Find The Area Of The Region That Lies Inside The First Curve And Outside The Second Curve.r = 11cos ,

Find The Area Of The Region That Lies Inside The First Curve And Outside The Second Curve: r = 11cos θ

Understanding the area enclosed between two polar curves is a fundamental problem in calculus and analytical geometry. When the curves are expressed in polar coordinates, the process involves setting up integrals based on the regions they define. In this article, we focus on a specific problem: finding the area of the region that lies inside the first curve and outside the second curve, where the second curve is given by $r = 11 \cos \theta$. This task requires careful analysis of the curves, intersection points, and the integration limits.

Understanding the Curves Involved

1. Description of the Second Curve: $r = 11 \cos \theta$

The curve $r = 11 \cos \theta$ is a well-known type of polar graph called a lemniscate or a circle that is symmetric about the polar axis. Here are key points:

- Type of curve: It is a circle with a radius of 5.5 units, offset along the x-axis.
- Shape and symmetry: Since r depends on $\cos \theta$, the curve is symmetric about the polar axis.
- Key features:

- At $\theta = 0$:

$$
r = 11 \times \cos 0 = 11 \times 1 = 11
$$

- At $\theta = \pi/2$:

$$
r = 11 \times \cos \frac{\pi}{2} = 0
$$

$$

- At $\theta = \pi$:

$$
r = 11 \times \cos \pi = -11
$$

Since negative r values in polar coordinates mean the point is in the opposite direction, the curve extends into the negative r domain, indicating that the curve crosses the origin and extends into the other quadrants.

Graphical interpretation: The curve forms a circle with diameter 11, centered at $(x, y) = (5.5, 0)$.

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2. The First Curve: Clarification Needed

The statement indicates "the first curve," but does not specify its equation. To proceed, we assume the first curve is another polar curve, potentially a circle or a different conic. For the purpose of this analysis, let's consider two common scenarios:

- Scenario A: The first curve is a circle centered at the origin with radius R, expressed as $r = R$.

- Scenario B: The first curve is a more complex curve, such as $r = a \sin \theta$ or $r = a \tan \theta$.

For clarity, we'll assume the first curve is the circle $r = R$, where R is a positive constant.

Determining the Region of Interest

1. The Region Inside the First Curve

- If the first curve is $r = R$, the interior region is all points with $r \leq R$.

2. The Region Outside the Second Curve

- The second curve $r = 11 \cos \theta$ defines a boundary; points with $r \geq 11 \cos \theta$ are outside the second curve.

3. The Desired Region

- The region inside the first curve and outside the second curve is the set of points satisfying:

$$
r \leq R \quad \text{and} \quad r \geq 11 \cos \theta
$$

- Graphically, this is the overlapping region where the circle $r = R$ encloses points that are outside the lemniscate $r = 11 \cos \theta$.

Finding Intersection Points of the Curves

1. Setting $r = R$ and $r = 11 \cos \theta$ equal

- To find the boundaries of the region, solve:

$$
R = 11 \cos \theta
$$

- This gives the values of θ where the circle and the lemniscate intersect.

2. Solving for θ:

$$
\cos \theta = \frac{R}{11}
$$

- Case 1: When $|R/11| \le 1$, real solutions exist.

- Case 2: When $|R/11| > 1$, the curves do not intersect, and the region's nature differs.

Assuming $R \le 11$, the intersection points are at:

$$
\theta = \pm \arccos \left(\frac{R}{11} \right)
$$

- Denote:

$$
\theta_1 = - \arccos \left(\frac{R}{11} \right), \quad \theta_2 = \arccos \left(\frac{R}{11} \right)
$$

Calculating the Area of the Region

1. General Approach in Polar Coordinates

- The area A enclosed between two curves $r = r_1(\theta)$ and $r = r_2(\theta)$ over $\theta \in [\alpha, \beta]$ is:

$$
A = \frac{1}{2} \int_{\alpha}^{\beta} \left[r_1^2(\theta) - r_2^2(\theta) \right] d\theta
$$

- In our case:

- $r_{\text{outer}}(\theta) = R$ (the first curve, assumed to be a circle)

- $r_{\text{inner}}(\theta) = 11 \cos \theta$ (second curve)

- The region of interest is where:

$$
11 \cos \theta \leq r \leq R
$$

- This occurs between the intersection points θ_1 and θ_2.

2. Setting Up the Integral

- The area of the region inside the first circle and outside the lemniscate is:

$$
A = \frac{1}{2} \int_{\theta_1}^{\theta_2} \left[R^2 - (11 \cos \theta)^2 \right] d\theta
$$

- The integral simplifies to:

$$
A = \frac{1}{2} \int_{-\arccos(R/11)}^{\arccos(R/11)} \left[R^2 - 121 \cos^2 \theta \right] d\theta
$$

Evaluating the Integral

1. Breaking Down the Integral

$$
A = \frac{1}{2} \left[R^2 \int_{-\arccos(R/11)}^{\arccos(R/11)} d\theta - 121 \int_{-\arccos(R/11)}^{\arccos(R/11)} \cos^2 \theta \, d\theta \right]
$$

- The first integral:

$$
\int_{-\arccos(R/11)}^{\arccos(R/11)} d\theta = 2 \arccos \left(\frac{R}{11} \right)
$$

- The second integral involves $\cos^2 \theta$, which can be integrated using the power-reduction formula:

$$
\cos^2 \theta = \frac{1 + \cos 2\theta}{2}
$$

2. Computing the $\cos^2 \theta$ integral

\[
\int \cos^2 \theta \, d\theta = \frac{1}{2} \int (1 + \cos 2\theta) d\theta =
\frac{1}{2} \left[\theta + \frac{1}{2} \sin 2\theta \right] + C
\]

- Applying limits:

\[
\int_{-\arccos(R/11)}^{\arccos(R/11)} \cos^2 \theta \, d\theta = \left.
\frac{1}{2} \left[\theta + \frac{1}{2} \sin 2\theta \right] \right|_{-
\arccos(R/11)}^{\arccos(R/11)}
\]

- Since \(\sin 2\theta \) is an odd function:

\[
\sin 2\theta \big|_{-\arccos(R/11)}^{\arccos(R/11)} = \sin 2 \arccos(R/11) -
\sin(-2 \arccos(R/11)) = 2 \sin 2 \arccos(R/11)
\]

- But:

\[
\sin 2 \arccos x = 2 x \sqrt{1 - x

Frequently Asked Questions

How do you interpret the problem of finding the area inside one polar curve and outside another?

It involves calculating the region bounded by the first curve, here r = 11cos(θ), and excluding the area occupied by the second curve, requiring identifying their intersection points and setting up appropriate integrals.

What is the significance of the curves r = 11cos(θ) in polar coordinates?

The curve r = 11cos(θ) represents a circle with a radius of 11/2 centered at (5.5, 0) in Cartesian coordinates, symmetric about the polar axis.

How do you find the points of intersection between the two curves?

You set the equations equal to each other or analyze the given second curve to find θ values where both curves meet, which helps determine the limits of integration for the area calculation.

What is the general approach to compute the area inside one polar curve but outside another?

Identify the intersection points, determine the relevant θ intervals, and integrate the difference of the squared radii divided by 2 over those intervals to find the desired area.

Can you provide the formula for calculating the area between two polar curves?

Yes, the area is given by ∫_{a}^{b} rac{1}{2} [r_1(heta)^2 - r_2(heta)^2] d heta, where r_1 and r_2 are the curves, and [a, b] are the limits between intersection points.

How does symmetry of the curves simplify the area calculation?

If the curves are symmetric about certain axes, you can compute the area over a portion of the curve and multiply accordingly, reducing the integral complexity.

What are common challenges when solving these types of area problems in polar coordinates?

Challenges include accurately finding intersection points, setting correct limits of integration, and correctly handling the regions where one curve lies outside the other, especially when curves intersect multiple times.

Additional Resources

Understanding the Problem: Finding the Area of the Region Inside the First Curve and Outside the Second Curve

When delving into the fascinating world of polar coordinates and curve analysis, a common and intriguing problem involves determining the area of a specific region defined by two curves. Specifically, the problem statement "Find the area of the region that lies inside the first curve and outside the second curve" invites a comprehensive exploration of how to approach such a task systematically. Here, the focus is on the curves described by the equation:

$$ r = 11 \cos \theta $$

While the problem mentions "the first curve" and "the second curve," it appears that only one explicit curve is provided. For completeness and clarity, I will assume a typical scenario where two curves are involved—possibly the given curve and another standard or derived curve—and explore methods to determine the specified area.

Deciphering the Given Curve: $r = 11 \cos \theta$

Understanding the Nature of the Curve

The equation $r = 11 \cos \theta$ represents a well-known polar curve called a lemniscate or limacon depending on the parameters. In this case, since r depends linearly on $\cos \theta$, it describes a limacon with specific features.

- Shape and Symmetry:
The curve is symmetric with respect to the polar axis (the line $\theta = 0$). When $\theta = 0$, $r = 11 \times 1 = 11$.
When $\theta = \pi$, $r = 11 \times (-1) = -11$, which in polar coordinates indicates the point $r = 11$ at $\theta = \pi$, but with the negative radius, the point is plotted in the opposite direction, effectively mirroring the curve across the origin.

- Key features:
- Maximum radius: $r_{max} = 11$ at $\theta = 0$.
- The curve passes through the origin when $r = 0$, i.e., when $\cos \theta = 0$ (i.e., at $\theta = \pi/2, 3\pi/2$).
- The shape resembles an elongated circle or dimpled limacon.

Plotting the Curve

Understanding the shape visually helps in grasping the area calculations:

- At $\theta = 0$, $r = 11$.
- At $\theta = \pi/2$, $r = 0$.
- At $\theta = \pi$, $r = -11$ (which corresponds to $r = 11$ at $\theta = \pi$ but in the opposite direction).
- The curve is symmetric with respect to the polar axis.

Identifying the Second Curve and the Region of Interest

Given the original problem's phrasing, a second curve is implied, but not explicitly provided. Common scenarios include:

- Comparing the given curve with a circle or another limacon.
- Finding the area inside $r = 11 \cos \theta$ and outside a circle of a certain radius.
- Or considering the same limacon and subtracting overlapping regions.

Assumption for clarity:
Let's assume the second curve is a circle of radius R, centered at the origin, i.e.,

$$r = R$$

This is a typical scenario where the goal is to find the area of the region that is inside the limacon $r = 11 \cos \theta$ and outside the circle $r = R$.

Mathematical Approach to the Area Calculation

General Formula for Area in Polar Coordinates

The area A enclosed by a polar curve $r = f(\theta)$ between angles $\theta = a$ and $\theta = b$ is given by:

$$A = \frac{1}{2} \int_{a}^{b} [f(\theta)]^2 d\theta$$

When dealing with two curves, the area of the region bounded by them involves integrating the difference of their squared radii over the relevant range:

$$\text{Area} = \frac{1}{2} \int_{\theta_1}^{\theta_2} \left([r_{outer}(\theta)]^2 - [r_{inner}(\theta)]^2 \right) d\theta$$

where:

- $r_{outer}(\theta)$ is the larger radius at each θ,
- $r_{inner}(\theta)$ is the smaller radius at each θ.

Determining the Intersection Points

To find the region that lies inside the first curve and outside the second, we need to:

1. Find the intersection points of the two curves by solving:

$$ 11 \cos \theta = R $$

2. Determine the angles θ where the two curves intersect:

$$ \cos \theta = \frac{R}{11} $$

Assuming $R \leq 11$, solutions exist at:

$$ \theta = \pm \arccos \left(\frac{R}{11} \right) $$

These points mark the boundaries of the region of interest.

Constructing the Region and Setting Up the Integral

Visualizing the Region

- For θ between $-\arccos \left(\frac{R}{11} \right)$ and $\arccos \left(\frac{R}{11} \right)$, the limacon $r = 11 \cos \theta$ is outside the circle $r = R$, i.e., it has larger radius.
- Outside the circle $r=R$ but inside the limacon corresponds to the angular region where:

$$ r_{limacon} (\theta) \ge R $$

- The region of interest is therefore bounded between the intersection points, where the limacon's radius exceeds R and outside the circle.

Formulating the Area Integral

The area of the region inside the limacon and outside the circle is obtained by integrating over the relevant angular range:

$$ \boxed{
A = \frac{1}{2} \int_{-\arccos(R/11)}^{\arccos(R/11)} \left([11 \cos \theta]^2 - R^2 \right) d\theta
} $$

This integral accounts for the difference in areas contributed by the two curves over the angular interval where the limacon is outside the circle.

Calculating the Integral Step-by-Step

Simplify the Expression

$$
A = \frac{1}{2} \int_{-\arccos(R/11)}^{\arccos(R/11)} \left(121 \cos^2 \theta - R^2 \right) d\theta
$$

Because the integrand involves $\cos^2 \theta$, which is an even function, and the limits are symmetric, the integral simplifies:

$$
A = \int_{0}^{\arccos(R/11)} \left(121 \cos^2 \theta - R^2 \right) d\theta
$$

since the integral from $-a$ to a of an even function is twice the integral from 0 to a.

Adjusted integral:

$$
A = 2 \times \frac{1}{2} \int_{0}^{\arccos(R/11)} \left(121 \cos^2 \theta - R^2 \right) d\theta = \int_{0}^{\arccos(R/11)} \left(121 \cos^2 \theta - R^2 \right) d\theta
$$

Evaluating the Integral

Break the integral into two parts:

$$
A = 121 \int_{0}^{\arccos(R/11)} \cos^2 \theta \, d\theta - R^2
$$

```
\int_{0}^{\arccos(R/11)} d\theta
\]

Use the identity:

\[
\cos^2 \theta = \frac{1 + \cos 2\theta}{2}
\]

So,

\[
A = 121 \times \frac{1}{2} \int_{0}^{\arccos(R/11)} (1 + \cos 2\theta)
d\theta - R^2 \times \arccos(R/11)
\]

Compute each integral:

1. \(\int_{0}^{a} 1 \, d\theta = a\)

2. \(\int_{0}^{a} \cos 2\theta \, d\theta = \frac{1}{2} \sin 2\theta
\Big|_{0}^{a} = \frac{1}{2} \sin 2a\)

Now, plugging back:

\[
A = \frac{121}{2} \left[ a +
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