# Find Both A Basis For The Row Space And A Basis For The Column Space Of The Given Matrix A. 5 OT 4 -3

**Find Both A Basis For The Row Space And A Basis For The Column Space Of The Given Matrix A. 5 OT 4 -3**

When working with matrices in linear algebra, understanding the concepts of row space and column space is fundamental. These spaces provide insight into the solutions of systems of linear equations, the rank of a matrix, and various applications across engineering, computer science, and data analysis. In this comprehensive guide, we will explore how to determine bases for both the row space and the column space of a specific matrix A, which is given as:

$$
A = \begin{bmatrix}
5 & 0 & 4 \\
-3 & 0 & 0
\end{bmatrix}
$$

By the end of this article, you will understand the step-by-step process of finding these bases, the significance of these subspaces, and their applications. Let’s dive into the details!

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## Understanding Row Space and Column Space in Matrices

Before we proceed with calculations, it’s essential to grasp what row space and column space mean in the context of a matrix.

### What is the Row Space?

- The row space of a matrix A consists of all linear combinations of its row vectors.
- It is a subspace of $\mathbb{R}^n$ (where n is the number of columns).
- The basis for the row space can be found by identifying the set of linearly independent rows.

### What is the Column Space?

- The column space of a matrix A includes all linear combinations of its column vectors.
- It is a subspace of $\mathbb{R}^m$ (where m is the number of rows).
- The basis for the column space can be identified by selecting the linearly independent columns.

# Step-by-Step Procedure to Find Bases for Row and Column Spaces

To find the bases, we typically perform row operations to reduce the matrix to a simpler form, such as row echelon form or reduced row echelon form. This process helps identify the pivot rows and columns, which form the bases.

## 1. Write the matrix A

\[
A = \begin{bmatrix}
5 & 0 & 4 \\
-3 & 0 & 0
\end{bmatrix}
\]

## 2. Perform row operations to simplify A

- The goal is to convert A into an echelon form to identify pivot positions.

Step 2.1: Make the leading coefficient of the first row a 1 (optional for basis, but often helpful).
Divide the first row by 5:

\[
R_1 \leftarrow \frac{1}{5} R_1
\]
\[
\Rightarrow \begin{bmatrix}
1 & 0 & \frac{4}{5} \\
-3 & 0 & 0
\end{bmatrix}
\]

Step 2.2: Eliminate the element below the leading 1 in the first column.
Add 3 times the first row to the second row:

\[
R_2 \leftarrow R_2 + 3 R_1
\]
\[
\Rightarrow R_2 = (-3 + 3 \times 1, 0 + 3 \times 0, 0 + 3 \times \frac{4}{5}) = (0, 0, \frac{12}{5})
\]

The matrix becomes:

\[
\begin{bmatrix}
1 & 0 & \frac{4}{5} \\

0 & 0 & \frac{12}{5}
\end{bmatrix}
\]

Step 2.3: If desired, make the second pivot a 1:

Divide the second row by \(\frac{12}{5}\):

\[
R_2 \leftarrow \frac{5}{12} R_2
\]
\[
\Rightarrow R_2 = (0, 0, 1)
\]

Now, the matrix is:

\[
\begin{bmatrix}
1 & 0 & \frac{4}{5} \\
0 & 0 & 1
\end{bmatrix}
\]

---

# Finding a Basis for the Row Space

Since row operations do not change the row space, the basis for the row space can be taken from the original rows or from the row echelon form, choosing the non-zero rows.

Step 3: Identify the non-zero rows in the echelon form:

\[
\begin{bmatrix}
1 & 0 & \frac{4}{5} \\
0 & 0 & 1
\end{bmatrix}
\]

These rows are linearly independent and span the row space.

Result:
The basis for the row space of A is:

\[
\boxed{
\left\{ \begin{bmatrix} 5 & 0 & 4 \end{bmatrix}, \quad \begin{bmatrix} -3 & 0 & 0 \end{bmatrix}
\right\}

}
\]

Note: Alternatively, the original rows are linearly independent, and since the row operations didn't combine the rows, these original rows can serve as the basis.

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# Finding a Basis for the Column Space

The basis for the column space can be identified by looking at the columns corresponding to the pivot positions in the row echelon form.

Step 4: Locate the pivot columns in the echelon form:

- The first pivot appears in column 1.
- The second pivot appears in column 3.

Step 5: Corresponding columns in the original matrix:

- Column 1: \(\begin{bmatrix} 5 \\ -3 \end{bmatrix}\)
- Column 3: \(\begin{bmatrix} 4 \\ 0 \end{bmatrix}\)

Result:
The basis for the column space of A is:

\[
\boxed{
\left\{ \begin{bmatrix} 5 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ 0 \end{bmatrix} \right\}
}
\]

These vectors are linearly independent and span the column space of A.

---

# Summary of Key Points

- The bases for the row and column spaces are derived from the pivot rows and pivot columns of the row echelon form.
- The row space basis is composed of the linearly independent rows in the echelon form.
- The column space basis comprises the original columns corresponding to the pivot columns.
- These bases are fundamental in understanding the rank, solutions of systems, and transformations involving the matrix.

## Applications of Bases for Row and Column Spaces

Understanding and computing bases for row and column spaces have numerous practical applications:

- Solving Linear Systems: Determining whether a system has solutions and finding particular solutions.
- Matrix Rank: The dimension of the row or column space gives the rank of the matrix, indicating the maximum number of linearly independent rows or columns.
- Data Analysis: In principal component analysis (PCA), the column space relates to the span of feature vectors.
- Engineering and Physics: Analyzing transformations, signals, and systems through their matrix representations.

## Conclusion

Finding bases for the row space and column space of a matrix is a crucial skill in linear algebra, with broad applications in science and engineering. By performing row operations, identifying pivot positions, and selecting linearly independent vectors, you can efficiently determine these bases. The matrix A given as $\begin{bmatrix} 5 & 0 & 4 \\ -3 & 0 & 0 \end{bmatrix}$ exemplifies how these techniques are applied in practice, leading to a deeper understanding of the matrix's structure and properties.

Remember: Mastery of these concepts enhances your ability to analyze complex systems, optimize solutions, and interpret data through the lens of linear algebra.

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Keywords: basis of row space, basis of column space, linear algebra, matrix rank, row operations, pivot columns, matrix analysis, linear independence, matrix transformations

## Frequently Asked Questions

### What is the first step in finding a basis for the row space of matrix A?

The first step is to write the matrix A and then perform row operations to reduce it to its row echelon form or reduced row echelon form.

### How do I find a basis for the column space of matrix A?

Identify the pivot columns in the row echelon form of A; the original columns corresponding to these pivot columns form a basis for the column space.

## Given the matrix A = [5, 0; 4, -3], what is its row echelon form?

The matrix is already in row echelon form: [[5, 0], [4, -3]]. You can perform row operations if needed for simplification.

## What are the basis vectors for the row space of matrix A = [5, 0; 4, -3]?

The basis for the row space is the set of non-zero rows in the row echelon form: {[5, 0], [4, -3]}.

## How do I determine the basis for the column space of matrix A = [5, 0; 4, -3]?

Identify the pivot columns in the row echelon form. Since the first column is a pivot, the first column of the original matrix forms a basis for the column space.

## Are the basis for the row space and column space always the same size?

Yes, both bases have the same number of vectors, which equals the rank of the matrix.

## Can the basis for the row space be different from the basis for the column space?

Yes, they are different sets of vectors, but both sets span their respective spaces. The basis for the row space consists of row vectors, while the basis for the column space consists of original column vectors corresponding to pivot columns.

## What is the rank of matrix A = [5, 0; 4, -3], and how does it relate to the bases?

The rank of A is 2, indicating the dimension of both the row and column spaces. Therefore, both bases will consist of 2 vectors.

# Additional Resources

Finding Both a Basis for the Row Space and a Basis for the Column Space of Matrix A: A Comprehensive Guide

Understanding the fundamental concepts of vector spaces associated with matrices is essential in linear algebra. Among these, the row space and the column space are two pivotal subspaces that reveal much about the matrix's structure and properties. When given a matrix, such as a 5×4 matrix with specific entries, identifying bases for these subspaces provides insights into the rank, linear independence, and the matrix's transformations. This detailed exploration will guide you step-by-step on how to find both bases for the row space and column space of a given matrix, exemplified by the matrix:

\[
A = \begin{bmatrix}
5 & 0 & 4 & -3
\end{bmatrix}
\]

(Note: The notation here suggests a 1×4 matrix, but the original prompt indicates a 5×4 matrix. For clarity, we'll assume the matrix is 5×4, with 5 rows and 4 columns, and interpret accordingly. If the matrix is indeed 5×4 with entries as specified, please clarify. For now, we'll proceed with the general case and a sample 5×4 matrix.)

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# Understanding the Row Space and Column Space

## What Is the Row Space?

The row space of a matrix A, denoted as Row(A), is the subspace of \(\mathbb{R}^n\) (where \(n\) is the number of columns) spanned by the row vectors of A. In simpler terms, it consists of all linear combinations of the rows.

- Key points:
- It is a subspace of \(\mathbb{R}^n\).
- Its dimension equals the rank of A.
- The basis for the row space can be found from the set of linearly independent rows.

## What Is the Column Space?

The column space of A, denoted as Col(A), is the subspace of \(\mathbb{R}^m\) (where \(m\) is the number of rows) spanned by the columns of A.

- Key points:
- It is a subspace of \(\mathbb{R}^m\).
- Its dimension also equals the rank of A.
- The basis for the column space can be derived from the set of linearly independent columns.

# Step-by-Step Process to Find Bases

The process involves several systematic steps:

1. Reduce the matrix to row echelon form or reduced row echelon form (RREF).
2. Identify the pivot rows and pivot columns.
3. Extract basis vectors for row space from the non-zero rows in RREF.
4. Identify the columns in the original matrix corresponding to the pivot columns in RREF to find the basis for the column space.

Let's walk through this process with a concrete example of a 5×4 matrix.

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# Example Matrix A

Suppose we have the following 5×4 matrix:

\[
A = \begin{bmatrix}
5 & 0 & 4 & -3 \\
2 & 1 & 3 & 0 \\
0 & 0 & 0 & 1 \\
4 & 2 & 8 & -6 \\
1 & 0 & 2 & -1 \\
\end{bmatrix}
\]

Note: The specific entries provided allow for a detailed demonstration.

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# Step 1: Row Reduce the Matrix to RREF

Applying Gaussian elimination or Gauss-Jordan elimination:

1. Identify the first pivot:
- The first non-zero element in the first row is 5 in position (1,1).
- Divide Row 1 by 5 to normalize:

\[
R_1 \leftarrow R_1 / 5
\]
\[
A \rightarrow \begin{bmatrix}
1 & 0 & 0.8 & -0.6 \\
2 & 1 & 3 & 0 \\
0 & 0 & 0 & 1 \\
4 & 2 & 8 & -6 \\
1 & 0 & 2 & -1 \\

\end{bmatrix}
\]

2. Eliminate entries below the pivot in column 1:

- \( R_2 \leftarrow R_2 - 2 R_1 \)
- \( R_4 \leftarrow R_4 - 4 R_1 \)
- \( R_5 \leftarrow R_5 - R_1 \)

Calculations:

- \( R_2: [2 - 2 \times 1, 1 - 2 \times 0, 3 - 2 \times 0.8, 0 - 2 \times (-0.6)] = [0, 1, 1.4, 1.2] \)
- \( R_4: [4 - 4 \times 1, 2 - 4 \times 0, 8 - 4 \times 0.8, -6 - 4 \times (-0.6)] = [0, 2, 4.8, -6 + 2.4] = [0, 2, 4.8, -3.6] \)
- \( R_5: [1 - 1 \times 1, 0 - 1 \times 0, 2 - 1 \times 0.8, -1 - 1 \times (-0.6)] = [0, 0, 1.2, -0.4] \)

Updated matrix:

\[
A = \begin{bmatrix}
1 & 0 & 0.8 & -0.6 \\
0 & 1 & 1.4 & 1.2 \\
0 & 0 & 0 & 1 \\
0 & 2 & 4.8 & -3.6 \\
0 & 0 & 1.2 & -0.4 \\
\end{bmatrix}
\]

3. Next, focus on the second pivot:

- The second row has a leading 1 in position (2,2).
- Use it to eliminate entries below:

- \( R_4 \leftarrow R_4 - 2 R_2 \)

Calculations:

- \( R_4: [0, 2 - 2 \times 1, 4.8 - 2 \times 1.4, -3.6 - 2 \times 1.2] = [0, 0, 4.8 - 2.8, -3.6 - 2.4] = [0, 0, 2, -6] \)

Updated matrix:

\[
A = \begin{bmatrix}
1 & 0 & 0.8 & -0.6 \\
0 & 1 & 1.4 & 1.2 \\
0 & 0 & 0 & 1 \\
0 & 0 & 2 & -6 \\
0 & 0 & 1.2 & -0.4 \\
\end{bmatrix}
\]

4. Focus on the third pivot in row 3:

- The third pivot is in position (3,4), but since row 3 has only a 1 in column 4, it's a pivot position.

- Use row 3 to eliminate entries in row 4 and row 5:

- \( R_4 \leftarrow R_4 - 2 R_3 \)

- \( R_5 \leftarrow R_5 - 1.2 R_3 \)

Calculations:

- \( R_4: [0, 0, 2 - 2 \times 0, -6 - 2 \times 1] = [0, 0, 2, -6 - 2] = [0, 0, 2, -8] \)

- \( R_5: [0, 0, 1.2 - 1.2 \times 0, -0.4 - 1.2 \times 1] = [0, 0, 1.2, -0.4 - 1.2] = [0, 0, 1.2, -1.6] \)

Updated matrix:

\[
A = \begin{bmatrix}
1 & 0 & 0.8 & -0.6 \\
0 & 1 & 1.4 & 1.2 \\
0 & 0 & 0 & 1 \\
0 & 0 & 2 & -8 \\
0 & 0 & 1.2 & -1.6 \\
\end{bmatrix}
\]

5. Normalize row 4:

- \( R_4 \leftarrow R_4 / 2 \)

\[
R_4: [0, 0, 1, -4]
\]

6. Eliminate above in column 3:

- \( R_5 \leftarrow R_5 - 1.2 R_4 \)

Calculations:

- \( R_5: [0, 0, 1.2 -

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