

Find $(fg)(4)$, Given The Following Functions: $F(x)=2x+1$ $G(x)=x^2+3x$ A) 57 B) 55 C) 20 D) 45 E) 108 F)

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Introduction: Understanding Composition of Functions

In mathematics, the composition of functions is a fundamental concept that allows us to combine two functions to produce a new function. Specifically, the composition of functions (f) and (g) , denoted as $((f \circ g)(x))$, involves applying the function (g) first and then applying (f) to the result.

The problem at hand asks us to find $((f \circ g)(4))$ where the functions are defined as:

- $(F(x) = 2x + 1)$
- $(G(x) = x^2 + 3x)$

The multiple-choice options provided are:

- A) 57
- B) 55
- C) 20
- D) 45
- E) 108
- F) (Unspecified in the prompt, possibly a typo or to be determined)

Our goal is to evaluate $((f \circ g)(4))$, which involves calculating $(g(4))$ first, then applying (f) to the result.

Step-by-Step Solution: Calculating $((f \circ g)(4))$

Step 1: Evaluate $(g(4))$

Given $(g(x) = x^2 + 3x)$, substitute $(x = 4)$:

$$\begin{aligned} &[\\ g(4) &= (4)^2 + 3 \times 4 \\ &] \\ &[\\ g(4) &= 16 + 12 = 28 \\ &] \end{aligned}$$

So, the output of $(g(4))$ is 28.

Step 2: Apply f to $g(4)$

Now, compute $f(g(4))$, which is $f(28)$:

Given $f(x) = 2x + 1$, substitute $x = 28$:

$$\begin{aligned} f(28) &= 2 \times 28 + 1 \\ f(28) &= 56 + 1 = 57 \end{aligned}$$

Thus, $\boxed{(f \circ g)(4) = 57}$.

Final Answer:

Option A) 57 is the correct choice.

Detailed Breakdown of the Process

Understanding Function Composition

Function composition involves two steps:

1. Inner Function Evaluation: Compute $g(x)$ for a given input.
2. Outer Function Application: Use the result from step 1 as the input for $f(x)$.

This process is crucial in various areas of mathematics, including calculus, algebra, and applied sciences. In real-world scenarios, compositions model complex systems where outputs of one process become inputs for another.

Mathematical Approach

The general approach for solving $(f \circ g)(x)$:

$$(f \circ g)(x) = f(g(x))$$

In our specific problem:

$$(f \circ g)(4) = f(g(4))$$

which involves:

- Calculating $g(4)$ to find an intermediate value.
- Substituting this value into $f(x)$.

This step-by-step process ensures clarity and accuracy.

Additional Practice: Applying the Same Technique to Different Inputs

To deepen understanding, consider other inputs and their compositions:

Example 1: Compute $(f \circ g)(2)$

- $g(2) = 2^2 + 3 \times 2 = 4 + 6 = 10$
- $f(10) = 2 \times 10 + 1 = 20 + 1 = 21$

Result: $(f \circ g)(2) = 21$

Example 2: Compute $(f \circ g)(0)$

- $g(0) = 0^2 + 3 \times 0 = 0 + 0 = 0$
- $f(0) = 2 \times 0 + 1 = 0 + 1 = 1$

Result: $(f \circ g)(0) = 1$

Understanding the Multiple-Choice Options

The options provided in the problem help verify the correctness of our calculations:

- A) 57 — matches our calculation
- B) 55 — close but incorrect
- C) 20 — not matching our result
- D) 45 — not matching
- E) 108 — too high compared to our result
- F) — unspecified or possibly incorrect

Given the step-by-step calculations, Option A) 57 is the correct answer.

Additional Considerations in Function Composition

Domain and Range

- Domain of $g(x)$: All real numbers, since $x^2 + 3x$ is defined for all real x .
- Range of $g(x)$: All real numbers greater than or equal to $-\frac{9}{4}$, since the minimum of $x^2 + 3x$ occurs at $x = -\frac{3}{2}$.
- Domain of $f(x)$: All real numbers, as $f(x) = 2x + 1$ is defined everywhere.
- Composition: Since g maps to all real numbers and f is defined everywhere, the composition

is valid for all real x .

Graphical Interpretation

Visualizing the functions can enhance understanding:

- $g(x) = x^2 + 3x$: A parabola opening upward.
- $f(x) = 2x + 1$: A straight line with slope 2 and intercept 1.
- The composition $f(g(x))$ involves plugging the parabola into the line, resulting in a quadratic function.

Real-World Applications of Function Composition

Understanding how to evaluate compositions like $(f \circ g)(x)$ is essential in various fields:

- Physics: Calculating the result of sequential processes, such as velocity and acceleration.
- Economics: Modeling layered decision-making processes.
- Computer Science: Function chaining and data processing pipelines.
- Engineering: System modeling where outputs of one component feed into another.

Summary and Key Takeaways

Main Points

- The problem involves evaluating $(f \circ g)(4)$, which requires computing $g(4)$ first.
- The evaluated $g(4)$ is 28.
- Applying f to 28 yields 57.
- The correct multiple-choice answer is Option A) 57.

Critical Skills Demonstrated

- Correct substitution into functions.
- Sequential evaluation of nested functions.
- Understanding the concept of composition in mathematics.

Final Note

Mastering function composition is vital for advanced mathematics and practical problem-solving. Always break down the problem into smaller steps, evaluate inner functions first, and then apply the outer functions accordingly for accurate results.

Frequently Asked Questions (FAQs)

Q1: What is the notation $(f \circ g)(x)$?

A1: It denotes the composition of functions f and g , meaning $f(g(x))$.

Q2: How do I evaluate $(f \circ g)(x)$?

A2: First, compute $g(x)$, then substitute this result into f .

Q3: Can the composition be applied to any functions?

A3: Only if the output of g lies within the domain of f . In our case, both functions are defined for all real numbers, so composition is valid everywhere.

Q4: Why is it important to understand function composition?

A4: It models complex processes in mathematics, science, and engineering, where multiple functions or operations are combined sequentially.

Final Thoughts

Understanding and evaluating composite functions like $(f \circ g)(x)$ is a foundational skill in mathematics. Through systematic evaluation, we determined that $(f \circ g)(4) = 57$, matching choice A. Mastery of these concepts will enhance problem-solving skills and deepen your understanding of mathematical relationships.

Keywords: function composition, evaluate $(f \circ g)(x)$, $g(4)$, $f(g(4))$, algebra, functions, mathematics, problem-solving, multiple-choice, step-by-step, calculus, algebraic functions

Frequently Asked Questions

What is the value of $(fg)(4)$ given the functions $F(x)=2x+1$ and $G(x)=x^2+3x$?

The value of $(fg)(4)$ is 45.

How do you compute $(fg)(4)$ using the functions F and G ?

You first find $G(4)$, then substitute that result into F . So, $(fg)(4) = F(G(4))$.

What is $G(4)$ for the function $G(x)=x^2+3x$?

$G(4) = 4^2 + 3 \cdot 4 = 16 + 12 = 28$.

After calculating $G(4)$, how do you find $F(G(4))$?

Substitute $G(4)=28$ into F : $F(28)=2 \cdot 28 + 1 = 56 + 1 = 57$.

What is the correct value of $(fg)(4)$ based on the calculations?

The correct value of $(fg)(4)$ is 57, which corresponds to option A.

Additional Resources

Find $(fg)(4)$: An In-Depth Analytical Approach to Composite Functions

Understanding how to evaluate composite functions is a foundational skill in algebra and calculus, underpinning more complex mathematical concepts and problem-solving strategies. Specifically, the task of finding $(fg)(4)$ —the composition of functions f and g at the input 4—requires a careful step-by-step analysis, especially when given explicit functions. This article aims to explore the process thoroughly, providing a detailed explanation of the concepts involved, the methods to compute such compositions, and an application of these principles with the given functions $F(x) = 2x + 1$ and $G(x) = x^2 + 3x$.

Understanding the Concept of Function Composition

What is a Function?

A function, in mathematics, is a relationship that assigns to each input exactly one output. It can be thought of as a machine: you feed in a number, and the machine processes it according to a rule to produce a new number.

For example:

- $F(x) = 2x + 1$ means that for any input x , the output is twice x plus one.
- $G(x) = x^2 + 3x$ indicates that for each input x , the output is x squared plus three times x .

What is Function Composition?

Function composition involves applying one function to the result of another function. It is denoted as $(f \circ g)(x)$, which reads as “ f composed with g at x ,” and mathematically expressed as:

$$\begin{aligned} \backslash \\ (f \circ g)(x) &= f(g(x)) \\ \backslash \end{aligned}$$

Similarly, $(g \circ f)(x) = g(f(x))$.

The key idea is that you first evaluate $g(x)$, then plug this result into f , leading to a nested evaluation.

Why is Function Composition Important?

Composing functions allows mathematicians and scientists to model complex systems where outputs of one process become inputs for another. It appears extensively in calculus, computer science, physics, and engineering to describe layered or sequential transformations.

Step-by-Step Process to Find $(fg)(4)$

Clarification of Notation

In the context of the problem, the notation $(fg)(4)$ can be a source of confusion. Typically, $(fg)(x)$ refers to the composition of functions f and g , i.e., $f(g(x))$.

Given the functions:

- $F(x) = 2x + 1$
- $G(x) = x^2 + 3x$

and the expression:

- $(fg)(4)$

It is most consistent to interpret this as:

$$\begin{array}{l} \backslash \\ (f \circ g)(4) = f(g(4)) \\ \backslash \end{array}$$

This is the standard notation for composition.

Calculating the Inner Function $g(4)$

Step 1: Evaluate G at x = 4

Given:

$$\begin{aligned} & \backslash \\ G(x) &= x^2 + 3x \\ & \backslash \end{aligned}$$

Substituting $x = 4$:

$$\begin{aligned} & \backslash \\ G(4) &= (4)^2 + 3 \times 4 = 16 + 12 = 28 \\ & \backslash \end{aligned}$$

Thus, $g(4) = 28$.

Applying the Outer Function F to the Result of G(4)

Step 2: Evaluate F at g(4) = 28

Given:

$$\begin{aligned} & \backslash \\ F(x) &= 2x + 1 \\ & \backslash \end{aligned}$$

Substituting $x = 28$:

$$\begin{aligned} & \backslash \\ F(28) &= 2 \times 28 + 1 = 56 + 1 = 57 \\ & \backslash \end{aligned}$$

Therefore, $(fg)(4) = F(g(4)) = 57$.

Final Result and Corresponding Multiple Choice Answer

From the calculations above, the value of $(fg)(4)$ is 57, which corresponds to answer choice:

This straightforward evaluation demonstrates the importance of understanding function composition, especially when functions are explicitly defined.

Analytical Insights into the Problem

The Significance of Function Composition in Mathematical Models

Function composition is more than an academic exercise; it is fundamental in modeling real-world systems. For example, in physics, the position of an object after a certain time may be modeled as a composition of functions representing velocity and acceleration. In computer science, nested functions or method calls are analogous to composition, enabling complex data transformations.

Potential Pitfalls and How to Avoid Them

- Misinterpreting the notation: Always clarify whether the problem asks for $f(g(x))$ or $g(f(x))$. The order significantly impacts the result.
- Substituting incorrectly: Ensuring correct substitution and order of operations is critical, especially with nested functions.
- Not verifying the domain: While not explicitly asked here, in more complex problems, checking the domain restrictions after composition ensures the result is valid.

Extensions and Variations

- Different inputs: Repeating similar calculations at different points, such as $x=0$ or $x=10$, helps understand the behavior of the composite functions.
- Inverse functions: Exploring inverses of F and G could deepen understanding of their relationships.
- Graphical interpretation: Plotting the functions F and G , and their composition, visually demonstrates how the functions interact and can aid intuition.

Conclusion: The Power of Systematic Evaluation

The evaluation of $(fg)(4)$, given explicit functions $F(x)$ and $G(x)$, exemplifies a core mathematical skill—function composition. By methodically evaluating the inner function first and then applying the

outer function, we obtain a precise result with minimal ambiguity. In this case, the answer is 57, matching choice A.

This process underscores the broader importance of understanding composition in mathematical modeling, problem-solving, and real-world applications. Whether working through algebraic problems, calculus exercises, or complex systems modeling, mastering the step-by-step approach to function composition is essential for clarity and accuracy.

In summary:

- The functions are $F(x) = 2x + 1$ and $G(x) = x^2 + 3x$.
- To find $(fg)(4)$, interpret as $F(g(4))$.
- Calculate $g(4)$: $(4^2 + 3 \times 4 = 16 + 12 = 28)$.
- Calculate $F(28)$: $(2 \times 28 + 1 = 57)$.
- The final answer is A) 57.

This detailed analysis not only provides the solution but also enhances comprehension of composite functions' role and calculation methodology in algebra and beyond.

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