# Find The Equation Of A Tangent Line To The Curve Y= 2x3 - 3x2 - 10x + 1 Which Is Perpendicular To The

**Find The Equation Of A Tangent Line To The Curve Y= 2x3 - 3x2 - 10x + 1 Which Is Perpendicular To The** a given curve, is a classic problem in calculus that combines understanding of derivatives, tangent lines, and perpendicular slopes. Whether you're a student seeking to deepen your understanding of differential calculus or a professional brushing up on fundamental concepts, mastering this problem involves several key steps: analyzing the function, finding the derivative to determine slopes, and applying the conditions for perpendicularity.

In this comprehensive guide, we will walk through the process of finding the equation of the tangent line to the curve $y = 2x^3 - 3x^2 - 10x + 1$ that is perpendicular to a certain line or direction. Since the statement is incomplete, we will explore common scenarios, such as finding the tangent line perpendicular to a given line or slope, and demonstrate the general approach applicable to similar problems.

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## Understanding the Given Curve and Its Derivative

### Analyzing the Function $y = 2x^3 - 3x^2 - 10x + 1$

The first step in solving the problem is to understand the behavior of the function. The curve is a cubic polynomial, which means it can have multiple inflection points, local maxima, and minima.

Key features of the function:

- Degree: 3 (cubic)
- Leading coefficient: 2
- Intercepts: The y-intercept occurs at $x=0$, where $y=1$

Understanding the shape of the curve helps in visualizing potential tangent lines and their slopes.

### Finding the Derivative $y'$ to Determine Slope

The derivative of the function gives the slope of the tangent line at any point $x$:

$$
y' = \frac{dy}{dx} = \frac{d}{dx}(2x^3 - 3x^2 - 10x + 1) = 6x^2 - 6x - 10
$$

This derivative function, $y' = 6x^2 - 6x - 10$, provides the slope of the tangent line at each point $x$.

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## Condition for Perpendicularity of Lines

Before proceeding, it’s important to recall the condition for two lines to be perpendicular:

- If the slope of the first line is $m_1$,
- and the slope of the second line is $m_2$,
- then the lines are perpendicular if and only if:

$$
m_1 \times m_2 = -1
$$

In this problem, we seek the tangent line at some point $x = x_0$ on the curve, with slope $m_t = y'(x_0)$, that is perpendicular to a given line or direction.

Note: Since the problem states "which is perpendicular to the," but does not specify the line, we will consider common cases:

1. The tangent line is perpendicular to a given line with a known slope $m$.
2. The tangent line is perpendicular to the curve's tangent at some point (less common but useful for understanding).

We will focus on scenario 1, where the tangent line is perpendicular to a line with slope $m$.

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## Finding the Slope of the Perpendicular Tangent Line

Suppose we are told the tangent line must be perpendicular to a line with slope $m$. Then, the slope of the tangent line $m_t$ must satisfy:

$$
m_t = -\frac{1}{m}
$$

Given that the tangent line to the curve at $x = x_0$ has slope $y'(x_0)$, the condition becomes:

$$
y'(x_0) = -\frac{1}{m}
$$

This means:

$$
6x_0^2 - 6x_0 - 10 = -\frac{1}{m}
$$

or equivalently,

$$
6x_0^2 - 6x_0 - 10 + \frac{1}{m} = 0
$$

This quadratic in $x_0$ can be solved for specific $m$.

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# Calculating the Coordinates and Equation of the Tangent Line

Once $x_0$ is found, the corresponding $y_0$ on the curve is:

$$
y_0 = 2x_0^3 - 3x_0^2 - 10x_0 + 1
$$

The tangent line at $(x_0, y_0)$ with slope $m_t$ has the equation:

$$
y - y_0 = m_t (x - x_0)
$$

or explicitly:

$$
y - y_0 = -\frac{1}{m} (x - x_0)
$$

This is the required tangent line perpendicular to the given line with slope $m$.

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# Step-by-Step Solution Strategy

To systematically find the equation of the tangent line perpendicular to a given line or condition, follow these steps:

1. **Identify the given line's slope:** Determine $m$, the slope of the line to which the tangent must be perpendicular.

2. **Find the slope of the tangent line:** Use $y'(x) = 6x^2 - 6x - 10$. Set $y'(x) = -1/m$ and solve for $x$ to find potential points $x_0$ on the curve.

3. **Calculate corresponding $y$-coordinates:** Compute $y_0 = 2x_0^3 - 3x_0^2 - 10x_0 + 1$.

4. **Write the tangent line equation:** Use point-slope form with $(x_0, y_0)$ and slope $m_t = -1/m$.

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# Example: Finding the Tangent Line Perpendicular to a Line with Slope 2

Suppose the line to which the tangent is perpendicular has slope $m = 2$. Then, the slope of the tangent line is:

$$
m_t = -\frac{1}{2}
$$

Set the derivative equal to $-1/2$:

$$
6x^2 - 6x - 10 = -\frac{1}{2}
$$

Multiply both sides by 2 to clear the denominator:

$$
2 \times (6x^2 - 6x - 10) = -1
$$

$$
12x^2 - 12x - 20 = -1
$$

Bring all to one side:

$$
12x^2 - 12x - 19 = 0
$$

Solve this quadratic:

$$
x = \frac{12 \pm \sqrt{(-12)^2 - 4 \times 12 \times (-19)}}{2 \times 12}
$$

Calculate discriminant:

$$
D = 144 - 4 \times 12 \times (-19) = 144 + 4 \times 12 \times 19
$$

$$
= 144 + 4 \times 228 = 144 + 912 = 1056
$$

Calculate $\sqrt{1056}$:

$$
\sqrt{1056} \approx 32.49
$$

Find $x$:

$$
x = \frac{12 \pm 32.49}{24}
$$

Two solutions:

1.

$$
x = \frac{12 + 32.49}{24} = \frac{44.49}{24} \approx 1.854
$$

2.

$$
x = \frac{12 - 32.49}{24} = \frac{-20.49}{24} \approx -0.854
$$

Calculate corresponding $y$-values:

- For $x \approx 1.854$:

$$
y \approx 2(1.854)^3 - 3(1.854)^2 - 10(1.854) + 1
$$

Calculate step by step:

$$
(1.854)^3 \approx 6.37
$$
$$
(1.854)^2 \approx 3.44
$$

Then,

$$
y \approx 2 \times 6.37 - 3 \times 3.44 - 10 \times 1.854 + 1
$$
$$
\approx 12.74 - 10.32 - 18.54 + 1
$$
$$
\approx (12.74 - 10.32) - 18.54 + 1 = 2.42 - 18.54 + 1 = -15.12 + 1 = -14.12
$$

- For $x \approx -0.854$:

$$
(-0.854)^3 \approx -0.62
$$
$$
(-0.854)^2 \approx 0.73
$$

# Frequently Asked Questions

## How do I find the equation of a tangent line to the curve y = 2x^3 - 3x^2 - 10x + 1 that is perpendicular to a given line?

First, find the derivative of the curve to get the slope of the tangent at any point, then determine the slope of the perpendicular line (the negative reciprocal), and finally use the point-slope form to write the equation of the tangent line.

## What is the process to determine the point where the

## tangent line to the curve $y = 2x^3 - 3x^2 - 10x + 1$ is perpendicular to a specific line?

You need to find the derivative $y' = 6x^2 - 6x - 10$, then set the slope of the tangent (y') equal to the negative reciprocal of the given line's slope to find the x-coordinate(s) of the point(s).

## If the line to which the tangent must be perpendicular has a slope of m, how do I find the corresponding point on the curve?

Set the derivative $y' = 6x^2 - 6x - 10$ equal to $-1/m$, solve for x, and then substitute x back into the original curve to find the y-coordinate.

## Can you demonstrate the step-by-step method to find the equation of the tangent line perpendicular to a line with slope m for the given curve?

Yes. First, find $y' = 6x^2 - 6x - 10$. Next, solve for x where $y' = -1/m$. Then, find y by plugging x into the original curve. Finally, use point-slope form with the point (x, y) and slope $-1/m$ to write the tangent line equation.

## What are common mistakes to avoid when finding a perpendicular tangent line to the curve $y = 2x^3 - 3x^2 - 10x + 1$?

Common mistakes include mixing up the derivative and the original function, not solving correctly for x, or using the wrong slope when applying the perpendicular condition. Always double-check calculations and ensure the correct slopes are used.

# Additional Resources

Find The Equation Of A Tangent Line To The Curve $Y= 2x^3 - 3x^2 - 10x + 1$ Which Is Perpendicular To The

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Introduction: The Significance of Tangent Lines in Calculus

In the realm of calculus, the concept of a tangent line serves as a fundamental bridge between algebraic functions and their geometric interpretations. The tangent line at a specific point on a curve provides a linear approximation of the function at that point, revealing insights into the function’s instantaneous rate of change, slope, and local behavior. Such lines are critical in fields ranging from physics to engineering, where understanding the behavior of dynamic systems hinges on the precise calculation of tangents.

One particular challenge that often arises in calculus involves determining the equation of a tangent line to a curve that satisfies specific geometric constraints—most notably, perpendicularity. This problem intertwines the calculus of derivatives with geometric concepts of orthogonality, necessitating a thorough analytical approach.

In this article, we explore this challenge in detail: Find The Equation Of A Tangent Line To The Curve Y= $2x^3 - 3x^2 - 10x + 1$ Which Is Perpendicular To The... (the statement is incomplete but implies a perpendicularity condition, likely to a given line or tangent). Our goal is to elucidate the step-by-step process of solving such a problem, grounding the discussion in rigorous mathematical principles.

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Understanding the Problem Context

Before diving into the solution, it is essential to clarify the problem's scope and what it entails:

- Given Curve: $y = 2x^3 - 3x^2 - 10x + 1$
- Objective: Find the equation of a tangent line to this curve that is perpendicular to a given line (or another tangent).

Note: Since the problem statement is incomplete, we can consider the typical case where the tangent line must be perpendicular to a specific line or to the tangent at some point on the curve. For clarity, we will assume the requirement is to find the tangent line to the curve at some point $(x_0, y_0)$ which is perpendicular to a given line, say, $y = mx + c$.

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Step 1: Deriving the Slope of the Tangent Line

The first step in solving such problems is to determine the slope of the tangent line at any arbitrary point on the curve.

The derivative $y'$:

Given the function:

$$
y = 2x^3 - 3x^2 - 10x + 1
$$

Applying differentiation term-by-term:

$$
\frac{dy}{dx} = y' = 6x^2 - 6x - 10
$$

This derivative represents the slope of the tangent line to the curve at any point $x$.

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Step 2: Connecting the Slope to Perpendicularity

The core geometric principle involved here is that two lines are perpendicular if and only if the product of their slopes is -1.

Assuming the tangent line at $x = x_0$ has slope $m_t$, and it is perpendicular to a given line with slope $m$, then:

$$
m_t \times m = -1
$$

which implies:

$$
m_t = -\frac{1}{m}
$$

In the absence of the specific line's slope, if the problem asks for a tangent line perpendicular to the tangent at some known point or to an external line, this step must be adapted accordingly.

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Step 3: Finding the Point $x_0$ on the Curve

Suppose the problem requires finding the point $(x_0, y_0)$ where the tangent line has a slope $m_t$ and is perpendicular to the given line.

Example scenario:

- The given line has slope $m$.

- The slope of the tangent line at $x_0$:

$$
m_t = 6x_0^2 - 6x_0 - 10
$$

- Set:

$$
6x_0^2 - 6x_0 - 10 = -\frac{1}{m}
$$

This quadratic equation can be solved for $x_0$:

$$
6x_0^2 - 6x_0 - 10 + \frac{1}{m} = 0
$$

which simplifies to:

$$
6x_0^2 - 6x_0 + \left( -10 + \frac{1}{m} \right) = 0
$$

Once $x_0$ is found, the corresponding $y_0$ can be computed by plugging into the original curve:

$$
y_0 = 2x_0^3 - 3x_0^2 - 10x_0 + 1
$$

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Step 4: Equation of the Tangent Line

The tangent line at $(x_0, y_0)$ with slope $m_t$ is given by the point-slope form:

$$
y - y_0 = m_t (x - x_0)
$$

This equation can be expanded or rearranged into slope-intercept form as needed.

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Additional Considerations and Special Cases

Case 1: The line is given explicitly.

- For example, if the line is $y = mx + c$, and the tangent's slope must be perpendicular to it, then the above steps apply directly.

Case 2: The tangent line must be perpendicular to the tangent at some point on the curve.

- In such a scenario, the problem involves finding points where the tangent line's slope is the negative reciprocal of the slope at another point, leading to a system of equations.

Case 3: The problem involves multiple solutions.

- Polynomial equations may yield multiple $x_0$ solutions, each corresponding to a different tangent line satisfying the perpendicularity condition.

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Practical Applications of Such Problems

Understanding how to find tangent lines with specific geometric properties is crucial in numerous disciplines:

- Physics: Calculating instantaneous velocities and directions of motion.
- Engineering: Designing components with precise geometric constraints.
- Economics: Analyzing marginal functions and their relationships.
- Computer Graphics: Rendering curves with tangent constraints.

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Example: A Hypothetical Complete Problem

Suppose the problem states:

"Find the equation of the tangent line to the curve $y = 2x^3 - 3x^2 - 10x + 1$ that is perpendicular to the line $y = 4x + 2$."

In this case:

- The slope of the given line: $m = 4$
- The slope of the tangent line: $m_t = -\frac{1}{4}$

Set the derivative equal to this:

$$
6x_0^2 - 6x_0 - 10 = -\frac{1}{4}
$$

Multiply through by 4:

$$
4(6x_0^2 - 6x_0 - 10) = -1
$$

$$
24x_0^2 - 24x_0 - 40 = -1
$$

Bring all to one side:

$$
24x_0^2 - 24x_0 - 39 = 0
$$

Solve for $x_0$:

$$
x_0 = \frac{24 \pm \sqrt{(-24)^2 - 4 \times 24 \times (-39)}}{2 \times 24}
$$

Calculate discriminant:

$$
\Delta = 576 - 4 \times 24 \times (-39) = 576 + 3744 = 4320
$$

$$
x_0 = \frac{24 \pm \sqrt{4320}}{48}
$$

Simplify $\sqrt{4320}$:

$$
\sqrt{4320} = \sqrt{144 \times 30} = 12 \sqrt{30}
$$

Thus:

$$
x_0 = \frac{24 \pm 12 \sqrt{30}}{48} = \frac{24}{48} \pm \frac{12 \sqrt{30}}{48} = \frac{1}{2} \pm \frac{\sqrt{30}}{4}
$$

Calculate $y_0$:

$$
y_0 = 2x_0^3 - 3x_0^2 - 10x_0 + 1
$$

Insert each $x_0$, then write the tangent line equations:

$$
y - y_0 = -\frac{1}{4} (x - x_0)
$$

This provides explicit tangent lines satisfying the perpendicularity condition.

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Conclusion: The Intertwined Nature of Geometry and Calculus

The problem of finding a tangent line to a curve that is perpendicular to a given line or another tangent is a quintessential example of the synergy between differential calculus and geometric reasoning. It demonstrates how derivatives serve as tools to translate geometric constraints into algebraic equations, which can then be systematically solved.

While the initial problem statement was incomplete, the methodology outlined provides a comprehensive framework for approaching such problems:

1. Derive the derivative to find the slope function.
2. Use perpendicularity conditions to establish relationships between slopes.
3. Solve resulting algebraic equations for the points of

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