Find The 4 Fourth Roots Of Z = 16 Cis 60 Where 0o < 0 <0< 03. A. WO = 2 Cis 15 B. W1 = 2 Cis

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Understanding how to find the roots of complex numbers is a fundamental aspect of complex analysis and can be incredibly useful in various fields such as engineering, physics, and mathematics. In this article, we will explore the process of finding the four fourth roots of the complex number $z = 16 \text{cis} 60^\circ$, particularly focusing on the given conditions where $0^\circ < \theta < 90^\circ$. We will also analyze the roots denoted as W_0 and W_1, expressed in polar form as $2 \text{cis} 15^\circ$ and another similar form, respectively.

Understanding Complex Numbers in Polar Form

Before diving into the root calculations, it is essential to understand how complex numbers are expressed in polar form. Any complex number z can be written as:

$$
z = r (\cos \theta + i \sin \theta) = r \text{cis} \theta
$$

where:
- r is the modulus (or magnitude) of the complex number.
- θ is the argument (or angle) of the complex number in degrees or radians.

This form is especially useful when calculating roots because of De Moivre's Theorem, which simplifies the process of raising complex numbers to powers or extracting roots.

De Moivre's Theorem and Roots of Complex Numbers

De Moivre's Theorem states that for any complex number $z = r \text{cis} \theta$ and any integer n:

$$
z^n = r^n \text{cis} (n \theta)
$$

When finding the n-th roots of z, we solve for w such that:

$$
w^n = z
$$

Expressed in polar form:

$$
w = r^{1/n} \text{cis} \left(\frac{\theta + 360^\circ k}{n} \right), \quad k = 0, 1, 2, ..., n-1
$$

This formula provides all n roots by varying k.

Step 1: Expressing $z = 16 \text{cis} 60^\circ$

Given:

$$
z = 16 \text{cis} 60^\circ
$$

- The modulus r is 16.
- The argument θ is 60°.

Our goal is to find all four fourth roots of z, i.e., find all w such that:

$$
w^4 = z
$$

Step 2: Calculating the Modulus of the Roots

Applying the root formula:

$$
|w| = r^{1/4} = 16^{1/4}
$$

Calculating:

$$
16^{1/4} = (2^4)^{1/4} = 2^{4/4} = 2^1 = 2
$$

Thus, each root has a modulus of 2.

Step 3: Determining the Arguments of the Roots

The arguments are given by:

$$
\theta_k = \frac{\theta + 360^\circ k}{4}, \quad k = 0, 1, 2, 3
$$

Substituting $\theta = 60^\circ$:

$$
\theta_k = \frac{60^\circ + 360^\circ k}{4}
$$

\]

Calculating each:

- For \(k=0 \):

\[
\theta_0 = \frac{60^\circ + 0}{4} = 15^\circ
\]

- For \(k=1 \):

\[
\theta_1 = \frac{60^\circ + 360^\circ}{4} = \frac{420^\circ}{4} = 105^\circ
\]

- For \(k=2 \):

\[
\theta_2 = \frac{60^\circ + 720^\circ}{4} = \frac{780^\circ}{4} = 195^\circ
\]

- For \(k=3 \):

\[
\theta_3 = \frac{60^\circ + 1080^\circ}{4} = \frac{1140^\circ}{4} = 285^\circ
\]

Step 4: Listing the Fourth Roots

Putting it all together, the four roots are:

\[
\boxed{
\begin{aligned}
W_0 &= 2 \text{cis} 15^\circ \\
W_1 &= 2 \text{cis} 105^\circ \\
W_2 &= 2 \text{cis} 195^\circ \\
W_3 &= 2 \text{cis} 285^\circ
\end{aligned}
}
\]

These roots are equally spaced around the circle of radius 2 in the complex plane, separated by 90° increments.

Analyzing the Roots \(W_0 \) and \(W_1 \)

From the calculations, the roots \(W_0 \) and \(W_1 \) are:

- \(W_0 = 2 \text{cis} 15^\circ \)

- $W_1 = 2 \text{cis} 105^\circ$

Significance of W_0

The root W_0 with argument 15° falls within the specified range $0^\circ < \theta < 90^\circ$, making it particularly interesting for applications where the principal root or the smallest positive argument root is desired.

Significance of W_1

Similarly, W_1 at 105° is just outside the first quadrant, but still a valid root in the set. Its position indicates a root located in the second quadrant.

Visualizing the Roots in the Complex Plane

Plotting these roots helps develop an intuitive understanding of their distribution:

- W_0 at 15° lies close to the positive real axis.
- W_1 at 105° resides in the second quadrant.
- W_2 at 195° is in the third quadrant.
- W_3 at 285° is in the fourth quadrant.

This symmetric distribution around the origin demonstrates the elegance of roots of unity and how complex roots are evenly spaced around a circle.

Applications of Finding Fourth Roots

Knowing the roots of complex numbers is crucial in various domains:

- Electrical Engineering: Analyzing AC circuits and impedance.
- Signal Processing: Fourier transforms involve roots of complex exponentials.
- Control Systems: Stability analysis often requires roots of characteristic equations.
- Mathematics: Solving polynomial equations and exploring complex functions.

Summary

To summarize, the process of finding the four fourth roots of $z = 16 \text{cis} 60^\circ$ involves:

1. Expressing the number in polar form with known magnitude and argument.
2. Calculating the modulus of the roots as the n-th root of the original modulus.
3. Determining the arguments of all roots by dividing the sum of the original argument and multiples of 360° by n.
4. Listing all roots in polar form, which are equally spaced around the circle.

The roots are:

\[
\boxed{
\begin{aligned}
W_0 &= 2 \text{cis} 15^\circ \\
W_1 &= 2 \text{cis} 105^\circ \\

W_2 &= 2 \text{cis} 195^\circ \\
W_3 &= 2 \text{cis} 285^\circ
\end{aligned}
}
\]

Understanding these roots provides a foundation for more advanced topics in complex analysis, signal processing, and related fields. Whether you're analyzing waveforms, solving polynomial equations, or exploring the geometry of complex numbers, mastering the calculation of roots is an invaluable skill.

Frequently Asked Questions

What is the general form for finding the fourth roots of a complex number expressed in polar form?

To find the fourth roots of a complex number in polar form, use the formula: for $Z = r (\cos \theta + i \sin \theta)$, the roots are given by $W_k = r^{1/4} [\cos ((\theta + 2\pi k)/4) + i \sin ((\theta + 2\pi k)/4)]$ where $k = 0, 1, 2, 3$.

Given Z = 16 Cis 60°, how do you compute its magnitude and argument for root calculation?

The magnitude is $r = 16$, and the argument is $\theta = 60°$. For the roots, $r^{1/4} = 16^{1/4} = 2$, and the arguments are $(60° + 360°k)/4$ for $k = 0, 1, 2, 3$.

What are the four fourth roots of Z = 16 Cis 60°?

The roots are: $W_k = 2 \text{ Cis } ((60° + 360°k)/4)$, for $k = 0, 1, 2, 3$. This yields approximately: 1. $W_0 = 2 \text{ Cis } 15°$, 2. $W_1 = 2 \text{ Cis } 105°$, 3. $W_2 = 2 \text{ Cis } 195°$, 4. $W_3 = 2 \text{ Cis } 285°$.

How are the roots evenly spaced on the complex plane?

The four roots are evenly spaced at 90° intervals around the circle of radius 2, corresponding to arguments 15°, 105°, 195°, and 285°.

Are the roots of Z = 16 Cis 60° symmetric about the origin?

Yes, the roots are symmetric about the origin, forming a square pattern on the complex plane with equal magnitude and evenly spaced angles.

What is the significance of the angles 15°, 105°, 195°, and 285° in the roots?

These angles represent the arguments of the four roots, calculated by dividing the original argument plus multiples of 360° by 4, ensuring the roots are evenly distributed.

How does the given notation 'W0 = 2 Cis 15' relate to the roots of Z?

W0 = 2 Cis 15° is the first root, corresponding to k=0 in the formula, with subsequent roots obtained by adding 90° increments to the argument.

Additional Resources

Complex Numbers and Roots: Exploring the Fourth Roots of $Z = 16 \text{cis} 60^\circ$

Understanding the roots of complex numbers is a fascinating journey into the world of advanced mathematics, blending algebra, geometry, and trigonometry. Today, we delve into the elegant problem of finding the four fourth roots of the complex number $Z = 16 \text{cis} 60^\circ$. This exploration not only illuminates fundamental concepts in complex analysis but also exemplifies the beauty of mathematical symmetry and structure.

Introduction to Complex Numbers in Polar Form

Before diving into root extraction, it’s essential to understand the representation of complex numbers in polar form. Unlike the rectangular form $a + bi$, polar form expresses a complex number as:

$$
Z = r \text{cis} \theta
$$

where:
- r is the modulus (absolute value) of the complex number, representing its distance from the origin in the complex plane.
- θ is the argument (angle), measured in degrees or radians, indicating the direction of the vector from the origin.

For the given $Z = 16 \text{cis} 60^\circ$:
- The modulus $r = 16$
- The argument $\theta = 60^\circ$

Expressed in exponential form, this becomes:

$$
Z = r e^{i \theta} = 16 e^{i 60^\circ}
$$

This representation simplifies the process of extracting roots, as it leverages De Moivre's theorem, which states:

$$
\sqrt[n]{r e^{i \theta}} = \sqrt[n]{r} e^{i (\theta + 360^\circ k)/n} \quad \text{for} \quad k = 0, 1, 2, ..., n-1
$$

This formula seamlessly captures all roots, considering the periodic nature of the complex exponential.

Understanding the Fourth Roots of a Complex Number

The problem of finding the fourth roots of Z involves solving:

$$
W^4 = Z
$$

Where W is the complex number we seek. Applying De Moivre's theorem, the process involves:

1. Computing the modulus of the roots.
2. Computing the arguments (angles) of the roots.

Step 1: Modulus of the roots

Since:

$$
|W|^4 = |Z| = r
$$

then:

$$
|W| = \sqrt[4]{r}
$$

Given $r = 16$:

$$
|W| = \sqrt[4]{16} = \sqrt{\sqrt{16}} = \sqrt{4} = 2
$$

Step 2: Arguments of the roots

The arguments are given by:

$$
\arg(W)_k = \frac{\theta + 360^\circ k}{4}
$$

\]

for \(k = 0, 1, 2, 3 \).

This generates four roots, equally spaced around the circle, separated by \(90^\circ \), due to the fourth root.

Calculating the Fourth Roots: Step-by-Step

Let's apply these principles to find the four roots explicitly.

Step 1: Basic parameters

- Modulus of roots: \(|W| = 2 \)
- Argument: \(\theta = 60^\circ \)

Step 2: Roots' arguments

For each \(k \), the argument is:

\[
\arg(W)_k = \frac{60^\circ + 360^\circ \times k}{4}
\]

Calculating for each \(k \):

- For \(k = 0 \):

\[
\arg(W)_0 = \frac{60^\circ + 0}{4} = 15^\circ
\]

- For \(k = 1 \):

\[
\arg(W)_1 = \frac{60^\circ + 360^\circ}{4} = \frac{420^\circ}{4} = 105^\circ
\]

- For \(k = 2 \):

\[
\arg(W)_2 = \frac{60^\circ + 720^\circ}{4} = \frac{780^\circ}{4} = 195^\circ
\]

- For \(k = 3 \):

\[

\arg(W)_3 = \frac{60^\circ + 1080^\circ}{4} = \frac{1140^\circ}{4} = 285^\circ
\]

Step 3: Express roots in polar form

Thus, the four roots are:

\[
\boxed{
\begin{aligned}
W_0 &= 2 \text{cis} 15^\circ \\
W_1 &= 2 \text{cis} 105^\circ \\
W_2 &= 2 \text{cis} 195^\circ \\
W_3 &= 2 \text{cis} 285^\circ
\end{aligned}
}
\]

Step 4: Convert to rectangular form (optional)

Using:

\[
\text{cis} \theta = \cos \theta + i \sin \theta
\]

we can compute approximate rectangular coordinates for each root:

- For \(W_0 \):

\[
W_0 = 2 (\cos 15^\circ + i \sin 15^\circ) \approx 2 (0.9659 + i 0.2588) \approx 1.9318 + 0.5176 i
\]

- For \(W_1 \):

\[
W_1 = 2 (\cos 105^\circ + i \sin 105^\circ) \approx 2 (-0.2588 + i 0.9659) \approx -0.5176 + 1.9318 i
\]

- For \(W_2 \):

\[
W_2 = 2 (\cos 195^\circ + i \sin 195^\circ) \approx 2 (-0.9659 - i 0.2588) \approx -1.9318 - 0.5176 i
\]

- For \(W_3 \):

\[
W_3 = 2 (\cos 285^\circ + i \sin 285^\circ) \approx 2 (0.2588 - i 0.9659) \approx 0.5176 - 1.9318 i
\]

Understanding the Geometric Distribution of Roots

The four roots are evenly spaced around a circle of radius 2 in the complex plane, with angles at $15^\circ, 105^\circ, 195^\circ, 285^\circ$. This symmetry reflects the fundamental nature of roots of unity, extended here to roots of an arbitrary complex number.

This distribution illustrates how complex roots form regular polygons when plotted, with each root separated by equal angular increments of 90°.

Summary of the Four Roots

Root	Polar Form	Approximate Rectangular Coordinates
W_0	$2 \text{cis} 15^\circ$	$1.9318 + 0.5176 i$
W_1	$2 \text{cis} 105^\circ$	$-0.5176 + 1.9318 i$
W_2	$2 \text{cis} 195^\circ$	$-1.9318 - 0.5176 i$
W_3	$2 \text{cis} 285^\circ$	$0.5176 - 1.9318 i$

Additional Clarifications: The Role of A and B Roots

In your problem statement, you mention:

- $A. \; W_0 = 2 \text{cis} 15^\circ$
- $B. \; W_1 = 2 \text{cis}$

While it appears some notation might be incomplete or stylized, the essence is that the roots are designated as W_0, W_1, W_2, W_3, with the first roots explicitly provided or derived.

The mention of "adopting the tone of a product review or expert feature" suggests emphasizing the elegance and utility of this mathematical process, highlighting how the roots' symmetry and structure are akin to a well-crafted product with balanced design.

Conclusion: The Significance of Understanding Roots in Complex Analysis

The process of finding the fourth roots of $Z = 16 \text{cis} 60^\circ$ exemplifies the power of polar representation and De Moivre’s theorem. It demonstrates how complex numbers, often perceived as abstract, possess a tangible geometric interpretation—roots evenly distributed around the circle, forming a beautiful symmetry.

Whether you're studying advanced mathematics, engineering, or physics, mastering these concepts enhances your understanding of oscillations, wave phenomena, signal processing, and more. The beauty of these roots lies not only in their mathematical precision but also in the harmony

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