Find The Area Of The Graph Of The Functionf(x, Y)=2/3(x3/2 +y3/2)that Lies Over The Domain [0, 3] [0,

Find The Area Of The Graph Of The Function f(x, y) = 2/3 (x^{3/2} + y^{3/2}) that Lies Over The Domain [0, 3] x [0, ?]

In this article, we will explore how to determine the surface area of the graph of the function $f(x, y) = \frac{2}{3} \left(x^{3/2} + y^{3/2} \right)$ over a specified domain. The domain is partially given as $[0, 3]$ along the x-axis, but the y-coordinate range is incomplete in the prompt; we will interpret it as $[0, 3]$ for the purposes of this analysis. Computing the surface area involves understanding the surface integral and applying the appropriate calculus techniques to evaluate it. This problem provides an insightful application of multivariable calculus, particularly the concepts of surface area calculation for functions of two variables.

Understanding the Problem and Setting Up the Domain

The Function and Its Graph

The function given is:

$$
f(x, y) = \frac{2}{3} \left(x^{3/2} + y^{3/2} \right)
$$

\]

This function describes a surface in three-dimensional space, where each point \((x, y)\) in the domain maps to a height \(z = f(x, y)\).

Domain Specification

The problem states the domain as \([0, 3]\) along both x and y axes, assuming symmetry and completeness, the domain is:

\[
\mathcal{D} = \left\{ (x, y) \mid 0 \leq x \leq 3, \quad 0 \leq y \leq 3 \right\}
\]

This is a square region in the xy-plane over which the surface area will be computed.

Surface Area of a Function of Two Variables

The General Formula

The surface area \(S\) of a surface \(z = f(x, y)\) over a domain \(\mathcal{D}\) is given by the surface integral:

\[
S = \iint_{\mathcal{D}} \sqrt{1 + \left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2} \, dx \, dy
\]

This formula accounts for the "slant" of the surface in all directions, integrating the infinitesimal surface elements across the domain.

The Partial Derivatives

To evaluate S, we need to compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

Calculate:

$$
\frac{\partial f}{\partial x} = \frac{2}{3} \times \frac{\partial}{\partial x} \left(x^{3/2} + y^{3/2} \right) = \frac{2}{3} \times \frac{\partial}{\partial x} x^{3/2} = \frac{2}{3} \times \frac{3}{2} x^{1/2} = x^{1/2}
$$

Similarly,

$$
\frac{\partial f}{\partial y} = y^{1/2}
$$

Thus, the expressions simplify significantly.

Formulating the Surface Area Integral

Plugging into the surface area formula:

$$
S = \iint_{0 \leq x \leq 3, \, 0 \leq y \leq 3} \sqrt{1 + (x^{1/2})^2 + (y^{1/2})^2} \, dx \, dy
$$

which simplifies to:

$$
S = \int_0^{3} \int_0^{3} \sqrt{1 + x + y} \, dy \, dx
$$

The symmetry allows us to write the integral as a double integral over the square domain.

Evaluating the Surface Area Integral

Inner Integral with Respect to y

Fix x, and integrate:

$$
I(x) = \int_0^{3} \sqrt{1 + x + y} \, dy
$$

Let $u = 1 + x + y$. Then, $du = dy$, and when $y=0$, $u = 1 + x$, when $y=3$, $u=4 + x$.

The integral becomes:

$$
I(x) = \int_{u=1 + x}^{4 + x} \sqrt{u} \, du
$$

Evaluate:

$$
I(x) = \int_{1 + x}^{4 + x} u^{1/2} \, du = \left[\frac{2}{3} u^{3/2} \right]_{u=1 + x}^{u=4 + x}
$$

So,

$$
I(x) = \frac{2}{3} \left((4 + x)^{3/2} - (1 + x)^{3/2} \right)
$$

Outer Integral with Respect to x

Now, the total surface area:

$$
S = \int_0^{3} I(x) \, dx = \frac{2}{3} \int_0^{3} \left((4 + x)^{3/2} - (1 + x)^{3/2} \right) dx
$$

Separate into two integrals:

$$
S = \frac{2}{3} \left[\int_0^{3} (4 + x)^{3/2} \, dx - \int_0^{3} (1 + x)^{3/2} \, dx \right]
$$

Calculating the Integrals

Integral of $(a + x)^{3/2}$

For a general a, the integral:

$$
\int (a + x)^{3/2} \, dx
$$

can be evaluated using substitution:

Let $t = a + x$, then $dt = dx$. When $x=0$, $t=a$; when $x=3$, $t= a+3$.

Thus,

$$
\int (a + x)^{3/2} dx = \int t^{3/2} dt = \frac{2}{5} t^{5/2} + C
$$

Apply limits:

$$
\int_{x=x_1}^{x=x_2} (a + x)^{3/2} dx = \frac{2}{5} \left[(a + x)^{5/2} \right]_{x_1}^{x_2}
$$

Applying to Our Cases

Calculate:

$$
\int_0^{3} (4 + x)^{3/2} dx = \frac{2}{5} \left[(4 + x)^{5/2} \right]_0^{3} = \frac{2}{5} \left[(7)^{5/2} - (4)^{5/2} \right]
$$

Similarly,

$$
\int_0^{3} (1 + x)^{3/2} dx = \frac{2}{5} \left[(4)^{5/2} - (1)^{5/2} \right]
$$

Note that:

- $(4)^{5/2} = (4^{1/2})^5 = (2)^5 = 32$
- $(7)^{5/2} = (7^{1/2})^5 = (\sqrt{7})^5$
- $(1)^{5/2} = 1$

Now, assemble the total:

$$
S = \frac{2}{3} \left[\frac{2}{5} \left(7^{5/2} - 4^{5/2} \right) - \frac{2}{5} \left(4^{5/2} - 1 \right) \right]
$$

Factor out $\frac{2}{5}$:

$$
S = \frac{2}{3} \times \frac{2}{5} \left[(7^{5/2} - 4^{5/2}) - (4^{5/2} - 1) \right]

\]

Simplify inside brackets:

$$
(7^{5/2} - 4^{5/2}) - (4^{5/2} - 1) = 7^{5/2} - 4^{5/2} - 4^{5/2} + 1 = 7^{5/2} - 2 \times 4^{5/2} + 1
$$

Recall:

- $4^{5/2} = 32$

Frequently Asked Questions

What is the function for which we are finding the area over the given domain?

The function is $f(x, y) = 2/3 (x^{3/2} + y^{3/2})$.

What is the specified domain for the function?

The domain is the rectangular region $[0, 3] \times [0, 3]$.

How do you set up the double integral to find the area under the graph of f(x, y)?

Since the area lies under the graph over the domain, the double integral is set up as $\iint_{[0,3]\times[0,3]} f(x, y)\, dy\, dx$.

What is the process to evaluate the integral of f(x, y) over the domain?

You evaluate the double integral by integrating $f(x, y)$ with respect to y first, then with respect to x, within the limits from 0 to 3.

How do you integrate the function $f(x, y) = 2/3\,(x^{3/2} + y^{3/2})$ with respect to y?

Since $f(x, y)$ separates into terms involving x and y, the integral with respect to y becomes $(2/3)\,[x^{3/2} y + (2/5)\, y^{5/2}]$ evaluated from 0 to 3.

What is the integral of f(x, y) with respect to y over [0, 3]?

It is $(2/3)\,[x^{3/2}\, 3 + (2/5)\, 3^{5/2}] = (2/3)\,[3\, x^{3/2} + (2/5)\, 3^{5/2}]$.

How do you proceed to integrate the result with respect to x?

You integrate the expression $(2/3)\,[3\, x^{3/2} + (2/5)\, 3^{5/2}]$ with respect to x from 0 to 3.

What is the final expression for the area under the graph over the domain?

The area is the integral from 0 to 3 of $(2/3)\,[3\, x^{3/2} + (2/5)\, 3^{5/2}]\, dx$, which simplifies to evaluate to a numerical value after computation.

Additional Resources

In-Depth Analysis of the Area Under the Graph of the Function $f(x, y) = \frac{2}{3} \left(x^{3/2} + y^{3/2} \right)$ over the Domain $[0, 3] \times [0, ?]$

When exploring the fascinating world of multivariable calculus, one of the key concepts that often surfaces is determining the area (or more precisely, the surface area) of a graph over a specified domain. In this case, we're examining the surface defined by the function:

$$
f(x, y) = \frac{2}{3} \left(x^{3/2} + y^{3/2} \right)
$$

over the domain $[0, 3] \times [0, ?]$. While the original problem seems to have a typo or incomplete notation for the domain, based on typical problems of this nature, we'll assume the domain is $[0, 3]$ for both x and y. This assumption allows us to proceed with a comprehensive analysis, ultimately aiming to find the area of the surface over this specified domain.

Understanding the Problem: Surface Area of a Function of Two Variables

Before diving into calculations, it's essential to clarify what the area of the graph of a function means in a multivariable context. Often, this refers to the surface area of the surface $z = f(x, y)$ over a given domain D.

Key Concept: Surface Area Surface Integral

The surface area S of a surface $z = f(x, y)$ over a domain D in the xy-plane is given by the surface integral:

$$
S = \iint_D \sqrt{1 + \left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2} \, dA
$$

where:

- $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are the partial derivatives of f with respect to x and y,
- dA represents the differential element of area in the domain D.

Step 1: Clarifying the Domain

Given the initial notation, it looks like the domain is $[0, 3]$ for x, but the domain for y is incomplete. Assuming the domain for y is also $[0, 3]$, the problem becomes symmetric, which is common in such exercises.

Assumed Domain:

$$
D = [0, 3] \times [0, 3]
$$

This square domain simplifies the calculations and is a typical setting for such problems.

Step 2: Computing Partial Derivatives

The function:

$$
f(x, y) = \frac{2}{3} \left(x^{3/2} + y^{3/2} \right)
$$

has derivatives:

$$
\frac{\partial f}{\partial x} = \frac{2}{3} \times \frac{3}{2} x^{1/2} = x^{1/2}
$$

Similarly,

$$
\frac{\partial f}{\partial y} = y^{1/2}
$$

These derivatives indicate how steeply the surface changes in the x and y directions.

Step 3: Setting Up the Surface Area Integral

Plugging the derivatives into the formula for surface area:

$$
S = \iint_D \sqrt{1 + \left(x^{1/2} \right)^2 + \left(y^{1/2} \right)^2} \, dA
$$

which simplifies to:

$$
S = \iint_D \sqrt{1 + x + y} \, dx \, dy
$$

This is because:

$$
\left(x^{1/2} \right)^2 = x, \quad \left(y^{1/2} \right)^2 = y
$$

Step 4: Computing the Double Integral

Our task reduces to evaluating:

$$
S = \int_{0}^{3} \int_{0}^{3} \sqrt{1 + x + y} \, dx \, dy
$$

Strategy:

- Integrate with respect to x first, then y,
- Use substitution to simplify the inner integral.

Step 5: Inner Integral with Respect to x

Fix y, and consider:

$$
I(y) = \int_{0}^{3} \sqrt{1 + x + y} \, dx
$$

Set:

\[
u = 1 + x + y \Rightarrow du = dx
\]

When \(x = 0\):

\[
u = 1 + 0 + y = 1 + y
\]

When \(x = 3\):

\[
u = 1 + 3 + y = 4 + y
\]

Thus,

\[
I(y) = \int_{u=1 + y}^{4 + y} \sqrt{u} \, du
\]

The integral of \(\sqrt{u} = u^{1/2}\):

\[
\int u^{1/2} du = \frac{2}{3} u^{3/2}
\]

Therefore,

\[

$$
I(y) = \left. \frac{2}{3} u^{3/2} \right|_{u=1 + y}^{4 + y} = \frac{2}{3} \left[(4 + y)^{3/2} - (1 + y)^{3/2} \right]
$$

Step 6: Outer Integral with Respect to y

Now, the total surface area:

$$
S = \int_{0}^{3} I(y) \, dy = \frac{2}{3} \int_{0}^{3} \left[(4 + y)^{3/2} - (1 + y)^{3/2} \right] dy
$$

Break this into two integrals:

$$
S = \frac{2}{3} \left[\int_{0}^{3} (4 + y)^{3/2} \, dy - \int_{0}^{3} (1 + y)^{3/2} \, dy \right]
$$

Step 7: Computing the Integrals

First integral:

$$
J_1 = \int_{0}^{3} (4 + y)^{3/2} \, dy
$$

Set:

$$
v = 4 + y \Rightarrow dv = dy
$$

When $y=0$, $v=4$; when $y=3$, $v=7$:

$$
J_1 = \int_{v=4}^{7} v^{3/2} \, dv = \int_{4}^{7} v^{3/2} dv
$$

Integral:

$$
\int v^{3/2} dv = \frac{2}{5} v^{5/2}
$$

Thus,

$$
J_1 = \left. \frac{2}{5} v^{5/2} \right|_{4}^{7} = \frac{2}{5} \left(7^{5/2} - 4^{5/2} \right)
$$

Second integral:

$$
J_2 = \int_{0}^{3} (1 + y)^{3/2} dy
$$

Set:

\[

$$
w = 1 + y \Rightarrow dw = dy
$$

When $y=0$, $w=1$; when $y=3$, $w=4$:

$$
J_2 = \int_{1}^{4} w^{3/2} dw = \frac{2}{5} \left(4^{5/2} - 1^{5/2} \right)
$$

Step 8: Final Calculation

Putting everything together:

$$
S = \frac{2}{3} \left[\frac{2}{5} \left(7^{5/2} - 4^{5/2} \right) - \frac{2}{5} \left(4^{5/2} - 1^{5/2} \right) \right]
$$

Factor out $\frac{2}{5}$:

$$
S = \frac{2}{3} \times \frac{2}{5} \left[(7^{5/2} - 4^{5/2}) - (4^{5/2} - 1^{5/2}) \right]
$$

Simplify:

$$
S = \frac{4}{15} \left[7^{5/2} - 4^{5/2} - 4^{5/2} + 1^{5/2} \right]
$$

\[

S = \frac{4}{15} \left[7^{5/2} - 2 \

Find The Area Of The Graph Of The Functionf X Y 2 3 X3 2 Y3 2 That Lies Over The Domain 0 3 0

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