Start with the matrix A:

$$
A = [a_1, a_2, ..., a_n]
$$

where each a_i is a column vector.

2. Consider rA:

$$
rA = [r a_1, r a_2, ..., r a_n]
$$

Each column vector is scaled by r.

3. Determinant and Column Operations:

The determinant is multilinear with respect to columns, which implies:

$$
\det(rA) = \det([r a_1, r a_2, ..., r a_n]) = r^n \det([a_1, a_2, ..., a_n]) = r^n \det(A)
$$

4. Conclusion:

The property holds for any scalar r and any $n \times n$ matrix A.

Implications and Applications

Understanding this formula is foundational in various areas of linear algebra, including:

- Transformation Analysis: When analyzing how scaling affects geometric transformations, the determinant provides a direct measure of volume change.
- Eigenvalue Scaling: The relationship between eigenvalues of A and rA is straightforward, with eigenvalues scaled by r, and the determinant being the product of eigenvalues.
- Matrix Inverses and Conditioning: Recognizing how scalar multiplication impacts determinants informs numerical stability considerations.
- Determinant Calculations: Simplifies computations when matrices are scaled, especially in problem-solving and proofs.

Choosing the Correct Answer

Given the options, the correct formula for the determinant of a scaled matrix rA when A is an $n \times n$ matrix is:

A. $\det(rA) = r^n \det(A)$

This formula captures the essence of how scalar multiplication influences the determinant and is supported both intuitively and rigorously by linear algebra principles.

Summary and Final Thoughts

The determinant’s behavior under scalar multiplication is a cornerstone concept that reveals the deep interplay between algebraic operations and geometric interpretations. Recognizing that multiplying an $n \times n$ matrix A by a scalar r results in the determinant being scaled by r^n provides a powerful tool in both theoretical analysis and practical computations.

In essence, the property:

$$
\boxed{
\det(rA) = r^n \det(A)
}
$$

serves as a fundamental rule in linear algebra, bridging the algebraic operation of scalar multiplication with the geometric intuition of volume scaling. Whether you're simplifying complex matrix expressions or analyzing transformations, this formula is an indispensable part of your mathematical toolkit.

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