Find An Equation Of The Tangent Line To The Given Curve At The Specified Point. Y = X 2 1 X 2 X 1 , (

Find An Equation Of The Tangent Line To The Given Curve At The Specified Point. Y = X^2 + 2X + 1

Understanding how to find the equation of a tangent line to a curve at a specific point is a fundamental concept in calculus. This process involves determining the slope of the curve at that point and then using point-slope form to write the tangent line's equation. In this comprehensive guide, we will explore the step-by-step method to find the tangent line for the given curve $y = x^2 + 2x + 1$, including detailed explanations, examples, and tips to master this essential skill.

Introduction to Tangent Lines and Their Significance

The tangent line to a curve at a specific point is a straight line that "just touches" the curve at that point without crossing it. It provides the best linear approximation to the curve near that point and is crucial in various applications such as physics, engineering, and computer graphics.

Key Concepts:

- Tangent Line: A line that touches the curve at a point and has the same slope as the curve at that point.
- Point of Tangency: The specific point on the curve where the tangent line is drawn.
- Slope of the Tangent Line: The derivative of the function at the point of interest.

Understanding the Given Curve

The curve in question is:

$$
y = x^2 + 2x + 1
$$

This is a quadratic function, which graphs as a parabola. The goal is to find the equation of the tangent line at a given point (x_0, y_0) on this curve.

Note: To find the tangent line at a specific point, you need to know either the x-coordinate of the point or the point itself.

Step-by-Step Method to Find the Tangent Line

The process involves several steps:

Step 1: Determine the Point of Tangency

Identify the point (x_0, y_0) where you want to find the tangent line. This could be given explicitly or derived from a specific x-value.

Example:

Suppose the point of tangency is at $x = 1$.

Calculate y at $x = 1$:

$$
y = (1)^2 + 2(1) + 1 = 1 + 2 + 1 = 4
$$

So, the point is $(1, 4)$.

Step 2: Find the Derivative of the Function

The derivative $y' = \frac{dy}{dx}$ gives the slope of the tangent line at any point x.

For $y = x^2 + 2x + 1$:

$$
y' = \frac{d}{dx}(x^2 + 2x + 1) = 2x + 2
$$

This derivative function gives the slope at any x.

Step 3: Calculate the Slope at the Point of Tangency

Plug in $x_0 = 1$ into the derivative:

$$

$$m = y'|_{x=1} = 2(1) + 2 = 2 + 2 = 4$$

The slope of the tangent line at $(1, 4)$ is 4.

Step 4: Write the Equation of the Tangent Line

Using the point-slope form:

$$y - y_0 = m (x - x_0)$$

Substitute $(x_0, y_0) = (1, 4)$ and $m = 4$:

$$y - 4 = 4 (x - 1)$$

Simplify:

$$y - 4 = 4x - 4$$

$$y = 4x$$

Result:

The equation of the tangent line at the point \((1, 4)\) is:

\[
\boxed{y = 4x}
\]

General Formula for the Tangent Line to a Curve

When given a function \(y = f(x) \) and a point \((x_0, y_0) \), the general process to find the tangent line is:

\[
\text{Tangent line:} \quad y = f'(x_0)(x - x_0) + y_0
\]

Where:

- \(f'(x_0) \) is the derivative evaluated at \(x_0 \),
- \(y_0 = f(x_0) \).

This formula simplifies the process for any function and point.

Additional Examples and Practice Problems

Example 1: Find the tangent line to $y = x^2 + 2x + 1$ at $x = -1$

Solution:

1. Find y at $x = -1$:

$$
y = (-1)^2 + 2(-1) + 1 = 1 - 2 + 1 = 0
$$

Point: $(-1, 0)$.

2. Find the derivative:

$$
y' = 2x + 2
$$

3. Calculate the slope at $x = -1$:

$$
m = 2(-1) + 2 = -2 + 2 = 0
$$

4. Write the tangent line:

$$
y - 0 = 0 (x + 1) \Rightarrow y = 0
$$

Answer:

The tangent line is $y = 0$.

Practice Problem:

Find the tangent line to $y = x^2 + 2x + 1$ at $x = 0$.

Tips and Common Mistakes to Avoid

- Always verify the point of tangency before calculating the slope.
- Double-check the derivative calculation, especially for more complex functions.
- Remember that the derivative gives the slope, but the tangent line passes through the specific point.
- Simplify equations carefully to avoid algebraic errors.

Applications of Finding Tangent Lines

Understanding how to find tangent lines is crucial in many fields:

- Physics: To determine instantaneous velocity at a specific point in time.
- Economics: To find marginal cost or revenue.
- Engineering: For designing curves and analyzing stress points.
- Computer Graphics: To compute shading and surface normals.

Conclusion

Finding the equation of a tangent line to a curve at a given point involves understanding derivatives, calculating the slope at that point, and applying the point-slope form of a linear equation. For the function $y = x^2 + 2x + 1$, the process is straightforward:

1. Identify the point of tangency.
2. Compute the derivative to find the slope.
3. Plug in the x-value to get the slope.
4. Use the point-slope form to derive the tangent line equation.

Mastering this process enhances your calculus skills and enables you to analyze curves effectively in various real-world scenarios.

Keywords: tangent line, derivative, calculus, curve, point of tangency, slope, equation of tangent, quadratic function, point-slope form, calculus tutorial

Frequently Asked Questions

How do I find the equation of the tangent line to the curve $y = x^2 + x$ at a given point?

First, compute the derivative $y' = 2x + 1$. Then, evaluate y' at the given point's x-coordinate to find the slope. Use the point-slope form with this slope and the point to find the tangent line equation.

What is the first step in finding the tangent line to $y = x^2 + x$ at point $(a, y(a))$?

Calculate the derivative $y' = 2x + 1$ and evaluate it at $x = a$ to determine the slope of the tangent line at that point.

If the point on the curve is (2, 6), how do I find the tangent line to $y = x^2 + x$ at that point?

Evaluate the derivative at $x = 2$: $y' = 2(2) + 1 = 5$. Use point-slope form: $y - 6 = 5(x - 2)$, which simplifies to $y = 5x - 4$.

Can I use the point-slope form to write the tangent line once I have the slope and point?

Yes, the point-slope form $y - y_1 = m(x - x_1)$ is ideal once you have the slope m and the point (x_1, y_1) on the curve.

What if the derivative at a point is zero? How does that affect the tangent line?

If the derivative at the point is zero, the tangent line is horizontal, and its equation is $y = y_1$, where y_1 is the y-coordinate of the point.

Is it necessary to simplify the tangent line equation after finding it?

Yes, simplifying the equation makes it easier to interpret and compare, but the key is to accurately find the slope and point first.

How do I verify that my tangent line equation is correct?

Check that the tangent line passes through the given point and that its slope matches the derivative value at that point.

Can this method be applied to any curve to find the tangent line at a specific point?

Yes, as long as the derivative exists at that point, you can find the tangent line by calculating the derivative and applying the point-slope form.

Additional Resources

Find An Equation Of The Tangent Line To The Given Curve At The Specified Point – an essential concept in calculus that combines the art of differentiation with geometric intuition. Whether you're a student grappling with derivatives or a professional seeking to understand the behavior of a curve at a specific point, mastering how to find the tangent line is fundamental. In this comprehensive guide, we'll walk through the step-by-step process of deriving the tangent line to the curve defined by the function $y = x^2 \ln x^2$ at a given point, illustrating key concepts and providing clear examples along the way.

Understanding the Problem: Find The Equation Of The Tangent Line To The Given Curve At The Specified Point

When asked to find the equation of the tangent line to a curve at a particular point, the goal is to determine a straight line that just "touches" the curve at that point and shares the same slope as the curve does there. The process involves two main steps:

1. Calculating the derivative of the function to find the slope of the tangent line at the specified point.
2. Using the point-slope form of a line to write the equation of the tangent line.

In this guide, we'll focus on the function:

$$y = x^2 \ln x^2$$

and find the tangent line at a specific point, say $x = a$.

Step 1: Clarify the Function and the Point of Tangency

Before diving into derivatives, it's crucial to understand the function's structure and the point at which we're finding the tangent line.

The Function: $y = x^2 \ln x^2$

- The function involves a composite expression, combining a polynomial x^2 and a natural logarithm $\ln x^2$.
- The domain of the function is $x > 0$ because $\ln x^2$ is defined for positive x.

The Point of Tangency: $x = a$

- To find the tangent line at $x = a$, we need both the point $(a, y(a))$ and the slope $y'(a)$.

Step 2: Computing the Derivative y'

The derivative y' gives the slope of the tangent line at any point x. Since our function is a product of two functions, $u(x) = x^2$ and $v(x) = \ln x^2$, we'll use product rule for differentiation:

$$
\frac{d}{dx}[u(x) \cdot v(x)] = u'(x) v(x) + u(x) v'(x)
$$

Derivatives of the Components:

- $u(x) = x^2 \Rightarrow u'(x) = 2x$
- $v(x) = \ln x^2$

Note that:

$$
v(x) = \ln x^2 = 2 \ln x
$$

since $\ln x^2 = 2 \ln x$, for $x > 0$.

Thus,

$$
v(x) = 2 \ln x
$$

and

$$
v'(x) = 2 \cdot \frac{1}{x} = \frac{2}{x}
$$

Applying the Product Rule:

$$
y'(x) = u'(x) v(x) + u(x) v'(x) = 2x \cdot 2 \ln x + x^2 \cdot \frac{2}{x}
$$

Simplify:

$$
y'(x) = 4x \ln x + 2x
$$

Step 3: Find the Slope at the Given Point $x = a$

To find the slope of the tangent line at $x = a$:

$$
m = y'(a) = 4a \ln a + 2a
$$

This expression gives the slope of the tangent line at the point $(a, y(a))$.

Step 4: Calculate the Point $(a, y(a))$

Now, find the y-coordinate corresponding to $x = a$:

$$
y(a) = a^2 \ln a^2
$$

Recall $\ln a^2 = 2 \ln a$, so:

\[

$$
y(a) = a^2 \cdot 2 \ln a = 2a^2 \ln a
$$

Therefore, the point of tangency is:

$$
\boxed{(a, \quad 2a^2 \ln a)}
$$

Step 5: Write the Equation of the Tangent Line

Using the point-slope form:

$$
y - y(a) = m (x - a)
$$

Substitute the known values:

$$
y - 2a^2 \ln a = (4a \ln a + 2a)(x - a)
$$

This is the equation of the tangent line at $x = a$.

Putting It All Together: Sample Calculation at $a = 1$

Let's choose a specific point $a = 1$ for clarity.

Step 1: Calculate $y(1)$:

$$
y(1) = 2 \times 1^2 \times \ln 1 = 2 \times 1 \times 0 = 0
$$

Step 2: Calculate the slope m:

$$
m = 4 \times 1 \times \ln 1 + 2 \times 1 = 4 \times 0 + 2 = 2
$$

Step 3: Write the equation:

$$
y - 0 = 2 (x - 1)
$$

or

$$
\boxed{ y = 2(x - 1) }
$$

This is the tangent line to the curve $y = x^2 \ln x^2$ at $x = 1$.

Additional Tips and Considerations

1. Domain Restrictions:

Always verify that the point of tangency lies within the domain of the original function. For $y = x^2 \ln x^2$, the domain is $x > 0$.

2. Simplify Logarithmic Expressions:

Using properties of logarithms can simplify derivatives and calculations, such as recognizing $\ln x^2 = 2 \ln x$.

3. Check Your Derivative:

Double-check derivatives, especially when applying product and chain rules, to avoid common mistakes.

4. Interpretation of the Tangent Line:

The tangent line provides the best linear approximation to the curve at the point $x = a$. Its slope indicates the instantaneous rate of change.

Summary of Steps to Find the Equation of the Tangent Line

1. Differentiate the function to find y'.
2. Evaluate y' at $x = a$ to find the slope m.
3. Calculate $y(a)$ to find the point of tangency.
4. Use the point-slope form to write the tangent line's equation.

Final Remarks

Finding the equation of the tangent line to a curve at a given point is a foundational skill in calculus that illustrates the connection between derivatives and the geometric behavior of functions. By carefully differentiating, evaluating, and applying the point-slope form, you can analyze the local linear approximation of complex functions like $y = x^2 \ln x^2$. Practice with different points and functions enhances understanding, allowing you to handle a wide range of problems involving tangents, slopes, and instantaneous rates of change.

Mastering this process not only helps in solving calculus problems but also deepens your understanding of the dynamic relationship between algebraic expressions, their derivatives, and their geometric interpretations.

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