Find The Centroid Of The Region Bounded By The Graphs Of The Given Equations. Y = X(16x^2), Y=0, X=4,

Find The Centroid Of The Region Bounded By The Graphs Of The Given Equations. Y = X(16x^2), Y=0, X=4, is a classic problem in calculus and analytical geometry that involves determining the centroid (or geometric center) of a specific region bounded by given curves and lines. Calculating the centroid of a plane region is fundamental in various fields such as engineering, physics, and architecture, as it helps in understanding the distribution of area and the balance point of the shape.

In this article, we will explore the process step-by-step to find the centroid of the region enclosed by the parabola $y = x(16x^2)$, the x-axis $y = 0$, and the vertical line $x = 4$. The detailed approach involves understanding the region, setting up the necessary integrals, performing calculations, and interpreting the results.

Understanding the Region Bounded by the Equations

Before diving into the calculations, it’s essential to comprehend the nature of the region described by the equations:

- Equation 1: $y = x(16x^2) = 16x^3$ — a cubic parabola opening upwards.
- Equation 2: $y = 0$ — the x-axis, serving as the lower boundary.
- Equation 3: $x = 4$ — a vertical boundary that limits the region in the x-direction.

Visualizing the Region

The curve $y = 16x^3$ intersects the x-axis at $x=0$ since $y=0$ when $x=0$. For $x > 0$, the cubic parabola rises steeply due to the cubic term. The vertical line $x=4$ cuts off the region at $x=4$.

Thus, the region is bounded:

- Below by $y=0$,
- On the right by $x=4$,
- On the left by $x=0$,
- Above by the curve $y=16x^3$.

Domain of Integration

The region of interest spans:

$$x \in [0, 4], \quad y \in [0, 16x^3]$$

Mathematical Foundations for Finding the Centroid

The centroid $(\bar{x}, \bar{y})$ of a region in the xy-plane is given by the formulas:

$$
\bar{x} = \frac{1}{A} \iint_R x \, dA
$$
$$
\bar{y} = \frac{1}{A} \iint_R y \, dA
$$

where A is the total area of the region:

$$
A = \iint_R dA
$$

For regions bounded by functions where the integration limits are easier in x, it’s convenient to set up the integrals as:

$$
A = \int_{x=a}^{b} \left(y_{top} - y_{bottom} \right) dx
$$

Similarly:

$$
\iint_R x \, dA = \int_{a}^{b} x \left(y_{top} - y_{bottom} \right) dx
$$

$$
\iint_R y \, dA = \int_{a}^{b} \frac{1}{2} \left(y_{top}^2 - y_{bottom}^2 \right) dx
$$

Alternatively, for y-coordinate, integrating y over the region:

$$
\bar{y} = \frac{1}{A} \int_{a}^{b} \left(\frac{1}{2} \left(y_{top}^2 - y_{bottom}^2 \right) \right) dx
$$

Calculating the Area of the Region

Step 1: Set up the integral for the area A:

$$
A = \int_{x=0}^{4} \left(y_{top} - y_{bottom} \right) dx
$$

Given:

- $y_{top} = 16x^3$,
- $y_{bottom} = 0$.

So:

$$
A = \int_{0}^{4} 16x^3 \, dx
$$

Step 2: Perform the integration:

$$
A = 16 \int_{0}^{4} x^3 \, dx = 16 \left[\frac{x^4}{4} \right]_0^4 = 16 \times \left(\frac{4^4}{4} \right)
$$

Calculate 4^4:

$$
4^4 = 4 \times 4 \times 4 \times 4 = 256
$$

Therefore:

$$
A = 16 \times \frac{256}{4} = 16 \times 64 = 1024
$$

Result: The area of the region is 1024 square units.

Finding the x-coordinate of the Centroid ($\bar{x}$)

Step 1: Set up the integral:

$$
\bar{x} = \frac{1}{A} \int_{0}^{4} x \times (y_{top} - y_{bottom}) dx = \frac{1}{1024}
$$

\int_{0}^{4} x \times 16x^3 \, dx
\]

Simplify the integrand:

\[
x \times 16x^3 = 16x^4
\]

Step 2: Calculate the integral:

\[
\bar{x} = \frac{1}{1024} \times 16 \int_{0}^{4} x^4 \, dx = \frac{16}{1024} \times \left[
\frac{x^5}{5} \right]_0^4
\]

Simplify the coefficient:

\[
\frac{16}{1024} = \frac{1}{64}
\]

Calculate the integral:

\[
\left[\frac{x^5}{5} \right]_0^4 = \frac{4^5}{5} = \frac{1024}{5}
\]

Putting it all together:

\[
\bar{x} = \frac{1}{64} \times \frac{1024}{5} = \frac{1024}{320} = \frac{1024}{320}
\]

Simplify:

\[
\frac{1024}{320} = \frac{1024/32}{320/32} = \frac{32}{10} = \frac{16}{5} = 3.2
\]

Result: The x-coordinate of the centroid is \(\bar{x} = 3.2\) units.

Finding the y-coordinate of the Centroid (\(\bar{y}\))

Step 1: Set up the integral:

\[

$$
\bar{y} = \frac{1}{A} \int_{0}^{4} \frac{1}{2} \left(y_{top}^2 - y_{bottom}^2 \right) dx
$$

Since $y_{bottom} = 0$, this simplifies to:

$$
\bar{y} = \frac{1}{2A} \int_0^4 y_{top}^2 dx
$$

Recall $y_{top} = 16x^3$, so:

$$
y_{top}^2 = (16x^3)^2 = 256 x^6
$$

Step 2: Calculate the integral:

$$
\bar{y} = \frac{1}{2 \times 1024} \int_0^4 256 x^6 dx = \frac{1}{2048} \times 256 \int_0^4 x^6 dx
$$

Simplify the coefficient:

$$
\frac{256}{2048} = \frac{1}{8}
$$

Calculate the integral:

$$
\int_0^4 x^6 dx = \left[\frac{x^7}{7} \right]_0^4 = \frac{4^7}{7}
$$

Calculate 4^7:

$$
4^7 = 4^{4} \times 4^{3} = 256 \times 64 = 16,384
$$

Thus:

$$
\bar{y} = \frac{1}{8} \times \frac{16,384}{7} = \frac{16,384}{56}
$$

Simplify:

$$
\frac{16,384}{56} = \frac{16,384/8}{56/8} = \frac{2048}{7}
$$

Finally, divide by $A=1024$:

$$
\bar{y} = \frac{1}{1024} \times \frac{2048}{7} = \frac{2048}{1024 \times 7} = \frac{2}{7}
$$

Result: The y-coordinate of the centroid is $\bar{y} = \frac{2}{7} \approx$

Frequently Asked Questions

What is the first step to find the centroid of the region bounded by the curves Y = X(16x^2), Y=0, and X=4?

The first step is to identify the region's boundaries and determine the limits of integration, which in this case are from x=0 to x=4, and then set up the integrals for the area and centroid coordinates.

How do you find the area of the region bounded by Y = X(16x^2), Y=0, and X=4?

The area is found by integrating the function Y = 16x^3 from x=0 to x=4: Area = $\int_0^4$ 16x^3 dx.

What is the formula to determine the x-coordinate of the centroid for this region?

The x-coordinate of the centroid, $\bar{x}$, is given by (1/Area) $\int_0^4$ x y dx, where y = 16x^3.

How do you compute the y-coordinate of the centroid for the bounded region?

The y-coordinate, $\bar{y}$, is calculated using $\bar{y}$ = (1/2A) $\int_0^4$ y^2 dx, where y = 16x^3, considering the centroid formula for regions bounded by curves.

Why is it necessary to set up integrals for both area and centroid coordinates when finding the centroid?

Because the centroid's coordinates depend on the shape's moments and area, which are obtained through integrals of the region's geometry; setting up these integrals ensures accurate calculation of the centroid position.

What is the significance of the bounds x=0 and x=4 in this problem?

These bounds define the interval over which the region is bounded and are essential for setting up the definite integrals to compute area and centroid coordinates accurately.

Additional Resources

Find The Centroid Of The Region Bounded By The Graphs Of The Given Equations. Y = X(16x^2), Y=0, X=4

Understanding the centroid of a region is a foundational concept in calculus and geometry, with applications spanning engineering, physics, architecture, and beyond. The centroid, often called the geometric center or center of mass (assuming uniform density), provides insights into the balance point of a shape. When dealing with irregular regions bounded by curves and lines, finding this point requires a blend of integration techniques and geometric reasoning.

In this article, we delve into a specific problem: determining the centroid of the region bounded by the curves $y = x(16x^2)$, $y = 0$, and the vertical line $x = 4$. This problem not only illustrates core calculus concepts but also offers a window into the practical process of analyzing complex regions.

Understanding the Region and Its Boundaries

Before diving into calculations, it’s crucial to grasp the geometric shape and boundaries of the region in question.

The Curves and Lines Involved

- The parabola $y = x(16x^2)$: Simplifying, $y = 16x^3$, which describes a cubic curve passing through the origin, with its shape influenced by the cubic function's behavior.
- The line $y=0$: The x-axis, serving as the lower boundary.
- The vertical line $x=4$: The right boundary of the region.

Note: The problem does not specify any other boundary on the left, but typically, the region of interest is from $x=0$ to $x=4$, assuming the region is bounded by the curve, the x-axis, and the vertical line at $x=4$.

Visualizing the Region

Imagine plotting the cubic curve $y=16x^3$ starting at the origin, rising as x increases, and then drawing a vertical line at $x=4$. The region bounded between these curves from $x=0$ to $x=4$ constitutes our area of interest. Since $y=0$ is the lower boundary, the region lies above the x-axis.

Setting Up the Mathematical Framework

Calculating the centroid involves determining its coordinates ($\bar{x}$, $\bar{y}$), which are given by the formulas:

- $\bar{x} = (1/A) \int x \, dA$
- $\bar{y} = (1/2A) \int y^2 \, dx$

where A is the area of the region.

For regions bounded by curves y=f(x) and y=g(x), the area and centroid calculations reduce to integrals over x, assuming the region is "well-behaved" (i.e., bounded and integrable).

Calculating the Area of the Region

The first step is to compute the total area (A), which is essential for finding the centroid.

Expressing the Area Integral

Since the region is bounded above by y=16x^3 and below by y=0, the area from x=0 to x=4 is:

$A = \int_0^4$ [upper curve - lower curve] dx
$A = \int_0^4 16x^3 \, dx$

Performing the Integration

Calculating:

$A = \int_0^4 16x^3 \, dx$
$= 16 \int_0^4 x^3 \, dx$
= 16 [x^4 / 4] evaluated from 0 to 4
= 16 [(4^4) / 4 - 0]
= 16 [(256) / 4]
= 16 64
= 1024

Result: The area of the region is 1024 square units.

Finding the x-Coordinate of the Centroid ($\bar{x}$)

The x-coordinate of the centroid ($\bar{x}$) is calculated by:

$\bar{x} = (1/A) \int_0^4 x$ (upper curve - lower curve) dx

Since the lower boundary is y=0, this simplifies to:

$\bar{x} = (1/A) \int_0^4 x\ 16x^3\ dx$
$= (1/1024) \int_0^4 16x^4\ dx$

Executing the Integral

Calculating:

$\int_0^4 16x^4\ dx = 16 \int_0^4 x^4\ dx$
$= 16\ [x^5 / 5]$ from 0 to 4
$= 16\ [(4^5) / 5 - 0]$
$= 16\ [1024 / 5]$
$= (16\ 1024) / 5$
$= (16\ 1024) / 5$

Simplify numerator:

16 1024 = 16 (1024) = 16,384

Thus,

$\bar{x} = (1/1024)\ (16{,}384 / 5)$
$= (16{,}384 / 1024)\ (1/5)$
$= 16\ (1/5)$
$= 16/5$
$= 3.2$

Result: The x-coordinate of the centroid is $\bar{x} = 3.2$ units.

Calculating the y-Coordinate of the Centroid ($\bar{y}$)

The y-coordinate involves integrating y^2 over the region:

$\bar{y} = (1 / (2A)) \int_0^4 [f(x)]^2\ dx$

Given $y = 16x^3$, then $y^2 = (16x^3)^2 = 256x^6$.

Therefore:

$\bar{y} = (1 / (2\ 1024)) \int_0^4 256x^6\, dx$
$= (1 / 2048)\ 256 \int_0^4 x^6\, dx$

Simplify constants:

$(256 / 2048) = 1 / 8$

Now,

$\bar{y} = (1/8) \int_0^4 x^6\, dx$

Calculate the integral:

$\int_0^4 x^6\, dx = [x^7 / 7]$ evaluated from 0 to 4 $= (4^7) / 7$

Calculate 4^7:

$4^7 = 4^4\ 4^3 = (256)\ (64) = 16{,}384$

Therefore:

$\int_0^4 x^6\, dx = 16{,}384 / 7$

Plug back into $\bar{y}$:

$\bar{y} = (1/8)\ (16{,}384 / 7) = (16{,}384) / (8\ 7) = 16{,}384 / 56$

Simplify numerator and denominator:

Divide numerator and denominator by 8:

$(16{,}384 / 8) = 2{,}048$

Then,

$\bar{y} = 2{,}048 / 7 \approx 292.57$

Result: The y-coordinate of the centroid is approximately $\bar{y} \approx 292.57$ units.

Summary of Results and Interpretation

- Area of the region: 1024 square units
- x-coordinate of the centroid ($\bar{x}$): 3.2 units
- y-coordinate of the centroid ($\bar{y}$): approximately 292.57 units

These coordinates give a precise location of the centroid relative to the axes. Notably, the x-coordinate is close to the right boundary at x=4, indicating the region's centroid is skewed towards the larger x-values due to the cubic growth of y. Meanwhile, the high y-coordinate reflects the significant vertical extent of the region, driven by the cubic function.

Applications and Significance of the Centroid

Determining the centroid of a region is more than an academic exercise; it has practical implications:

- Structural Engineering: The centroid indicates where a load or force may be balanced, critical in designing stable structures.
- Physics: The center of mass for a uniform lamina corresponds to the centroid, influencing motion and stability analyses.
- Manufacturing: In materials science and manufacturing, understanding the centroid helps in material distribution and design optimization.
- Computational Geometry: Algorithms for computer graphics and modeling often rely on centroid calculations for object positioning and manipulation.

Conclusion

The process of finding the centroid of a region bounded by curves and lines combines geometric visualization with integral calculus. In this particular problem, the key steps involved calculating the area, then applying the centroid formulas to determine the coordinates. The region bounded by y=16x^3, y=0, and x=4 exhibits a skewed centroid towards larger x-values and high y-values, reflecting its shape and extent.

This exercise exemplifies how calculus provides powerful tools to analyze complex regions, transforming curves and boundaries into meaningful quantitative data. Whether applied in engineering, physics, or design, understanding the centroid is fundamental for balancing, stability, and optimization. As mathematical techniques evolve, so too does our capacity to interpret and manipulate the shapes and regions that form the backbone of countless scientific and technological endeavors.

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