Find The Distance Between The Line With Equation (x1)/2=(y+1)/3=(z2)/5 , And The Line With Parametric

Find The Distance Between The Line With Equation (x1)/2=(y+1)/3=(z2)/5 , And The Line With Parametric is a common problem in three-dimensional coordinate geometry that involves calculating the shortest distance between two skew lines. Understanding how to approach this problem is essential for students and professionals working in fields such as engineering, physics, and computer graphics. In this comprehensive guide, we will explore the methodologies, formulas, and step-by-step procedures to determine the minimum distance between a line given in symmetric form and another line expressed parametrically.

Understanding Lines in 3D Space

Before diving into the calculation techniques, it's essential to understand how lines are represented in three-dimensional space.

Line in Symmetric Form

A line can be expressed in symmetric form as:
$$
\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}
$$
where (x_0, y_0, z_0) is a point on the line, and (a, b, c) is the direction vector of the line.

In the problem statement, the line is given as:
$$
\frac{x - 0}{2} = \frac{y + 1}{3} = \frac{z - 2}{5}
$$
which can be interpreted as passing through the point $A(0, -1, 2)$ with a direction vector $\vec{d}_1 = (2, 3, 5)$.

Line in Parametric Form

A line can also be expressed parametrically as:
$$
\begin{cases}
x = x_1 + t \cdot a' \\
y = y_1 + t \cdot b' \\
z = z_1 + t \cdot c'
\end{cases}

where (x_1, y_1, z_1) is a point on the line, and (a', b', c') is its direction vector.

Suppose the parametric form of the second line is:

$$
\begin{cases}
x = x_2 + s \cdot a'' \\
y = y_2 + s \cdot b'' \\
z = z_2 + s \cdot c''
\end{cases}
$$

with known point (x_2, y_2, z_2) and direction vector $\vec{d}_2 = (a'', b'', c'')$.

Methodology for Calculating the Distance Between Two Skew Lines

If the two lines are skew (neither intersecting nor parallel), the shortest distance between them can be found using vector algebra. The key idea is to find the length of the common perpendicular— the segment perpendicular to both lines.

Step 1: Identify Points and Direction Vectors

- For the first line:
- Point $A = (x_1, y_1, z_1)$
- Direction vector $\vec{d}_1 = (a_1, b_1, c_1)$

- For the second line:
- Point $B = (x_2, y_2, z_2)$
- Direction vector $\vec{d}_2 = (a_2, b_2, c_2)$

In our specific example:

$$
A = (0, -1, 2), \quad \vec{d}_1 = (2, 3, 5)
$$

and assume the parametric form of the second line is:

$$
\begin{cases}
x = x_2 + s \\
y = y_2 + 2s \\
z = z_2 + 3s
\end{cases}
$$

where $B = (x_2, y_2, z_2)$ is known, and $\vec{d}_2 = (1, 2, 3)$.

Step 2: Calculate the Cross Product of Direction Vectors

Compute $\vec{d}_1 \times \vec{d}_2$. This vector is perpendicular to both lines and indicates the direction of the shortest segment connecting the lines.

$$
\vec{n} = \vec{d}_1 \times \vec{d}_2
$$

Step 3: Find a Vector Connecting the Two Lines

Calculate the vector $\vec{AB} = \vec{B} - \vec{A}$, connecting points on each line.

Step 4: Calculate the Distance

The shortest distance D between the skew lines is given by:

$$
D = \frac{|\left(\vec{AB} \cdot \vec{n} \right)|}{|\vec{n}|}
$$

where:
- $\vec{AB} \cdot \vec{n}$ is the scalar (dot) product,
- $|\vec{n}|$ is the magnitude of $\vec{n}$.

Step-by-Step Example Calculation

Let’s work through an example with specific points and vectors:

- First line: $\frac{x}{2} = \frac{y + 1}{3} = \frac{z - 2}{5}$

- Point on line: $A = (0, -1, 2)$
- Direction vector: $\vec{d}_1 = (2, 3, 5)$

- Second line: Suppose in parametric form:

$$
\begin{cases}
x = 1 + s \\
y = 4 + 2s \\
z = -1 + 3s
\end{cases}
$$

- Point on second line: $B = (1, 4, -1)$

- Direction vector: $\vec{d}_2 = (1, 2, 3)$

Calculations:

1. Cross product $\vec{n} = \vec{d}_1 \times \vec{d}_2$:

$$
\vec{n} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 3 & 5 \\
1 & 2 & 3
\end{vmatrix}
= \mathbf{i}(3 \times 3 - 5 \times 2) - \mathbf{j}(2 \times 3 - 5 \times 1) + \mathbf{k}(2 \times 2 - 3 \times 1)
$$

$$
= \mathbf{i}(9 - 10) - \mathbf{j}(6 - 5) + \mathbf{k}(4 - 3) = \mathbf{i}(-1) - \mathbf{j}(1) + \mathbf{k}(1)
$$
$$
\Rightarrow \vec{n} = (-1, -1, 1)
$$

2. Vector $\vec{AB} = B - A = (1 - 0, 4 - (-1), -1 - 2) = (1, 5, -3)$

3. Dot product $\vec{AB} \cdot \vec{n} = (1)(-1) + (5)(-1) + (-3)(1) = -1 - 5 - 3 = -9$

4. Magnitude of $\vec{n}$:

$$
|\vec{n}| = \sqrt{(-1)^2 + (-1)^2 + 1^2} = \sqrt{1 + 1 + 1} = \sqrt{3}
$$

5. Distance D:

$$
D = \frac{| -9 |}{\sqrt{3}} = \frac{9}{\sqrt{3}} = 3 \sqrt{3}
$$

Result:

The shortest distance between the two lines is $3 \sqrt{3}$ units.

Additional Considerations and Special Cases

While the above method is effective for skew lines, it’s important to recognize specific scenarios:

Parallel Lines

If the direction vectors are scalar multiples, the lines are parallel. In such cases, the shortest distance reduces to the distance between a point on one line and the other line.

Intersecting Lines

If the lines intersect, the shortest distance is zero, and the point of intersection can be found by solving the equations simultaneously.

Lines in Different Forms

Sometimes, lines are given in different forms, such as symmetric and parametric simultaneously, necessitating conversion to a common form before applying the formula.

Conclusion

Determining the distance between a line given in symmetric form and another in parametric form involves understanding the geometric relationships and applying vector calculus techniques. The key steps include identifying points and direction vectors, computing the cross product to find a normal vector, and then calculating the scalar projection to find the shortest segment connecting the two lines. This approach is not only fundamental in academic settings but also has

Frequently Asked Questions

How do you find the distance between two skew lines given in symmetric form and parametric form?

To find the distance between skew lines, identify their direction vectors and points on each line, then use the formula: Distance = $|(P2 - P1) \cdot (d1 \times d2)| / |d1 \times d2|$, where P1 and P2 are points on each line, and d1 and d2 are their direction vectors.

What is the direction vector of the line given by

(x1)/2 = (y+1)/3 = (z2)/5?

The direction vector can be extracted from the symmetric equations as (2, 3, 5).

How do you convert the symmetric form (x1)/2 = (y+1)/3 = (z2)/5 into parametric equations?

Set the common ratio as t, then express each coordinate: x = 2t, y = 3t - 1, z = 5t. These are the parametric equations of the line.

How do you find a point on the line given in symmetric form (x1)/2 = (y+1)/3 = (z2)/5?

Choose a value for t, for example t=0: then x=0, y=-1, z=0; so the point (0, -1, 0) lies on the line.

What is the formula to calculate the shortest distance between a line in symmetric form and a line in parametric form?

Identify points and direction vectors on each line, then compute the vector between points, and use the cross product with the direction vectors to find the shortest distance using the scalar projection formula.

Can the distance between two lines be zero? Under what condition?

Yes, the distance is zero if the lines intersect or are coincident; otherwise, they are skew lines with a positive distance.

What is the significance of the cross product of the direction vectors in finding the distance between skew lines?

The cross product of the direction vectors gives a vector perpendicular to both lines, and its magnitude relates to the area spanned, which is used in calculating the shortest distance between the lines.

How do you verify if two lines are parallel, intersecting, or skew?

Compare their direction vectors: if scalar multiples, they are parallel; if they share a common point, they intersect; otherwise, they are skew lines.

Additional Resources

Finding the Distance Between Two Skew Lines: A Comprehensive Guide

Understanding the concept of the shortest distance between two lines in three-dimensional space is fundamental in vector calculus, analytical geometry, and various engineering and physics applications. This detailed review aims to elucidate the process of calculating the minimum distance between a line given in symmetric form and another line expressed in parametric form. We will dissect each step, explore the underlying theory, and provide illustrative examples to cement understanding.

Understanding the Problem Statement

The core problem involves two lines in 3D space:

1. Line 1: Given in symmetric form:

$$
\frac{x_1}{2} = \frac{y + 1}{3} = \frac{z_2}{5}
$$

2. Line 2: Expressed in parametric form:

$$
\text{Line 2:} \quad \mathbf{r} = \mathbf{r}_0 + t \mathbf{d}
$$

where $\mathbf{r}_0$ is a point on the line, $\mathbf{d}$ is the direction vector, and t is a real parameter.

The goal is to find the shortest (perpendicular) distance between these two lines, which is a classic problem involving vector operations, notably the cross product and dot product.

Deciphering the Symmetric Equation of the First Line

The symmetric form of a line in 3D is generally written as:

$$
\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}
$$

\]

In your provided equation:

\[
\frac{x_1}{2} = \frac{y + 1}{3} = \frac{z_2}{5}
\]

we interpret it as:

- The line passes through a point where all expressions are equal to some parameter \(s\), i.e.,

\[
\frac{x_1}{2} = s,\quad \frac{y + 1}{3} = s,\quad \frac{z_2}{5} = s
\]

which leads to parametric equations:

\[
\begin{cases}
x = 2s \\
y = 3s - 1 \\
z = 5s
\end{cases}
\]

Note: The variables \(x_1\) and \(z_2\) seem to be placeholders or typographical errors. Assuming standard notation, the line can be represented as:

\[
x = 2s,\quad y = 3s - 1,\quad z = 5s
\]

Expressing the First Line in Parametric Form

Based on the above, the Line 1 is:

\[
\boxed{
\mathbf{r}_1(s) = (0, -1, 0) + s (2, 3, 5)
}
\]

where:

- Point on the line: $\mathbf{P}_1 = (0, -1, 0)$
- Direction vector: $\mathbf{d}_1 = (2, 3, 5)$

This simplifies calculations and allows us to compare with the second line.

Expressing the Second Line in Parametric Form

Suppose Line 2 is given explicitly in parametric form as:

$$
\mathbf{r}_2(t) = \mathbf{P}_2 + t \mathbf{d}_2
$$

where:

- $\mathbf{P}_2 = (x_0, y_0, z_0)$ is a known point on Line 2
- $\mathbf{d}_2 = (a, b, c)$ is the direction vector of Line 2

Note: The user mentions "with parametric," but does not specify the exact parametric equations. For completeness, assume typical form:

$$
\mathbf{r}_2(t) = (x_0 + a t, y_0 + b t, z_0 + c t)
$$

Once specific points and directions are known, the calculation proceeds.

Approach to Find the Shortest Distance Between Two Lines

The shortest distance D between two skew lines (lines that are neither parallel nor intersecting) can be obtained via vector operations:

$$
D = \frac{|(\mathbf{P}_2 - \mathbf{P}_1) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)|}{|\mathbf{d}_1 \times \mathbf{d}_2|}
$$

Key steps:

1. Find a vector connecting points on both lines, $\mathbf{P}_2 - \mathbf{P}_1$.

2. Compute the cross product of the direction vectors, $\mathbf{d}_1 \times \mathbf{d}_2$.
3. Take the dot product of the connecting vector with the cross product.
4. Calculate the magnitude of the cross product.
5. Divide the absolute value of the scalar triple product by the magnitude to get the shortest distance.

Detailed Step-by-Step Calculation

1. Choose Points and Direction Vectors

From the earlier assumptions:

- Point on Line 1: $\mathbf{P}_1 = (0, -1, 0)$
- Direction vector of Line 1: $\mathbf{d}_1 = (2, 3, 5)$

Suppose Line 2 passes through point:

$$
\mathbf{P}_2 = (x_0, y_0, z_0)
$$

and has direction:

$$
\mathbf{d}_2 = (a, b, c)
$$

These should be given explicitly, but in absence, let's assume:

$$
\mathbf{P}_2 = (x_2, y_2, z_2), \quad \mathbf{d}_2 = (a, b, c)
$$

---

2. Compute the Connecting Vector $\mathbf{P}_2 - \mathbf{P}_1$

$$
\mathbf{R} = \mathbf{P}_2 - \mathbf{P}_1 = (x_2 - 0, y_2 + 1, z_2 - 0) = (x_2, y_2 + 1, z_2)
$$

---

3. Calculate the Cross Product $\mathbf{d}_1 \times \mathbf{d}_2$

$$
\mathbf{d}_1 \times \mathbf{d}_2 =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 3 & 5 \\
a & b & c
\end{vmatrix}
= \mathbf{i}(3 c - 5 b) - \mathbf{j}(2 c - 5 a) + \mathbf{k}(2 b - 3 a)
$$

which yields the vector:

$$
\mathbf{N} = (3 c - 5 b,\, -(2 c - 5 a),\, 2 b - 3 a)
$$

---

4. Compute the Scalar Triple Product $\mathbf{R} \cdot \mathbf{N}$

$$
\text{Scalar triple product} = \mathbf{R} \cdot \mathbf{N} = x_2 (3 c - 5 b) + (y_2 + 1) (-(2 c - 5 a)) + z_2 (2 b - 3 a)
$$

This scalar quantity measures the volume of the parallelepiped defined by the vectors and is crucial in calculating the shortest distance.

---

5. Calculate the Magnitude of $\mathbf{N}$

$$
|\mathbf{N}| = \sqrt{(3 c - 5 b)^2 + (2 c - 5 a)^2 + (2 b - 3 a)^2}
$$

Final Distance Formula

Putting it all together, the shortest distance \(D\) is:

\[
D = \frac{\left| \mathbf{R} \cdot \mathbf{N} \right|}{|\mathbf{N}|}
\]

This formula gives the minimal perpendicular distance between the two lines.

Special Cases and Considerations

- Parallel Lines: When \(\mathbf{d}_1 \times \mathbf{d}_2 = \mathbf{0}\), the lines are either parallel or coincident. In this case, the formula simplifies, and the distance is the length of the perpendicular segment connecting the lines.

- Intersecting Lines: If the lines intersect, the shortest distance is zero.

- Skew Lines: When the lines are neither parallel nor intersecting, the above formula applies directly.

Worked Example with Specific Data

Suppose Line 2 passes through point \(\mathbf{P}_2 = (4, 2, 1)\) with direction vector \(\mathbf{d}_2 = (1, 2, 3)\).

Step 1: \(\mathbf{P

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