Find The Equation For The Following Parabola. Focus (3,4) Directrix Y 2 A. (x-3)2-2 (y-2.5) B. (x-3)2

Find The Equation For The Following Parabola. Focus (3,4) Directrix Y=2 A. (x-3)^2-2 (y-2.5) B. (x-3)^2

Understanding the equation of a parabola is fundamental in conic sections, a key topic in algebra and coordinate geometry. This article aims to guide you through the process of deriving the equations of parabolas given specific focus points and directrices. We will explore the concepts step-by-step, focusing on the given focus point (3,4) and the directrices Y=2, as well as the other conic forms provided in the question.

Understanding the Basic Concepts of Parabolas

Before diving into the specifics of the problem, it’s essential to grasp the fundamental properties of parabolas.

What is a Parabola?

A parabola is a symmetric, U-shaped curve that can be described geometrically as the set of all points equidistant from a fixed point called the focus and a fixed line called the directrix.

Key Components of a Parabola

- **Focus**: A fixed point located inside the parabola.
- **Directrix**: A fixed straight line outside the parabola.
- **Vertex**: The point where the parabola changes direction, located midway between the focus and the directrix.
- **Axis of symmetry**: The line passing through the focus and vertex, about which the parabola is symmetric.

Standard Equation Forms

- For a parabola opening upwards or downwards: \((x - h)^2 = 4p(y - k)\)

- For a parabola opening sideways: $(y - k)^2 = 4p(x - h)$

Where (h, k) is the vertex and p is the distance from the vertex to the focus (or equivalently, from the vertex to the directrix).

Deriving the Equation of a Parabola with Focus (3,4) and Directrix Y=2

Given the focus and directrix, the goal is to find the equation of the parabola that satisfies the geometric definition: the set of points equidistant from the focus and the directrix.

Step 1: Understand the Given Data

- **Focus**: $(3,4)$
- **Directrix**: $Y=2$

Since the directrix is a horizontal line $Y=2$, and the focus is at $(3,4)$, the parabola opens upward because the focus is above the directrix.

Step 2: Find the Vertex

The vertex lies midway between the focus and the directrix along the axis of symmetry.

- The focus's y-coordinate: 4
- The directrix's y-coordinate: 2
- The vertex's y-coordinate: $\frac{4 + 2}{2} = 3$

The x-coordinate of the vertex matches that of the focus (since the parabola opens vertically):

- The vertex's x-coordinate: 3

Vertex: $(3,3)$

Step 3: Calculate the Distance p

- p is the distance from the vertex to the focus (or the directrix).
- $p = 4 - 3 = 1$

Since the parabola opens upward:

- $p = 1$

Step 4: Write the Equation

Using the standard form for a parabola opening upward:

$$
(x - h)^2 = 4p(y - k)
$$

Substitute $h=3$, $k=3$, and $p=1$:

$$
(x - 3)^2 = 4 \times 1 \times (y - 3)
$$
$$
(x - 3)^2 = 4(y - 3)
$$

Final Equation:

$$
\boxed{(x - 3)^2 = 4(y - 3)}
$$

This is the equation of the parabola with focus at $(3,4)$ and directrix $Y=2$.

Analysis of the Equation and Its Features

Understanding the characteristics of this parabola helps in visualization and further applications.

Vertex

- Located at $(3,3)$
- It is the minimum point for the parabola opening upward.

Focus

- At $(3,4)$, which is exactly $p=1$ unit above the vertex.

Directrix

- The line $Y=2$, exactly 1 unit below the vertex.

Axis of Symmetry

- Vertical line $x=3$.

Opening Direction

- Upward, because the focus is above the vertex.

Deriving the Equation for the Second Set: (x-3)^2 - 2 (y - 2.5)

The provided expression $(x-3)^2 - 2 (y - 2.5)$ appears to be an incomplete or alternative form of a parabola's equation. Let’s analyze and interpret it.

Possible Interpretation

Suppose the expression is part of an equation set equal to zero:

$$
(x - 3)^2 - 2(y - 2.5) = 0
$$

Rearranged as:

$$
(x - 3)^2 = 2(y - 2.5)
$$

This resembles the standard form $(x - h)^2 = 4p (y - k)$, where:

$$
4p = 2 \Rightarrow p = \frac{1}{2}
$$

Vertex:

- The form indicates the vertex at $(h, k) = (3, 2.5)$.

Focus:

- Since $p = 0.5$, the focus is located at:

$$
(3, 2.5 + p) = (3, 3.0)
$$

Directrix:

- Located at:

$$
y = 2.5 - p = 2.0
$$

This parabola opens upward, with vertex at $(3, 2.5)$, focus at $(3, 3.0)$, and directrix at $y=2.0$.

Summary:

- Equation:

$$
(x - 3)^2 = 2(y - 2.5)
$$

- Key features:
- Vertex: $(3, 2.5)$
- Focus: $(3, 3.0)$
- Directrix: $y=2.0$

Understanding the Third Expression: $(x-3)^2$

The expression $(x-3)^2$ alone is the basic form of a parabola that opens upward or downward depending on its context.

Implication in Parabola Equations

- The simplest form:

$$
(y - k) = a (x - h)^2
$$

- For example, $(x - 3)^2 = 4(y - 4)$ describes a parabola opening upward with vertex at $(3,4)$.

- It indicates symmetry about the vertical line $x=3$.

Application in the Given Data

- The expression $(x - 3)^2$ suggests a parabola centered horizontally at $x=3$, with the vertex and focus depending on the coefficient.

Summary of Key Concepts and Formulas

To effectively find the equation of a parabola given focus and directrix, follow these steps:

1. Identify the focus and directrix.
2. Determine the parabola's orientation:
- If the directrix is horizontal, parabola opens upward or downward.
- If the directrix is vertical, parabola opens sideways.
3. Find the vertex:
- The midpoint between focus and directrix along the axis of symmetry.
4. Calculate p:
- Distance from the vertex to the focus (or directrix).
- Sign indicates the opening direction.
5. Write the standard form equation:
- For vertical opening: $(x - h)^2 = 4p(y - k)$
- For horizontal opening: $(y - k)^2 = 4p(x - h)$

Practical Applications of Parabola Equations

Understanding how to derive and manipulate parabola equations has numerous practical applications, including:

- Satellite dishes and radio telescopes: Designed as parabolic reflectors to focus signals.
- Car headlights: Use parabolic mirrors to direct light beams.
- Physics: Trajectory analysis of projectiles.
- Engineering and architecture: Structural designs involving parabolic arches.

Frequently Asked Questions

How do you find the equation of a parabola with focus (3,4) and directrix y = 2?

Use the definition of a parabola: the set of points equidistant from the focus and the directrix. The equation becomes $(x - 3)^2 = 4p(y - 4)$, where p is the distance from the vertex to the focus or directrix. Since the directrix y=2 is 2 units below the focus y=4, p=2. Thus, the equation is $(x - 3)^2 = 8(y - 4)$.

What is the standard form of a parabola with focus at (3,4) and directrix y = 2?

The standard form is $(x - h)^2 = 4p(y - k)$, where (h,k) is the vertex. Here, the vertex is midway between the focus and directrix at (3, 3), and $p=1$. Therefore, the equation is $(x - 3)^2 = 4(y - 3)$.

How can I derive the parabola equation given focus (3,4) and directrix y=2?

Calculate the vertex as the midpoint between focus and directrix: (3, 3). The distance p from the vertex to the focus is 1. Using the parabola formula, $(x - 3)^2 = 4(1)(y - 3)$, simplifying to $(x - 3)^2 = 4(y - 3)$.

What is the parabola equation with focus at (3,4) and directrix y=2, in vertex form?

The vertex is at (3, 3), and $p=1$. The parabola opens upward, so the equation in vertex form is $(x - 3)^2 = 4(y - 3)$.

How does the focus and directrix determine the parabola's equation?

The focus and directrix define the parabola as the locus of points equidistant to both. This leads to the standard form equation by setting the distance from any point to the focus equal to its distance to the directrix, resulting in a quadratic equation consistent with the parabola's shape.

What is the effect of changing the focus position on the parabola's equation?

Changing the focus position shifts the vertex and alters p, which affects the parabola's width and orientation. The equation adjusts accordingly, with the vertex at the midpoint between focus and directrix and the parameter p reflecting the focus's distance from the vertex.

Can you provide the equation of the parabola with focus (3,4) and directrix y=2 in general form?

Yes. Starting from $(x - 3)^2 = 4(y - 3)$, expanding gives $x^2 - 6x + 9 = 4y - 12$, which simplifies to $x^2 - 6x - 4y + 21 = 0$.

What are the key steps to find the parabola's equation given focus and directrix?

1. Find the vertex as the midpoint between focus and directrix. 2. Calculate p as the distance from vertex to focus. 3. Write the standard form of the parabola using $(x - h)^2 = 4p(y - k)$. 4. Simplify to obtain the explicit equation.

How does the parabola equation change if the directrix is y = -2 instead of y=2, with focus at (3,4)?

The vertex shifts to (3, 1), midway between focus and directrix, with p=3. The equation becomes (x - 3)^2 = 12(y - 1), opening upward with a wider shape due to larger p.

Additional Resources

Find The Equation For The Following Parabola. Focus (3,4) Directrix Y=2 A. (x-3)^2 - 2 (y - 2.5) B. (x-3)^2

When tackling the problem of finding the equation for the following parabola, especially with specific focus and directrix details, it’s essential to understand the fundamental properties of parabolas. In this guide, we will analyze the given focus and directrix, then systematically derive the parabola's equations, providing a comprehensive step-by-step approach.

Understanding these concepts is crucial not only for solving specific problems but also for grasping the geometric foundations that underpin conic sections in general.

Introduction to Parabolas: Focus and Directrix

A parabola is a conic section that can be defined as the locus of points equidistant from a fixed point called the focus and a fixed line called the directrix.

Key Properties:
- Every point on the parabola is equidistant from the focus and the directrix.
- The parabola opens in the direction perpendicular to the directrix.
- The vertex is the midpoint between the focus and the directrix along the line perpendicular to the directrix.

Knowing the focus and directrix allows us to derive the parabola’s equation directly, especially when the focus and directrix are aligned in standard orientations.

Step 1: Understanding the Given Data

Let's analyze the components provided:

- Focus: (3, 4)
- Directrix: Y = 2
- Additional Options:
- A. (x-3)^2 - 2
- B. (x-3)^2

The problem appears to involve two potential parabola equations, likely related to different orientations or transformations based on the focus and directrix.

Step 2: Recognizing the Parabola's Orientation

Given the focus at (3, 4) and the directrix at Y=2:

- The focus lies above the directrix (since $y=4 > y=2$).
- The parabola opens upward, because the focus is above the directrix.

This aligns with the standard form of a parabola opening upward:

$$ (x - h)^2 = 4p (y - k) $$

where:
- (h, k) is the vertex,
- p is the distance from the vertex to the focus (or directrix).

Step 3: Computing the Vertex

Since the parabola opens upward:

- The vertex is midway between the focus and the directrix along the axis perpendicular to the directrix.

Calculate:

- Vertex’s y-coordinate:

$$
k = \frac{y_{focus} + y_{directrix}}{2} = \frac{4 + 2}{2} = 3
$$

- Vertex’s x-coordinate:

$$
h = x_{focus} = 3
$$

Thus, the vertex is at (3, 3).

Step 4: Determining the Parameter p

The focal length p is the distance from the vertex to the focus:

$$
p = y_{focus} - y_{vertex} = 4 - 3 = 1
$$

This confirms that the parabola opens upward, with $p = 1$.

Step 5: Deriving the Equation

Using the standard form:

$$
(x - h)^2 = 4p (y - k)
$$

Plug in $h=3$, $k=3$, and $p=1$:

$$
(x - 3)^2 = 4 \times 1 \times (y - 3)
$$

Simplify:

$$
(x - 3)^2 = 4(y - 3)
$$

This is the equation of the parabola with focus at (3, 4) and directrix Y=2.

Step 6: Interpreting the Options

Now, compare this to the options provided:

- Option A: $(x-3)^2 - 2$
- Option B: $(x-3)^2$

Our derived equation:

$$
(x - 3)^2 = 4(y - 3)
$$

can be rewritten as:

$$
(y - 3) = \frac{(x - 3)^2}{4}
$$

which is in the form:

$$
(y - k) = \frac{1}{4} (x - h)^2
$$

\]

Alternatively, solving for \(y \):

\[
y = \frac{1}{4}(x - 3)^2 + 3
\]

Neither option A nor B directly matches this form, but understanding the structure helps clarify.

Step 7: Additional Considerations – The Other Options

Option A: \((x-3)^2 - 2\)

This resembles a parabola in quadratic form but lacks the explicit \(y \) variable. To match the standard form, it would need to be expressed as:

\[
(y - k) = \text{something}
\]

which suggests that it might be an incomplete or alternative form.

Option B: \((x-3)^2\)

This is a basic quadratic expression, but without a \(y \) term, it doesn't represent a parabola in the typical form unless it's part of a larger expression.

Conclusion: The Correct Equation

The precise equation of the parabola with focus at (3, 4) and directrix at Y=2 is:

\[
\boxed{
(x - 3)^2 = 4(y - 3)
}
\]

or equivalently:

\[
y = \frac{1}{4}(x - 3)^2 + 3
\]

This formula explicitly describes the parabola opening upward with vertex at (3, 3).

Final Notes and Additional Tips

- Always identify the focus and directrix positions to determine the parabola’s orientation.
- The vertex is midway between the focus and directrix along the axis of symmetry.
- The parameter p is the focal distance and determines the parabola's “width.”
- Standard forms of the parabola are:

- Opening upward or downward: $(x - h)^2 = 4p(y - k)$
- Opening left or right: $(y - k)^2 = 4p(x - h)$

- When in doubt, derive the equation from the focus-directrix definition for accuracy.

Summary

In this guide, we've thoroughly analyzed the problem of finding the equation of a parabola given a focus at (3, 4) and a directrix at Y=2. By calculating the vertex and the focal length p, we derived the equation:

$$
\boxed{
(x - 3)^2 = 4(y - 3)
}
$$

This process illustrates the importance of understanding the geometric properties of conic sections, enabling precise derivations and clear comprehension of parabola equations in various contexts.

Remember: Mastering the derivation of conic section equations enhances your analytical skills and deepens your understanding of geometric relationships—a vital foundation in advanced mathematics and related fields.

Find The Equation For The Following Parabola Focus 3 4 Directrix Y 2 A X 3 2 2 Y 2 5 B X 3 2

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