Find The Area Of A Triangle Bounded By The Y Axis, The Line F(x)=10 1/6 X, And The Line Perpendicular

Find The Area Of A Triangle Bounded By The Y Axis, The Line F(x)=10 1/6 X, And The Line Perpendicular

Understanding how to find the area of a triangle formed by specific lines is a fundamental skill in coordinate geometry. In this article, we will explore a detailed step-by-step process to determine the area of a triangle bounded by the y-axis, the line $f(x) = 10 \frac{1}{6} x$, and a line perpendicular to this given line. This problem not only enhances your understanding of geometric properties but also sharpens your skills in handling algebraic equations and their intersections within the coordinate plane.

Understanding the Problem and Setting Up the Equations

Before diving into calculations, it's essential to interpret the problem correctly and understand the geometric figures involved.

The Given Lines and Constraints

- Y-axis (x=0): This is the vertical line where all points have an x-coordinate of zero.

- Line $f(x) = 10 \frac{1}{6} x$: This is a straight line with a positive slope. The mixed fraction $10 \frac{1}{6}$ can be converted to an improper fraction for easier calculations.

- Line Perpendicular to $f(x)$: This line will have a slope that is the negative reciprocal of the slope of $f(x)$.

Converting the Slope to an Improper Fraction

The slope m of the line $f(x)$ is:

$$
m = 10 \frac{1}{6} = 10 + \frac{1}{6} = \frac{60}{6} + \frac{1}{6} = \frac{61}{6}
$$

So,

$$
f(x) = \frac{61}{6}x
$$

Finding the Equation of the Perpendicular Line

Since the original line has a slope $m = \frac{61}{6}$, the perpendicular line will have a slope:

$$
m_{\perp} = -\frac{6}{61}
$$

The perpendicular line passes through a specific point—often the origin or a point on the original line. For this problem, assuming the perpendicular line passes through the origin (since the problem specifies the triangle is bounded by the y-axis and these lines), its equation will be:

$$
L_{\perp}: y = -\frac{6}{61} x
$$

Note: If the problem states a different point of intersection, adjustments should be made accordingly.

Determining the Intersection Points

The vertices of the triangle are points where the lines intersect each other and the axes.

Intersection of $f(x)$ with the y-axis

- On the y-axis, $x=0$, so:

$$
f(0) = \frac{61}{6} \times 0 = 0
$$

- Point: $(0,0)$

Intersection of $L_{\perp}$ with the y-axis

- When $x=0$:

$$
y = -\frac{6}{61} \times 0 = 0
$$

- Point: $(0,0)$

So, both lines intersect at the origin.

Intersection of $f(x)$ and $L_{\perp}$

- To find this point, set their equations equal:

$$
\frac{61}{6} x = -\frac{6}{61} x
$$

- Simplify:

$$
\frac{61}{6} x + \frac{6}{61} x = 0
$$

- Find common denominator:

$$
\left(\frac{61^2}{6 \times 61}\right) x + \left(\frac{6^2}{6 \times 61}\right) x = 0
$$

- Alternatively, multiply both sides by the common denominator $6 \times 61 = 366$:

$$
(61 \times 61) x + (6 \times 6) x = 0
$$

$$
(3721) x + 36 x = 0
$$

$$
(3721 + 36) x = 0
$$

$$
3757 x = 0
$$

- Therefore,

\[

$$
x=0
$$

- Substituting back into either line:

$$
y = \frac{61}{6} \times 0 = 0
$$

Conclusion: The lines intersect at the origin, which we've already established.

Note: Since both lines intersect at the origin and extend outward, and both pass through the origin, the triangle will be bounded between the axes and these lines at some positive x-value.

Identifying the Triangle's Vertices

Considering the lines and axes:

- Vertex A: At the origin $(0,0)$

- Vertex B: Intersection of $f(x)$ with the x-axis

- Vertex C: Intersection of $L_{\perp}$ with the x-axis

Finding the x-intercepts of the lines

- For $f(x) = \frac{61}{6} x$:

- Set $f(x) = 0$:

$$
0 = \frac{61}{6} x \Rightarrow x=0
$$

- The line passes through the origin, so its x-intercept is at 0.

- For $L_{\perp} = -\frac{6}{61} x$:

- Set $L_{\perp} = 0$:

$$
0 = -\frac{6}{61} x \Rightarrow x=0
$$

- Also passes through the origin.

Implication: Both lines meet at the origin, and their positive intercepts with the axes are at different points. To form a triangle bounded by the axes and these lines, consider the points where the lines intersect the axes at some positive x-values.

Important Note: Since both lines pass through the origin, and are linear, the triangle formed is between the lines and the axes at some positive x-value where they intersect the axes or at specific points they cross the axes.

Determining the Points of Intersection with the Axes

Line $f(x) = \frac{61}{6} x$

- Y-intercept: At $x=0$, $y=0$

- X-intercept: Set $y=0$:

$$
0 = \frac{61}{6} x \Rightarrow x=0
$$

- Y-axis intercept: $(0,0)$

Line $L_{\perp} = -\frac{6}{61} x$

- Y-intercept: At $x=0$, $y=0$

- X-intercept: Set $y=0$:

$$
0 = -\frac{6}{61} x \Rightarrow x=0
$$

- Y-axis intercept: $(0,0)$

Observation: Both lines intersect at the origin, and both pass through the axes at the origin, so the triangle must be formed at some point where the lines extend into the first quadrant, intersecting the axes at some positive x and y values.

Alternative Approach: Find the Intersection of the Lines with the Y-Axis and the X-Axis at a Specific Point

Given the initial assumptions, the problem probably intends for us to find the area of the triangle bounded by:

- The y-axis ($x=0$)
- The line $f(x) = \frac{61}{6} x$
- The perpendicular line passing through a point on the x-axis or y-axis, possibly at some point $x=a$

To proceed, let's assume the perpendicular line intersects the x-axis at some point $x=b$, and the original line intersects the y-axis at a point $y=c$, forming a triangle.

Alternative assumption for clarity:
Suppose the perpendicular line intersects the x-axis at $x=a$, where $a > 0$, and the line $f(x)$ intersects the y-axis at $y=0$. The triangle is formed between these points and the axes.

Calculating the Coordinates of the Triangle

Assuming the perpendicular line intersects the x-axis at a point $x=a$:

$$
L_{\perp}: y = -\frac{6}{61} x
$$

- At the x-axis, $y=0$:

$$
0 = -\frac{6}{61} x \Rightarrow x=0
$$

- So, at $x=0$, point is $(0,0)$. For a non-zero point, the line extends into the positive x-axis at some $x=a$, but given the slope, the y-coordinate at $x=a$ is:

$$
y = -\frac{6}{61} a

\]

- Since \(y\) is negative for positive \(a\), the line crosses the x-axis at \((a

Frequently Asked Questions

How do I find the area of a triangle bounded by the y-axis, the line f(x) = 10 1/6 x, and a perpendicular line?

First, convert 10 1/6 to an improper fraction (say, 61/6). Find the point where the line intersects the y-axis (x=0), which is at the origin. Then, determine the equation of the perpendicular line and find its intersection with f(x). Use these intersection points to calculate the triangle's base and height, and apply the formula: Area = 1/2 × base × height.

What is the slope of the line f(x) = 10 1/6 x, and how does it affect the triangle's area?

The slope of the line f(x) = 10 1/6 x is 61/6. This slope determines how steep the line is and influences where the perpendicular line intersects it. Knowing the slope helps in writing the equation of the perpendicular line and finding the intersection points needed to compute the area.

How do I find the equation of the line perpendicular to f(x) = 10 1/6 x at a given point?

The slope of the perpendicular line is the negative reciprocal of 61/6, which is -6/61. To find the equation, use point-slope form with the point of intersection and this perpendicular slope.

How can I determine the intersection point of the line f(x) = 10 1/6 x and its perpendicular?

Set the equations equal: f(x) = perpendicular line. Substitute the known slope and solve for x to find the intersection point. Then, plug x back into either equation to find y, giving the intersection coordinates.

What is the significance of the y-axis in calculating the triangle's area?

The y-axis acts as one boundary of the triangle. The intersection point on the y-axis (at x=0) is crucial for determining the base or height of the triangle when calculating its area.

Can I use coordinate geometry formulas directly to find the area of this triangle?

Yes, by identifying the vertices of the triangle through intersection points, you can use the coordinate geometry formula for the area: Area = $1/2 |x1(y2 - y3) + x2(y3 - y1) + x3(y1 - y2)|$.

What are the steps to solve this problem systematically?

1. Convert mixed number to improper fraction. 2. Find the intersection of the given line with the y-axis (at $x=0$). 3. Determine the equation of the perpendicular line at the relevant point. 4. Find the intersection point of the perpendicular line with the original line. 5. Use these points to calculate the base and height of the triangle. 6. Apply the formula for the area of a triangle.

Additional Resources

Triangle Area Calculation

Understanding the Geometric Framework: The Triangle in Context

In the realm of coordinate geometry, the task of finding the area of a triangle bounded by specific lines and axes serves as a fundamental yet insightful exercise. It synthesizes algebraic manipulation with geometric intuition, offering an excellent platform for exploring the interplay of linear functions, perpendicularity, and bounded regions.

The problem at hand involves a triangle enclosed by three boundary lines:

1. The Y-axis (which is the line $x = 0$)
2. The Line given by $f(x) = 10 \frac{1}{6} x$
3. A Perpendicular Line to the given line passing through a specific point (usually the origin or another reference point)

At first glance, this problem may seem straightforward, but delving into each component reveals an intricate structure that requires careful analysis and precise calculations. Let's explore each boundary in detail, understand their relationships, and methodically determine the area of the enclosed triangle.

Deciphering the Boundary Lines

1. The Y-Axis: The Vertical Boundary

The y-axis, described by the equation $x=0$, functions as a natural boundary in the coordinate plane. It extends infinitely in the positive and negative y-directions, but within our bounded region, it serves as one of the triangle's sides.

Significance:
- Acts as a vertical boundary limiting the region on the left side.
- The intersection points with other lines will be crucial in determining vertices.

2. The Line $f(x) = 10 \frac{1}{6} x$

The given function involves a mixed number: $10 \frac{1}{6}$. To work efficiently, convert this to an improper fraction:

$$
10 \frac{1}{6} = \frac{60}{6} + \frac{1}{6} = \frac{61}{6}
$$

So, the line simplifies to:

$$
f(x) = \frac{61}{6} x
$$

Characteristics:

- Slope $m = \frac{61}{6}$, which is a steep positive slope.
- Passes through the origin $(0,0)$, assuming the function is defined at $x=0$.

Implication:
- The line extends infinitely in both directions, but within the bounded region, we're concerned with its intersection points with other boundary lines.

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3. The Perpendicular Line

The third boundary is the line perpendicular to $f(x)$. To understand its placement:

- It must have a slope that is the negative reciprocal of $\frac{61}{6}$.

Calculating the perpendicular slope:

$$
m_{\perp} = -\frac{1}{m} = -\frac{6}{61}
$$

Line Equation:

Without loss of generality, assume the perpendicular line passes through the origin (or another key point). For simplicity, let's consider it passing through the origin, so its equation is:

$$
g(x) = -\frac{6}{61} x
$$

Note: If the problem specifies a different passing point, adjustments will be necessary, but for now, the origin is the natural choice given the symmetry and the typical setup.

Identifying the Vertices of the Triangle

The triangle's vertices are determined by the intersection points of the boundary lines:

- Vertex A: Intersection of the y-axis ($x=0$) and the line $f(x)$
- Vertex B: Intersection of the y-axis ($x=0$) and the perpendicular line $g(x)$
- Vertex C: Intersection of the two lines $f(x)$ and $g(x)$

Let's analyze each in turn:

1. Vertex A: Intersection with the Y-Axis

At $x=0$:

- $f(0) = \frac{61}{6} \times 0 = 0$
- $g(0) = -\frac{6}{61} \times 0 = 0$

Thus, both lines pass through the origin. Therefore, the point of intersection is at:

$$
A = (0, 0)
$$

This point acts as a common vertex for the triangle.

2. Vertex B: Intersection of the Perpendicular Line with the Y-Axis

Since $g(0) = 0$, the perpendicular line also passes through the origin, leading to:

$$
B = (0, 0)
$$

This confirms that the origin is a shared point for both lines, forming a common vertex.

3. Vertex C: Intersection of $f(x)$ and $g(x)$

To find the intersection point C, set the two line equations equal:

$$
f(x) = g(x)
$$

$$
\frac{61}{6} x = -\frac{6}{61} x
$$

Bring all to one side:

$$
\frac{61}{6} x + \frac{6}{61} x = 0
$$

Factor out x:

$$
x \left(\frac{61}{6} + \frac{6}{61} \right) = 0
$$

Calculate the sum inside parentheses:

$$
\frac{61}{6} + \frac{6}{61}
$$

Find common denominator $6 \times 61 = 366$:

$$
\frac{61 \times 61}{6 \times 61} + \frac{6 \times 6}{61 \times 6} = \frac{3721}{366} + \frac{36}{366} = \frac{3757}{366}
$$

So,

$$
x \times \frac{3757}{366} = 0
$$

Therefore,

$$
x = 0
$$

Plugging back into either line:

$$
y = \frac{61}{6} \times 0 = 0
$$

The intersection point is at the origin:

$$
C = (0, 0)
$$

Conclusion:
All three lines intersect at the same point, the origin. This suggests that the three boundaries are not forming a bounded triangle in the usual sense but rather a degenerate region collapsing into a single point.

Reconciling the Geometric Structure: Clarifying the Boundaries

Given that all three lines intersect at the origin, the initial assumption that these lines form a bounded triangle appears invalid. This indicates that there might be additional context or different boundary points intended, such as:

- The line $f(x) = \frac{61}{6} x$ and the perpendicular line $g(x) = -\frac{6}{61} x$ are valid boundaries only within a certain domain.
- The problem may involve a specific segment of these lines, perhaps bounded by a vertical line (like a vertical boundary at some $x = a$).

Possible correction or clarification:

Suppose the problem intended to consider the region bounded by:

- The y-axis ($x=0$)

- The line $f(x) = \frac{61}{6} x$, but only from $x=0$ to a certain $x=a$
- The perpendicular line $g(x) = -\frac{6}{61} x$, again only from $x=0$ to $x=a$
- A vertical boundary at $x=a$

This setup would form a triangle with vertices at:

- $(0, 0)$
- $(a, 0)$
- $(a, f(a))$ or $(a, g(a))$

Alternatively, the problem might involve the triangle bounded by the axes and the lines in a different configuration, such as:

- The y-axis ($x=0$)
- The line $y = 10 \frac{1}{6} x$
- The line perpendicular to it passing through a point on the x-axis, say at $x = x_0$

Assuming a Typical Scenario: Constructing the Triangle with Finite Boundaries

Given the ambiguity, let's consider a common and instructive case:

- The triangle is formed by the y-axis ($x=0$)
- The line $y = \frac{61}{6} x$
- The line perpendicular to this passing through $(a, 0)$, where $a > 0$

Vertices:

- $A = (0, 0)$ (origin)
- $B = (a, 0)$ (on x-axis)
- $C = (a, f(a))$

Area Calculation:

The area of the triangle is:

$$
\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}
$$

Choosing the base as the segment along the x-axis from $(0,0)$ to $(a,0)$:

$$
\text{base} = a
$$

The height corresponds to the y-coordinate at \(x=a \):

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