Find An Equation Of The Tangent Plane To The Surface Z=4y22x2z=4y22x2 At The Point (4, -2, -16).z=___

Find An Equation Of The Tangent Plane To The Surface Z=4y^2 - 2x^2 At The Point (4, -2, -16).

Understanding how to find the equation of a tangent plane to a surface at a specific point is a fundamental concept in multivariable calculus. This process involves calculating the gradient of the surface's defining function and then using it to formulate the tangent plane equation. In this article, we will walk through each step in detail, illustrating how to derive the tangent plane equation for the surface $Z = 4y^2 - 2x^2$ at the point $(4, -2, -16)$. This guide aims to provide clarity and depth, making it accessible for students and enthusiasts alike.

Understanding the Surface and Its Equation

Surface Equation Overview

The surface in question is defined implicitly by the function:

$$
Z = 4y^2 - 2x^2
$$

This is a quadratic surface involving both x and y, defining a three-dimensional shape in space. The goal is to find the equation of the tangent plane at a specific point on this surface.

Point of Interest

The point provided is:

$$
(x_0, y_0, z_0) = (4, -2, -16)
$$

Before proceeding, it's prudent to verify that this point indeed lies on the surface. Substituting $x = 4$ and $y = -2$ into the surface equation:

$$
Z = 4(-2)^2 - 2(4)^2 = 4(4) - 2(16) = 16 - 32 = -16
$$

The calculated Z matches the given $z_0 = -16$. Therefore, the point lies on the surface, confirming the appropriateness of the point for tangent plane calculation.

Mathematical Foundations for Finding the Tangent Plane

Implicit Function and Gradient Vector

In multivariable calculus, the tangent plane to a surface at a point can be found if the surface is expressed as an implicit function:

$$
F(x, y, z) = 0
$$

Rearranged from the given surface equation:

$$
F(x, y, z) = 4y^2 - 2x^2 - z = 0
$$

The gradient vector of F, denoted as ∇F, points in the direction of the greatest rate of increase of F, and it is perpendicular to the surface at the point.

Calculating the gradient:

$$
\nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right)
$$

This vector will serve as the normal vector to the tangent plane.

Formula for the Tangent Plane

The general equation of the tangent plane at a point (x_0, y_0, z_0) is:

$$
\nabla F (x_0, y_0, z_0) \cdot \left((x - x_0), (y - y_0), (z - z_0) \right) = 0
$$

or explicitly,

$$
\left. \frac{\partial F}{\partial x} \right|_{(x_0, y_0, z_0)} (x - x_0) + \left. \frac{\partial F}{\partial y} \right|_{(x_0, y_0, z_0)} (y - y_0) + \left. \frac{\partial F}{\partial z} \right|_{(x_0, y_0, z_0)} (z - z_0) = 0
$$

This formula provides a straightforward method to derive the tangent plane once the gradient is known.

Calculating Partial Derivatives of the Surface Function

Partial Derivatives of $F(x, y, z)$

Given:

$$
F(x, y, z) = 4y^2 - 2x^2 - z
$$

Calculate each partial derivative:

- With respect to x:

$$
\frac{\partial F}{\partial x} = -4x
$$

- With respect to y:

$$
\frac{\partial F}{\partial y} = 8y
$$

- With respect to z:

$$
\frac{\partial F}{\partial z} = -1
$$

Evaluate these derivatives at the point $(4, -2, -16)$:

- $\frac{\partial F}{\partial x} \big|_{(4, -2, -16)} = -4(4) = -16$

- $\frac{\partial F}{\partial y} \big|_{(4, -2, -16)} = 8(-2) = -16$

- $\frac{\partial F}{\partial z} \big|_{(4, -2, -16)} = -1$

The gradient vector at the point is therefore:

$$
\nabla F (4, -2, -16) = (-16, -16, -1)
$$

Formulating the Equation of the Tangent Plane

Applying the Gradient to the Point

Using the formula:

$$
-16 (x - 4) + (-16)(y + 2) + (-1)(z + 16) = 0
$$

Note: Since $y_0 = -2$, $y - y_0 = y + 2$; similarly for x, $x - x_0 = x - 4$; and for z, $z - z_0 = z + 16$.

Simplifying the Equation

Distribute:

$$
-16x + 64 - 16y - 32 - z - 16 = 0
$$

Combine like terms:

$$
-16x - 16y - z + (64 - 32 - 16) = 0
$$

$$
-16x - 16y - z + 16 = 0
$$

Rearranged:

$$
-16x - 16y - z = -16
$$

To make the equation more standard, multiply through by -1:

$$
16x + 16y + z = 16
$$

This is the equation of the tangent plane to the surface at the point $(4, -2, -16)$.

Final Equation of the Tangent Plane

\[
\boxed{
16x + 16y + z = 16

}
\]

This linear equation describes the tangent plane at the specified point. It provides a precise geometric representation of how the surface behaves in the immediate vicinity of $(4, -2, -16)$.

Conclusion and Additional Insights

Understanding how to find the tangent plane to a surface is essential for analyzing local behavior in multivariable calculus. The process involves:

1. Confirming the point lies on the surface.
2. Expressing the surface as an implicit function.
3. Calculating the gradient vector of the function at the point.
4. Using the gradient vector as the normal vector to the tangent plane.
5. Formulating the tangent plane equation using the point-normal form.

This method is broadly applicable to various surfaces, including spheres, cylinders, and more complex shapes. Mastery of these techniques enhances spatial reasoning and enables deeper insights into the geometry of multivariable functions.

Whether you're studying for exams, working on engineering projects, or simply exploring the fascinating world of calculus, understanding how to derive tangent planes equips you with a powerful tool to analyze and visualize three-dimensional surfaces effectively.

If you'd like further assistance with related topics such as normal vectors, derivatives of multivariable functions, or applications in physics and engineering, numerous resources and tutorials are available online to deepen your understanding.

Frequently Asked Questions

How do you find the equation of the tangent plane to the surface z = 4y^2 - 2x^2 at a given point?

To find the tangent plane, first compute the gradient vector of the surface at the point, then use the point-normal form of the plane equation: $n \cdot (r - r_0) = 0$.

What is the gradient vector of the surface z = 4y^2 - 2x^2?

The gradient vector is $\nabla z = (\partial z/\partial x, \partial z/\partial y, -1) = (-4x, 8y, -1)$.

How do you evaluate the gradient at the point (4, -2, -16)?

Substitute x=4 and y=-2 into the gradient: $\nabla z = (-44, 8(-2), -1) = (-16, -16, -1)$.

What is the equation of the tangent plane at the point (4, -2, -16)?

Using the normal vector (-16, -16, -1) and point (4, -2, -16), the tangent plane equation is -16(x - 4) -16(y + 2) -1(z + 16) = 0.

How do you simplify the tangent plane equation to standard form?

Expand and simplify: -16x + 64 -16y - 32 - z - 16 = 0, which simplifies to -16x -16y - z + 16 = 0.

What is the final equation of the tangent plane to the surface at the given point?

The equation is -16x -16y - z + 16 = 0.

Why is it important to verify the point lies on the tangent plane?

Verifying ensures the point satisfies the tangent plane equation, confirming correctness of the calculation.

Can the tangent plane be used to approximate the surface near the point? How?

Yes, the tangent plane provides a linear approximation of the surface near the point, useful for estimating values close to (4, -2, -16).

Additional Resources

Find An Equation Of The Tangent Plane To The Surface $z=4y^2-2x^2$ At The Point (4, -2, -16)

Understanding how to find an equation of the tangent plane to a surface at a given point is a fundamental skill in multivariable calculus. This process involves applying concepts of partial derivatives, gradients, and the equation of planes to approximate the surface locally. Whether you're analyzing a physical phenomenon, solving a math problem, or preparing for exams, mastering this technique is essential. This guide provides a comprehensive, step-by-step approach to finding the tangent plane to the surface defined by \(z = 4y^2 - 2x^2 \) at the specific point \((4, -2, -16) \).

Introduction to the Problem

Suppose you're given a surface described by a function \(z = f(x, y) \). The goal is to find the equation of the tangent plane at a particular point \((x_0, y_0, z_0) \) on this surface. The specific problem is:

> Find the equation of the tangent plane to the surface $z = 4y^2 - 2x^2$ at the point $(4, -2, -16)$.

The process can be summarized in a few key steps:

1. Verify that the point lies on the surface.
2. Compute the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point.
3. Use the gradient vector to determine the normal to the tangent plane.
4. Write the equation of the tangent plane using the point-normal form.

Let's delve into each step with detailed explanations.

Step 1: Verify the Point Lies on the Surface

Before proceeding, confirm that the point $(4, -2, -16)$ is indeed on the surface $z = 4y^2 - 2x^2$.

Calculation:

Plug $x=4$ and $y=-2$ into the surface equation:

$$
z = 4(-2)^2 - 2(4)^2 = 4(4) - 2(16) = 16 - 32 = -16
$$

Since the computed z value is -16, which matches the point's z-coordinate, the point lies on the surface.

Step 2: Compute Partial Derivatives

The tangent plane at a point depends on the gradient of the surface function. For the surface $z = f(x, y)$, the partial derivatives are:

$$
f_x(x, y) = \frac{\partial z}{\partial x}
$$
$$
f_y(x, y) = \frac{\partial z}{\partial y}
$$

Calculating $f_x(x, y)$:

Given

$$
z = 4y^2 - 2x^2

\]

Differentiate with respect to \(x \):

\[
f_x(x, y) = \frac{\partial}{\partial x} (4y^2 - 2x^2) = 0 - 4x = -4x
\]

Calculating \(f_y(x, y) \):

Differentiate with respect to \(y \):

\[
f_y(x, y) = \frac{\partial}{\partial y} (4y^2 - 2x^2) = 8y - 0 = 8y
\]

Evaluate at \((x_0, y_0) = (4, -2) \):

\[
f_x(4, -2) = -4 \times 4 = -16
\]
\[
f_y(4, -2) = 8 \times (-2) = -16
\]

Step 3: Find the Normal Vector to the Tangent Plane

The normal vector to the tangent plane at the point can be constructed from the gradient of the surface function. For the surface \(z = f(x, y) \), the gradient vector is:

\[
\nabla F(x, y, z) = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right)
\]

However, since the surface is given as \(z = f(x, y) \), it's often helpful to define a function:

\[
F(x, y, z) = z - f(x, y) = 0
\]

Then,

\[
\nabla F(x, y, z) = \left(-f_x(x, y), -f_y(x, y), 1 \right)
\]

Computing the normal vector:

At \((4, -2, -16) \):

\[
\vec{n} = \left(-(-16), -(-16), 1 \right) = (16, 16, 1)
\]

This vector is normal to the surface at the given point.

Step 4: Write the Equation of the Tangent Plane

The equation of a plane given a point \((x_0, y_0, z_0) \) and a normal vector \(\vec{n} = (A, B, C) \) is:

\[
A(x - x_0) + B(y - y_0) + C(z - z_0) = 0
\]

Plugging in the values:

- \((x_0, y_0, z_0) = (4, -2, -16) \)
- \(\vec{n} = (16, 16, 1) \)

We get:

\[
16(x - 4) + 16(y + 2) + 1(z + 16) = 0
\]

Simplify:

\[
16x - 64 + 16y + 32 + z + 16 = 0
\]

Combine like terms:

\[
16x + 16y + z (- 64 + 32 + 16) = 0
\]

Calculating the constants:

\[
-64 + 32 + 16 = -16
\]

So, the equation of the tangent plane becomes:

\[

$$
16x + 16y + z - 16 = 0
$$

Or, in a more standard form:

$$
16x + 16y + z = 16
$$

Final Answer

The equation of the tangent plane to the surface $z = 4y^2 - 2x^2$ at the point $(4, -2, -16)$ is:

$$\boxed{16x + 16y + z = 16}$$

Additional Tips and Insights

Why is the normal vector important?

The normal vector $\vec{n}$ encapsulates the orientation of the tangent plane. It is perpendicular to the plane, and knowing it allows you to write the plane's equation directly.

Alternative forms:

You might see the tangent plane written in different forms, such as solving for z:

$$
z = -16x -16y + 16
$$

which is just the original plane equation rearranged.

Visualizing the tangent plane:

Imagine the surface as a smooth hill or valley. The tangent plane at a point touches the surface exactly at that point and best approximates the surface near that point. Visualizing this can enhance understanding of how derivatives and gradients relate to the geometry of surfaces.

Conclusion

Finding the equation of the tangent plane to a surface involves verifying the point lies on the surface, computing the relevant derivatives, and constructing the plane's equation from the normal vector. This detailed approach can be applied broadly to various surfaces and points, providing a powerful tool for analyzing multivariable functions and their geometric properties. With practice, these steps become intuitive, enabling you to navigate complex surfaces with confidence and precision.

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