Find The Equation Of The Line That Goes Through The Point (0 , -3) And Is Perpendicular To The Line

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Understanding how to find the equation of a line that passes through a specific point and is perpendicular to another line is a fundamental concept in coordinate geometry. This process involves working with the slopes of lines, the point-slope form of a line's equation, and the concept of perpendicularity in the context of slopes. In this article, we will explore the step-by-step approach to determine such a line, delve into the underlying mathematical principles, and provide illustrative examples to clarify the process.

Understanding the Basic Principles

The Slope-Intercept Form of a Line

Before diving into the problem, it is essential to recall the slope-intercept form of a line:

$$y = mx + b$$

where:
- m is the slope of the line,
- b is the y-intercept (the point where the line crosses the y-axis).

This form is particularly useful because it explicitly displays the slope, which is crucial for our problem.

The Concept of Perpendicular Lines

Two lines are perpendicular if they intersect at a right angle (90 degrees). In coordinate geometry, the slopes of two perpendicular lines are negative reciprocals of each other, provided neither line is vertical or horizontal:

- If one line has slope m,
- the perpendicular line has slope $m_{\perp} = -\frac{1}{m}$.

This relationship is fundamental when determining the slope of the line perpendicular to a given line.

Given Point and the Need to Find the Perpendicular Line

In our problem, we are given:
- A point $(0, -3)$,
- A line (whose equation is not specified explicitly in the prompt but is implied to be given or known).

Our goal is to find the equation of a line that:
- Passes through $(0, -3)$,
- Is perpendicular to the given line.

The approach involves:
1. Determining the slope of the original line,
2. Calculating the slope of the perpendicular line,
3. Using the point $(0, -3)$ and the perpendicular slope to write the equation of the new line.

Step 1: Identifying the Slope of the Given Line

Case 1: The Equation of the Given Line Is Known

If the original line's equation is provided, for example:

$$ y = m_{original} x + c $$

then the slope of this line is simply:

$$ m_{original} $$

For example, if the line is $y = 2x + 5$, then its slope is 2.

Case 2: The Equation of the Given Line Is Not Provided

If the original line's equation is not explicitly given but a graph or some additional information is available, you can find the slope by:

- Selecting two points on the line,
- Calculating the slope using the difference quotient:

$$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

For instance, if the line passes through points (x_1, y_1) and (x_2, y_2), then:

$$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

Step 2: Calculating the Perpendicular Slope

Once the original slope $m_{original}$ is known, the slope of the perpendicular line $m_{\perp}$ is calculated as:

$$m_{\perp} = -\frac{1}{m_{original}}$$

Important considerations:

- If $m_{original} = 0$ (a horizontal line), then the perpendicular line is vertical, and its equation is of the form $x = x_0$.
- If $m_{original}$ is undefined (a vertical line), then the perpendicular line is horizontal with equation $y = y_0$.

Step 3: Writing the Equation of the Perpendicular Line

Given:
- The point $(x_0, y_0) = (0, -3)$,
- The perpendicular slope $m_{\perp}$,

we can use the point-slope form of the line:

$$y - y_0 = m_{\perp} (x - x_0)$$

Substituting:

$$ y - (-3) = m_{\perp} (x - 0) $$

$$ y + 3 = m_{\perp} x $$

Rearranged to slope-intercept form:

$$ y = m_{\perp} x - 3 $$

This is the equation of the line passing through $(0, -3)$ and perpendicular to the original line.

Examples Illustrating the Process

Example 1: Original Line Has a Known Equation

Suppose the original line is:

$$ y = 4x + 1 $$

- The slope of this line:

$$ m_{original} = 4 $$

- The perpendicular slope:

$$ m_{\perp} = -\frac{1}{4} $$

- Equation of the perpendicular line passing through $(0, -3)$:

$$ y + 3 = -\frac{1}{4} x $$

or

$$ y = -\frac{1}{4} x - 3 $$

Result:

$$
\boxed{
\text{Equation of the perpendicular line:} \quad y = -\frac{1}{4} x - 3
}
$$

Example 2: Original Line Is Vertical

Suppose the original line is vertical:

$$ x = 2 $$

- Its slope is undefined.

- The perpendicular line, in this case, is horizontal, with the equation:

$$ y = y_0 $$

Since the line passes through $(0, -3)$, the perpendicular line is:

$$ y = -3 $$

Result:

$$
\boxed{
\text{Equation of the perpendicular line:} \quad y = -3
}
$$

Special Cases and Additional Considerations

Vertical and Horizontal Lines

- When the original line is horizontal ($y = c$), the perpendicular line is vertical ($x = x_0$).
- When the original line is vertical, the perpendicular is horizontal.

Lines with Undefined or Zero Slopes

- For a zero slope ($m=0$), the perpendicular line's slope is undefined (vertical).
- For an undefined slope, the perpendicular has zero slope (horizontal).

Using Coordinate Geometry Tools

- Graphing calculators or coordinate geometry software can help visualize the lines and confirm the equations.
- When working by hand, plotting carefully and selecting points on the original line can aid in understanding the relationships.

Summary of the Procedure

To find the equation of a line passing through $(0, -3)$ and perpendicular to a given line:

1. Determine the slope of the given line:
- If given explicitly, extract $m_{original}$.
- If not, find two points on the line and compute the slope.

2. Calculate the perpendicular slope:
- $m_{\perp} = -\frac{1}{m_{original}}$.

3. Use the point-slope form to write the new line's equation:
- $y - y_0 = m_{\perp} (x - x_0)$,
- where $(x_0, y_0) = (0, -3)$.

4. Simplify to slope-intercept form if desired.

This method ensures a systematic approach to solving the problem and can be applied to various scenarios involving different given lines and points.

Conclusion

Finding the equation of a line that passes through a specific point and is perpendicular to another line involves understanding the relationship between slopes of perpendicular lines, employing the point-slope form, and simplifying the resulting equation. Whether the original line's equation is known or can be deduced from a graph, the core principles remain the same. Mastery of this process enhances one's ability to analyze geometric relationships and solve a wide array of problems in coordinate geometry. With practice, identifying perpendicular lines and writing their equations becomes an intuitive and valuable skill in mathematics.

Frequently Asked Questions

How do I find the equation of a line passing through the point (0, -3) and perpendicular to a given line?

First, determine the slope of the given line, then find the negative reciprocal to get the perpendicular slope. Use the point-slope form with the point (0, -3) and this slope to find the equation.

What is the significance of the point (0, -3) when finding the line's equation?

The point (0, -3) is the y-intercept of the line, making it easy to write the equation in slope-intercept form once the slope is known.

How do I determine the slope of the line perpendicular to a given line?

Find the slope of the given line first. The perpendicular line's slope is the negative reciprocal of that slope. For example, if the original slope is m, the perpendicular slope is -1/m.

Can I find the equation of the perpendicular line without knowing the original line's equation?

No, you need at least the slope of the original line to find its perpendicular slope. If the original line's equation is known, you can determine its slope and then find the perpendicular slope accordingly.

What is the formula to write the equation of a line passing through a point with a known slope?

Use the point-slope form: $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point and m is the slope.

If the given line's equation is y = 2x + 1, what is the equation of the line passing through (0, -3) and perpendicular to it?

The original slope is 2, so the perpendicular slope is -1/2. Using the point (0, -3), the equation is y + 3 = -1/2(x - 0), simplifying to y = -1/2 x - 3.

Why is the point (0, -3) used directly in the line equation?

Because the point's x-coordinate is 0, it directly gives the y-intercept in the equation, simplifying the process of writing the line's equation once the slope is known.

Can the line passing through (0, -3) and perpendicular to another line be written in slope-intercept form?

Yes, once the perpendicular slope is calculated, substitute into y = mx + b using the point (0, -3) to find b, resulting in the slope-intercept form.

Additional Resources

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Introduction

In the realm of coordinate geometry, understanding how to find the equation of a line that satisfies specific conditions is fundamental. One common problem involves determining the line that passes through a given point and is perpendicular to another line. This task combines knowledge of slopes, perpendicularity, and point-slope forms to arrive at the desired equation. Today, we delve into this problem, dissecting each step with precision and clarity, providing a comprehensive guide for students and enthusiasts alike.

Understanding the Basics: Lines, Slopes, and Perpendicularity

The Equation of a Line

At the core of the problem lies the concept of a line's equation. In the Cartesian plane, a straight line can typically be expressed in the slope-intercept form:

$$y = mx + b$$

where:
- m is the slope of the line,
- b is the y-intercept.

Alternatively, the point-slope form:

$$ y - y_1 = m(x - x_1) $$

is useful when passing through a known point (x_1, y_1).

Understanding Slope

The slope m of a line indicates its steepness and direction. It is calculated as:

$$ m = \frac{\Delta y}{\Delta x} $$

for two points (x_1, y_1) and (x_2, y_2).

Perpendicular Lines and Their Slopes

Two lines are perpendicular if they intersect at a right angle (90°). The key property here is that their slopes are negative reciprocals of each other:

$$ m_1 \times m_2 = -1 $$

or equivalently,

$$ m_2 = -\frac{1}{m_1} $$

This relationship is fundamental in deriving the equation of the required line.

Deciphering the Given Problem

The problem states:

> Find the equation of the line that passes through the point (0, -3) and is perpendicular to the line.

However, the statement as given appears incomplete because it references "the line" but does not specify which line the new line should be perpendicular to. For the purpose of this analysis, we will assume that the original line's equation is known, or perhaps it is implied that the line is perpendicular to a familiar or provided line.

Possible scenarios include:

1. The line is perpendicular to a specific line, say $y = mx + c$.
2. The line is perpendicular to a line passing through the origin with a known slope.
3. The problem is to find the general form of a line through (0, -3) perpendicular to a given line with known equation.

For clarity and completeness, we will analyze the most common and instructive scenario: finding the equation of a line passing through (0, -3) and perpendicular to a given line with a known slope.

Step-by-Step Approach to Solving the Problem

1. Determining the Slope of the Original Line

Suppose the original line is given in slope-intercept form:

$$y = m_x x + c_x$$

where m_x is the slope of the given line.

For example, let's consider a specific case:

- The given line: $y = 2x + 5$

This line has a slope $m_x = 2$.

2. Finding the Slope of the Perpendicular Line

Using the property of perpendicular lines:

$$m_{perp} = -\frac{1}{m_x}$$

In our example:

$$m_{perp} = -\frac{1}{2}$$

This is the slope of the line we are seeking, which passes through (0, -3).

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3. Formulating the Equation of the Perpendicular Line

With the slope m_{perp} and the point $(x_1, y_1) = (0, -3)$, we can write the equation using the point-slope form:

$$y - y_1 = m_{perp} (x - x_1)$$

Plugging in the known values:

$$y - (-3) = -\frac{1}{2} (x - 0)$$

Simplifies to:

$$ y + 3 = -\frac{1}{2} x $$

Or, in slope-intercept form:

$$ y = -\frac{1}{2} x - 3 $$

This is the equation of the line passing through (0, -3) and perpendicular to $y = 2x + 5$.

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4. Generalizing the Approach for Any Given Line

Suppose the original line has an arbitrary slope m. The steps are:

- Identify the slope m of the given line.
- Calculate the perpendicular slope:

$$ m_{perp} = -\frac{1}{m} $$

- Use the point $(0, -3)$ and the slope m_{perp} to write the line's equation:

$$ y + 3 = m_{perp} (x - 0) $$

Simplify:

$$ y = m_{perp} x - 3 $$

This method applies universally, provided the original line's slope is known.

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5. Alternative Scenarios and Additional Considerations

When the Original Line is Vertical

If the original line is vertical, say $x = k$, its slope is undefined. The perpendicular to a vertical line is a horizontal line:

$$y = y_1$$

Since the line passes through (0, -3), the perpendicular line is:

$$y = -3$$

When the Original Line is Horizontal

If the line is horizontal, $y = c$, its slope is 0. The perpendicular line is vertical:

$$x = x_1$$

In this case, passing through (0, -3):

$$x = 0$$

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6. Visualizing the Geometric Relationships

Understanding the geometric relationships enhances comprehension. Visual aids show how lines with slopes m and $-1/m$ intersect at right angles. For example:

- The original line $y = 2x + 5$ has a positive slope.

- Its perpendicular line has a negative reciprocal slope, $-\frac{1}{2}$.
- The point (0, -3) lies on the perpendicular line, which can be visualized as crossing the original line at a right angle.

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7. Practical Applications and Real-World Relevance

The ability to find perpendicular lines is not purely academic; it has practical applications such as:

- Engineering Design: Ensuring components are orthogonal.
- Navigation: Calculating paths perpendicular to existing routes.
- Computer Graphics: Rendering perpendicular vectors for shading or object orientation.
- Physics: Analyzing forces acting at right angles.

Mastery of these concepts enables precise problem-solving in various scientific and engineering fields.

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8. Summary and Key Takeaways

- The equation of a line passing through a point and perpendicular to another line depends fundamentally on the slopes.
- The slope of the perpendicular line is the negative reciprocal of the original line's slope.
- The point-slope form simplifies the derivation when passing through a known point.
- Special cases (vertical or horizontal lines) require adapted approaches.
- Understanding the geometric intuition deepens comprehension and aids in problem-solving.

Conclusion

The process of finding the equation of a line that passes through a specific point and is perpendicular to another line exemplifies the elegance and logical structure of coordinate geometry. Whether dealing with standard slopes, vertical or horizontal lines, or more complex scenarios, the principles remain consistent. By mastering these techniques, students and professionals can confidently analyze and solve a wide array of geometric problems, bridging theoretical concepts with practical applications.

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