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Understanding how to find the coordinate vector of a vector relative to a given basis is fundamental in linear algebra, especially when working with vector spaces like $\mathbb{R}^2$. In this article, we will explore the process step-by-step, using a specific basis $S = (u, U)$ and a vector W. Our goal is to determine the coordinate vector of W relative to the basis S. We will cover the concepts thoroughly, including basis formation, vector representation, and the calculation process, ensuring clarity for learners at all levels.

Introduction to Vector Spaces and Bases in $\mathbb{R}^2$

Before delving into the specific problem, it is essential to understand some foundational concepts.

Vector Spaces and $\mathbb{R}^2$

- $\mathbb{R}^2$ is the set of all ordered pairs of real numbers (x, y).
- It is a vector space, meaning it allows vector addition and scalar multiplication.

What Is a Basis?

- A basis of $\mathbb{R}^2$ is a set of two vectors $\{u, U\}$ that are linearly independent and span the entire space.
- Any vector $W \in \mathbb{R}^2$ can be expressed uniquely as a linear combination of the basis vectors.

Coordinate Vector Relative to a Basis

- The coordinate vector of W relative to the basis $S = (u, U)$ is the ordered pair (c_1, c_2) such that:

$$
W = c_1 u + c_2 U
$$

- Finding this coordinate vector involves solving a system of linear equations.

Given Data and Problem Statement

Let's clarify the given vectors and the task at hand.

Vectors Provided

- $u = (a_1, a_2)$
- $U = (4, -3)$
- $W = (w_1, w_2)$

Note: There appears to be a typographical error in the prompt, as both basis vectors are labeled U. Typically, a basis S in $\mathbb{R}^2$ is formed by two distinct vectors, say, u and U.

Assumption: The basis vectors are $u = (a_1, a_2)$ and $U = (4, -3)$. The vector $W = (w_1, w_2)$ is given or needs to be specified.

Step-by-Step Process to Find the Coordinate Vector of W Relative to $S = (u, U)$

The main goal is to find scalars c_1 and c_2 such that:

$$
W = c_1 u + c_2 U
$$

This equation translates into a system of linear equations.

Step 1: Write the System of Equations

Suppose:

$$
u = (a_1, a_2), \quad U = (4, -3), \quad W = (w_1, w_2)
$$

Then:

$$
(w_1, w_2) = c_1 (a_1, a_2) + c_2 (4, -3)
$$

which gives two equations:

$$
w_1 = c_1 a_1 + c_2 \times 4
$$
$$
w_2 = c_1 a_2 + c_2 \times (-3)
$$

Step 2: Set Up the Matrix Equation

Expressing the system in matrix form:

$$
\begin{bmatrix}
a_1 & 4 \\
a_2 & -3
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2
\end{bmatrix}
=
\begin{bmatrix}
w_1 \\
w_2
\end{bmatrix}
$$

Denote:

$$
A = \begin{bmatrix}
a_1 & 4 \\
a_2 & -3
\end{bmatrix}
,\quad
\mathbf{c} = \begin{bmatrix}
c_1 \\
c_2
\end{bmatrix}
,\quad
\mathbf{w} = \begin{bmatrix}
w_1 \\
w_2
\end{bmatrix}
$$

Our task reduces to solving:

$$
A \mathbf{c} = \mathbf{w}
$$

Step 3: Calculate the Determinant of A

The determinant, $\det(A)$, is crucial to determine if the system has a unique solution:

$$
\det(A) = a_1 \times (-3) - a_2 \times 4
$$

- If $\det(A) \neq 0$, the system has a unique solution.

- If $\det(A) = 0$, the vectors are linearly dependent, and W may or may not lie in the span.

Step 4: Find the Inverse of A and Solve for $\mathbf{c}$

Using Cramer's rule or matrix inversion:

$$
\mathbf{c} = A^{-1} \mathbf{w}
$$

The inverse of A (for a 2x2 matrix) is:

$$
A^{-1} = \frac{1}{\det(A)} \begin{bmatrix}
-3 & -4 \\
-a_2 & a_1
\end{bmatrix}
$$

Hence, the coordinate vector:

$$
\begin{bmatrix}
c_1 \\
c_2
\end{bmatrix}
= \frac{1}{\det(A)} \begin{bmatrix}
-3 & -4 \\
-a_2 & a_1
\end{bmatrix}
\begin{bmatrix}
w_1 \\
w_2
\end{bmatrix}
$$

Practical Example with Specific Vectors

To illustrate, let's assume specific values for u and W.

Example Data

- $u = (1, 2)$
- $U = (4, -3)$
- $W = (5, 4)$

Step 1: Set Up the System

$$
\begin{bmatrix}
1 & 4 \\
2 & -3
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2
\end{bmatrix}
=
\begin{bmatrix}
5 \\
4
\end{bmatrix}
$$

Step 2: Compute the Determinant

$$
\det(A) = (1)(-3) - (2)(4) = -3 - 8 = -11
$$

Since $-11 \neq 0$, the system has a unique solution.

Step 3: Find A^{-1}

$$
A^{-1} = \frac{1}{-11} \begin{bmatrix}
-3 & -4 \\
-2 & 1
\end{bmatrix}
= \begin{bmatrix}
3/11 & 4/11 \\
2/11 & -1/11
\end{bmatrix}
$$

Step 4: Calculate the Coordinates

$$
\mathbf{c} = A^{-1} \mathbf{w} = \begin{bmatrix}
3/11 & 4/11 \\
2/11 & -1/11
\end{bmatrix}
$$

\begin{bmatrix}
5 \\
4
\end{bmatrix}
\]

Calculations:

\[
c_1 = \frac{3}{11} \times 5 + \frac{4}{11} \times 4 = \frac{15}{11} + \frac{16}{11} = \frac{31}{11}
\]

\[
c_2 = \frac{2}{11} \times 5 - \frac{1}{11} \times 4 = \frac{10}{11} - \frac{4}{11} = \frac{6}{11}
\]

Result:

\[
\boxed{
\text{Coordinate vector of } W \text{ relative to } S = \left(\frac{31}{11}, \frac{6}{11} \right)
}
\]

Interpreting the Result and Applications

Knowing the coordinate vector of \(W\) relative to a basis \(S\) allows us to:

- Express \(

Frequently Asked Questions

How do I find the coordinate vector of W relative to the basis S = (U, U)?

To find the coordinate vector of W relative to the basis S = (U, U), express W as a linear combination of U and U (which are the basis vectors). Solve for scalars a and b such that W = aU + bU. The coordinate vector is then (a, b).

Given U = (4, -3), U = (2, 6), and W = ?, how do I compute the coordinate vector of W?

First, clarify whether the basis vectors are U = (4, -3) and V = (2, 6). Then, set up the equation W = aU + bV and solve for a and b using the components of W. The coordinate vector is (a, b).

What is the importance of basis vectors in finding coordinate vectors in R2?

Basis vectors provide a reference frame for expressing other vectors. The coordinate vector of a vector W relative to a basis S = (u, v) gives its unique representation as a linear combination of u and v.

How can I determine if the vectors U and V form a valid basis for R2?

Check if U and V are linearly independent by calculating their determinant or verifying they are not scalar multiples. If they are linearly independent, they form a basis for R2.

If W is given, and the basis vectors are U = (4, -3) and V = (2, 6), how do I find the coordinate vector of W?

Set up the equation W = aU + bV, then solve for a and b using systems of equations based on the components of W. The solutions (a, b) form the coordinate vector of W relative to the basis.

Can I find the coordinate vector of W relative to the basis (U, U) if both basis vectors are the same?

No, if both basis vectors are identical, they do not form a valid basis for R2, and you cannot uniquely express W in terms of that basis. A basis requires linearly independent vectors.

Additional Resources

Coordinate Vector Calculation

Understanding the Problem: Finding the Coordinate Vector of W Relative to Basis S = (U, U)

When working with vectors in a two-dimensional space $\mathbb{R}^2$, a fundamental task is expressing a given vector W in terms of a basis $S = (U, V)$. This process involves finding the coordinate vector of W relative to the basis S, which essentially describes how W can be represented as a linear combination of the basis vectors.

In this particular scenario, the basis S consists of the vectors $U = (4, -3)$ and $U = (2, 6)$. However, note that both basis vectors are labeled with the same letter U, which might be a typographical oversight. Typically, a basis in $\mathbb{R}^2$ is formed by two distinct vectors, often denoted as U and V. For clarity, we will assume that the basis vectors are:

$$
U = (4, -3), \quad V = (2, 6)
$$

and that the vector W is given as some specific vector in $\mathbb{R}^2$. Since the problem statement is incomplete regarding W, we will proceed with the general approach to find the coordinate vector of an arbitrary $W = (w_1, w_2)$ relative to the basis $S = (U, V)$. If a specific W is provided later, the process remains the same.

The Significance of Coordinate Vectors in $\mathbb{R}^2$

Why Do We Need Coordinate Vectors?

Coordinate vectors provide a powerful way to represent vectors relative to a particular basis. They serve as the "coordinates" or "weights" that tell us how to reconstruct the original vector from the basis vectors. This is crucial for:

- Simplifying vector operations by working in a basis where the vectors have a simple form
- Changing coordinate systems (basis transformations)

- Solving systems of linear equations
- Understanding the structure of vector spaces

The Concept of a Basis

A basis in $\mathbb{R}^2$ is a set of two vectors $\{U, V\}$ that are linearly independent, meaning neither can be written as a scalar multiple of the other. Every vector W in $\mathbb{R}^2$ can then be uniquely expressed as a linear combination:

$$
W = c_1 U + c_2 V
$$

where c_1, c_2 are the coordinates of W relative to the basis S.

Step-by-Step Approach to Find the Coordinate Vector

1. Confirm the Basis Vectors Are Linearly Independent

Before proceeding, verify that $U = (4, -3)$ and $V = (2, 6)$ form a basis by checking their linear independence.

Method:

Calculate the determinant of the matrix formed by placing U and V as columns:

$$
\det \begin{bmatrix}
4 & 2 \\
-3 & 6
\end{bmatrix} = (4)(6) - (-3)(2) = 24 + 6 = 30
$$

Since the determinant is non-zero ($30 \neq 0$), the vectors are linearly independent and hence form a basis.

---

2. Express W as a Linear Combination of U and V

Suppose $W = (w_1, w_2)$. Then, the goal is to find scalars c_1, c_2 such that:

$$
W = c_1 U + c_2 V
$$

which gives the system of equations:

$$
\begin{cases}
w_1 = 4 c_1 + 2 c_2 \\
w_2 = -3 c_1 + 6 c_2
\end{cases}
$$

3. Solve the System of Linear Equations

To find c_1 and c_2, we use methods such as substitution or matrix algebra.

Matrix form:

$$
\begin{bmatrix}
4 & 2 \\
-3 & 6
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2
\end{bmatrix}
=
\begin{bmatrix}
w_1 \\
w_2
\end{bmatrix}
$$

The solution involves inverting the coefficient matrix or using Cramer's rule:

$$
\mathbf{C} = \mathbf{A}^{-1} \mathbf{W}
$$

where:

$$
\mathbf{A} = \begin{bmatrix}
4 & 2 \\
-3 & 6
\end{bmatrix}
,\quad
\mathbf{W} = \begin{bmatrix}
w_1 \\
w_2
\end{bmatrix}
$$

---

4. Computing the Inverse of the Coefficient Matrix

Given:

$$
\det \mathbf{A} = 30
$$

the inverse of $\mathbf{A}$ is:

$$
\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{bmatrix}
6 & -2 \\
3 & 4
\end{bmatrix}
= \frac{1}{30} \begin{bmatrix}
6 & -2 \\
3 & 4
\end{bmatrix}
$$

\]

Thus,

$$
\begin{bmatrix}
c_1 \\
c_2
\end{bmatrix}
= \frac{1}{30} \begin{bmatrix}
6 & -2 \\
3 & 4
\end{bmatrix}
\begin{bmatrix}
w_1 \\
w_2
\end{bmatrix}
$$

or explicitly,

$$
c_1 = \frac{1}{30}(6 w_1 - 2 w_2), \quad c_2 = \frac{1}{30}(3 w_1 + 4 w_2)
$$

Practical Example: Computing the Coordinate Vector for a Specific W

Suppose the vector $W = (5, 7)$. Let's compute its coordinate vector relative to the basis $S = (U, V)$.

Step 1: Write the system:

$$
\begin{cases}
5 = 4 c_1 + 2 c_2 \\
7 = -3 c_1 + 6 c_2
\end{cases}
$$

Step 2: Compute c_1:

$$
c_1 = \frac{1}{30}(6 \times 5 - 2 \times 7) = \frac{1}{30}(30 - 14) = \frac{16}{30} = \frac{8}{15}
$$

Step 3: Compute c_2:

$$
c_2 = \frac{1}{30}(3 \times 5 + 4 \times 7) = \frac{1}{30}(15 + 28) = \frac{43}{30}
$$

Result:

$$
\boxed{
\text{Coordinate vector of } W = (5, 7) \text{ relative to } S = \left(\frac{8}{15}, \frac{43}{30} \right)
}
$$

Summary and Key Takeaways

The General Formula

Given basis vectors $U = (u_1, u_2)$ and $V= (v_1, v_2)$ and an arbitrary vector $W = (w_1, w_2)$, the coordinate vector (c_1, c_2) can be computed via:

$$
c_1 = \frac{1}{\det \mathbf{A}} (v_2 w_1 - v_1 w_2)
$$

$$
c_2 = \frac{1}{\det \mathbf{A}} (-u_2 w_1 + u_1 w_2)
$$

where

$$
\det \mathbf{A} = u_1 v_2 - u_2 v_1
$$

This approach leverages the properties of determinants and linear algebra to efficiently determine the coordinates.

Final Thoughts: Why This Matters

Mastering how to find the coordinate vector of a vector relative to a basis is a foundational skill in linear algebra, with applications spanning computer graphics, physics, engineering, and more. Whether transforming vectors between different coordinate systems or simplifying complex linear transformations, understanding this process provides clarity and power in analyzing multidimensional data.

In this exploration, we've demonstrated a systematic approach, verified the basis's validity, and provided a concrete example, all of which serve as a blueprint for tackling similar problems in advanced mathematics and applied sciences.

Find The Coordinate Vector Of W Relative To The Basis S U U For R2 Let U 4 3 U 2 6 W

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