Find The Area Between The Curves. X=5,x=1,y=2x,y=x^23 The Area Between The Curves Is (Type An Integer

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Understanding the concept of finding the area between curves is fundamental in calculus and plays a vital role in various applications across science, engineering, and mathematics. In this comprehensive guide, we will explore how to determine the area enclosed between two curves, specifically focusing on the curves defined by y=2x and y=x^23, bounded between x=1 and x=5. By the end of this article, you will be able to confidently calculate the area between these two curves and understand the underlying principles involved.

Introduction to Area Between Curves

The area between two curves is the region enclosed by the graphs of the functions y=f(x) and y=g(x) over a specific interval [a, b]. Calculating this area involves integrating the difference between the functions across the interval where they overlap.

Why Is Calculating Area Between Curves Important?

- Real-world applications: Physics (work done, center of mass), economics (cost and revenue analysis), engineering (stress analysis).
- Mathematical insights: Understanding the behavior of functions, analyzing volumes of revolution, and solving optimization problems.

Given Curves and Limits

Let’s consider the specific problem:

- Curves: y=2x and y=x^23
- Interval: from x=1 to x=5

Our goal is to find the area enclosed between these curves over the specified interval.

Step-by-Step Solution Approach

To find the area between the curves, follow these steps:

1. Determine Which Curve Is on Top

Compare y=2x and y=x^23 over the interval [1, 5]:

- At x=1: y=2(1)=2, y=1^23=1
- At x=2: y=4, y=2^23, which is a large number (~8 million), so y=x^23 is on top.
- At x=3: y=6, y=3^23, which is enormous (~3.7×10^{10}), so y=x^23 remains on top.
- At x=5: y=10, y=5^23, which is about 1.2×10^{16}, still much larger.

Since y=x^23 grows faster and is larger than y=2x throughout the interval, the top curve is y=x^23, and the bottom is y=2x.

2. Set Up the Integral for the Area

The area between the curves from x=1 to x=5 is:

$$
A = \int_{1}^{5} [\text{top curve} - \text{bottom curve}] dx
$$

$$
A = \int_{1}^{5} [x^{23} - 2x] dx
$$

3. Compute the Integral

Break down the integral:

$$
A = \int_{1}^{5} x^{23} dx - \int_{1}^{5} 2x dx
$$

Calculate each separately:

- $\int x^{23} dx = \frac{x^{24}}{24}$
- $\int 2x dx = x^{2}$

Now evaluate:

\[
A = \left[\frac{x^{24}}{24}\right]_{1}^{5} - \left[x^{2}\right]_{1}^{5}
\]

Substitute the limits:

\[
A = \left(\frac{5^{24}}{24} - \frac{1^{24}}{24}\right) - (25 - 1)
\]

Simplify:

\[
A = \frac{5^{24} - 1}{24} - (24)
\]

Final Calculation and Interpretation

The value \(5^{24}\) is a very large number, but since the problem asks for the area as an integer, we can analyze the expression:

\[
A = \frac{5^{24} - 1}{24} - 24
\]

Note that \(5^{24}\) is divisible by 24:

- 24 factors as \(2^3 \times 3\).
- 5^24 mod 24 is such that the numerator \(5^{24} - 1\) is divisible by 24 because of properties of modular arithmetic.

Indeed, \(5^{24} \equiv 1 \pmod{24}\), so \(5^{24} - 1\) is divisible by 24, making the division exact, resulting in an integer.

Thus, the area \(A\) is an integer value:

\[
A = \frac{5^{24} - 1}{24} - 24
\]

This confirms that the area between the curves is an integer, as per the problem statement.

Conclusion

Calculating the area between the curves y=2x and y=x^23 over the interval x=1 to x=5 involves understanding the relative positions of the curves, setting up the integral of the difference, and performing the integration. The key steps include identifying the top and bottom functions, simplifying the integral, and evaluating the bounds. The resulting area, expressed as an integer, demonstrates the power of calculus in solving complex area problems and highlights the importance of precise calculation and understanding of function behavior.

Summary of Key Points

- The area between two curves is found by integrating their difference over the interval.
- Determine which function is on top within the interval.
- Set up the integral accordingly.
- Perform the integration, evaluate at the bounds, and simplify.
- Confirm the result fits the problem's conditions, such as being an integer.

By mastering these steps, you can confidently approach similar problems involving complex curves and their enclosed areas, strengthening your calculus skills and mathematical reasoning.

Additional Resources

- Textbook chapters on Integration and Areas Between Curves
- Online calculus tutorials and videos
- Practice problems for calculating areas between various types of curves
- Software tools like WolframAlpha or Desmos for visualization and computation

Remember, practicing with different functions and intervals will deepen your understanding and improve your problem-solving efficiency in calculus.

Frequently Asked Questions

How do you find the area between the curves y = 2x and y = x^2 from x = 1 to x = 5?

You calculate the definite integral of the difference between the functions over the interval [1, 5], i.e., ∫ from 1 to 5 of (2x - x^2) dx.

What is the first step in setting up the integral to

find the area between y = 2x and y = x^2 between x=1 and x=5?

Determine which function is on top over the interval; since 2x > x^2 between 1 and 5, set up the integral as ∫ (2x - x^2) dx from 1 to 5.

How do you evaluate the integral ∫ (2x - x^2) dx from x=1 to x=5?

Integrate term by term: ∫ 2x dx = x^2, and ∫ x^2 dx = (1/3)x^3. Then, evaluate from 1 to 5: [x^2 - (1/3)x^3] from 1 to 5.

What is the numerical value of the area between the curves y=2x and y=x^2 from x=1 to x=5?

The area is 40.

Why is the area between the curves calculated as the integral of (top function - bottom function)?

Because the integral of the difference gives the total 'vertical' area between the curves over the specified interval.

Additional Resources

Finding the area between curves is a fundamental concept in calculus that combines geometric intuition with analytical rigor. It involves calculating the region enclosed between two or more curves within specified bounds, often requiring integration techniques to determine the exact measure of that region. This process is essential in various fields such as physics, engineering, economics, and environmental sciences, where understanding the spatial relationship between different functions or data sets can inform critical decision-making or theoretical insights.

Introduction to the Problem: The Area Between Curves

When analyzing the area enclosed between two curves, the primary goal is to find a numerical value representing the size of the region bounded by these curves over a specific interval. This is a common task in calculus, often encountered in problems involving geometry, optimization, and modeling physical phenomena. The general approach involves identifying the functions that define the curves, determining their points of intersection, and then integrating the difference of the functions over the relevant interval.

In the specific problem under discussion, the goal is to find the area between the curves defined by the functions:

- $y = 2x$
- $y = x^{23}$

within the bounds $x = 1$ and $x = 5$.

This problem encapsulates key aspects of calculus: understanding the behavior of polynomial functions, setting up proper integrals, and executing the calculations accurately.

Understanding the Curves and Their Behavior

The Line $y = 2x$

The function $y = 2x$ is a straight line passing through the origin with a slope of 2. Its graph is a linear function, increasing steadily as x increases, with no points of inflection or curvature. This line is straightforward to interpret and is often used as a baseline in many geometric problems.

The Polynomial $y = x^{23}$

The function $y = x^{23}$ is a highly steep polynomial for $x > 1$, due to the large exponent. Since the exponent is odd and high, the function:

- Passes through the origin
- Is increasing for $x > 0$
- Grows rapidly as x increases

At $x = 1$, $y = 1^{23} = 1$, and at $x = 5$, $y = 5^{23}$, which is an astronomically large value. This indicates that the polynomial curve is significantly steeper than the line $y=2x$ as x grows larger.

Determining the Points of Intersection

Before calculating the area, it is critical to identify where the two curves intersect within the specified bounds \(x = 1 \) and \(x = 5 \). The intersection points define the limits of integration and ensure that the area calculation is accurate.

Equating the Curves

Set the functions equal to find the intersection:

\[
2x = x^{23}
\]

Rearranged:

\[
x^{23} - 2x = 0
\]

Factor out \(x \):

\[
x (x^{22} - 2) = 0
\]

This yields two solutions:

1. \(x = 0 \)
2. \(x^{22} = 2 \)

The first solution, \(x=0 \), is outside our interval of interest (\(1 \leq x \leq 5 \)), so we focus on:

\[
x^{22} = 2
\]

Solving for \(x \):

\[
x = \sqrt[22]{2}
\]

Since 22 is a large exponent, \(\sqrt[22]{2} \) is approximately:

\[

x \approx 2^{1/22}
\]

Using logarithmic approximation:

\[
x \approx e^{\frac{\ln 2}{22}} \approx e^{0.6931/22} \approx e^{0.0315} \approx 1.032
\]

Therefore, the two curves intersect at approximately \(x \approx 1.032 \) within the interval \([1,5] \).

Setting Up the Integral for the Area Calculation

To find the area between the curves, we need to consider the relative position of the functions over the interval.

- For \(x \) between 1 and approximately 1.032, \(y = 2x \) is above \(y = x^{23} \).
- For \(x \) from approximately 1.032 to 5, \(y = x^{23} \) surpasses \(y = 2x \).

Given this, the total area \(A \) between the curves over \(x \in [1,5] \) can be expressed as:

\[
A = \int_{1}^{x_0} [2x - x^{23}] \, dx + \int_{x_0}^{5} [x^{23} - 2x] \, dx
\]

where \(x_0 \approx 1.032 \).

Calculating the Integrals

Integral from 1 to \(x_0 \): \(\int_{1}^{x_0} [2x - x^{23}] \, dx \)

First, compute the indefinite integrals:

\[
\int 2x \, dx = x^2 + C
\]

\[
\int x^{23} \, dx = \frac{x^{24}}{24} + C
\]

Therefore, the definite integral:

\[
\int_{1}^{x_0} [2x - x^{23}] \, dx = \left[x^2 - \frac{x^{24}}{24}
\right]_{1}^{x_0}
\]

Plugging in limits:

\[
= \left(x_0^2 - \frac{x_0^{24}}{24} \right) - \left(1^2 - \frac{1^{24}}{24}
\right) = \left(x_0^2 - \frac{x_0^{24}}{24} \right) - \left(1 -
\frac{1}{24} \right)
\]

\[
= x_0^2 - \frac{x_0^{24}}{24} - 1 + \frac{1}{24}
\]

Integral from \(x_0 \) to 5: \(\int_{x_0}^{5} [x^{23} - 2x] \, dx \)

Similarly:

\[
\int x^{23} \, dx = \frac{x^{24}}{24}
\]

\[
\int 2x \, dx = x^2
\]

So,

\[
\int_{x_0}^{5} [x^{23} - 2x] \, dx = \left[\frac{x^{24}}{24} - x^2
\right]_{x_0}^{5}
\]

\[
= \left(\frac{5^{24}}{24} - 25 \right) - \left(\frac{x_0^{24}}{24} - x_0^2 \right) = \frac{5^{24}}{24} - 25 - \frac{x_0^{24}}{24} + x_0^2
\]

Combining the Results and Approximations

The total area:

\[
A \approx \left(x_0^2 - \frac{x_0^{24}}{24} - 1 + \frac{1}{24} \right) + \left(\frac{5^{24}}{24} - 25 - \frac{x_0^{24}}{24} + x_0^2 \right)
\]

Simplify:

\[
A \approx 2x_0^2 - \frac{2x_0^{24}}{24} + \frac{5^{24}}{24} - 26 + \frac{1}{24}
\]

\[
A \approx 2x_0^2 - \frac{x_0^{24}}{12} + \frac{5^{24}}{24} - 26 + \frac{1}{24}
\]

Given that:

- \(x_0 \approx 1.032 \)
- \(x_0^2 \approx (1.032)^2 \approx 1.065 \)
- \(x_0^{24} \approx (x_0^{22}) \times x_0^{2} \)

Recall that earlier, \(x_0^{22} = 2 \), so:

\[
x_0^{24} = x_0^{22} \times x_0^2 = 2 \times 1.065 \approx 2.13
\]

Now, approximate the big term:

\[
\frac{5^{24}}{24}
\]

Considering the magnitude:

\[

```
5^{24} = (5^{4})^{6} = (625)^{6}
\]

which is an extremely large number. Specifically:

\[
5^{24} \approx 5.96 \
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