

Find The Equation Of The Line That Is Parallel To $2x + 5y = 8$, And Passes Through The Point $(-1,0)$. 2

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Understanding how to find the equation of a line that is parallel to a given line and passes through a specific point is a fundamental skill in coordinate geometry. This process involves several steps, including converting the given line to slope-intercept form, identifying the slope, and then applying the point-slope form of a line's equation. In this article, we will explore these concepts in detail, providing a comprehensive guide to solving such problems.

Understanding the Fundamentals of Line Equations

Before delving into the problem, it is essential to understand some basic concepts related to line equations.

Standard Form of a Line: $Ax + By = C$

The given line equation, $2x + 5y = 8$, is in standard form. This form is useful for identifying the coefficients of x and y , which are crucial for calculating the slope.

Slope-Intercept Form: $y = mx + b$

The slope-intercept form expresses a line's equation with the slope (m) and y-intercept (b). It is often more intuitive for understanding the line's steepness and position.

Parallel Lines and Their Slopes

Two lines are parallel if they have the same slope but different y-intercepts. Therefore, finding a line parallel to a given one involves determining its slope and then using that slope to find the new line's equation.

Step 1: Convert the Given Equation to Slope-Intercept Form

The first step in solving the problem is to rewrite the given line's equation in slope-intercept form to identify its slope.

Given Equation:

$$2x + 5y = 8$$

Rearranging for y:

- Subtract $2x$ from both sides:

$$5y = -2x + 8$$

- Divide both sides by 5:

$$y = (-2/5)x + 8/5$$

Identify the Slope:

From the slope-intercept form $y = mx + b$, the slope (m) is:

$$m = -2/5$$

This slope indicates the steepness and direction of the original line.

Step 2: Find the Slope of the Parallel Line

Since parallel lines share the same slope:

$$m_{\text{parallel}} = -2/5$$

The new line passing through a specific point will have this same slope.

Step 3: Use the Point-Slope Form to Find the Equation

The point-slope form of a line's equation is:

$$y - y_1 = m(x - x_1)$$

Where (x_1, y_1) is the point the line passes through, and m is the slope.

Given Point:

$(-1, 0)$

Applying the formula:

$$y - 0 = (-2/5)(x - (-1))$$

Simplify:

$$y = (-2/5)(x + 1)$$

Step 4: Write the Equation in Slope-Intercept or Standard Form

To make the equation more explicit, expand and simplify:

$$y = (-2/5)x - (2/5)(1)$$

$$y = (-2/5)x - 2/5$$

This is the slope-intercept form of the line passing through $(-1, 0)$ and parallel to $2x + 5y = 8$.

Alternatively, to express it in standard form:

- Multiply both sides by 5 to clear the denominator:

$$5y = -2x - 2$$

- Rearranged:

$$2x + 5y = -2$$

This is the standard form of the required line.

Summary of the Solution

- The original line's slope: $-2/5$
- The parallel line passes through $(-1, 0)$
- Equation in slope-intercept form: $y = (-2/5)x - 2/5$
- Equation in standard form: $2x + 5y = -2$

Additional Insights and Tips

Understanding how to manipulate equations to find the desired line is vital. Here are some tips:

Convert to Slope-Intercept Form for Clarity

Transforming equations into $y = mx + b$ makes it easier to identify slopes and y-intercepts.

Always Verify the Slope

When dealing with parallel lines, double-check that the slopes are the same before proceeding.

Use Point-Slope Form for Passing Through Specific Points

This form simplifies plugging in the point and slope to find the new line's equation.

Check Your Work

Always verify whether the point satisfies the derived line equation to ensure correctness.

Additional Examples for Practice

To solidify understanding, consider the following practice problems:

1. Find the equation of the line parallel to $3x - 4y = 7$ that passes through the point $(2, -1)$.
2. Determine the line passing through $(0, 3)$ and parallel to $y = (1/2)x + 4$.
3. Find the equation of the line passing through $(-3, 5)$ and parallel to $5x + 2y = 10$.

Each problem involves similar steps: convert the given line to slope-intercept form, identify the slope, and then use the point-slope form with

the given point.

Common Mistakes to Avoid

While solving these types of problems, be cautious of the following pitfalls:

- **Incorrectly identifying the slope:** Always convert the line to slope-intercept form first.
- **Mixing up the point coordinates:** Double-check the point provided to avoid errors.
- **Not simplifying equations:** Always simplify your final answer for clarity and correctness.
- **Using the wrong form:** Remember, different forms serve different purposes; choose the one suitable for your needs.

Conclusion

Finding the equation of a line parallel to a given line and passing through a specific point involves understanding the relationship between the slope, point, and line equations. By converting the original line to slope-intercept form, identifying the slope, and then applying the point-slope formula, you can efficiently derive the desired line's equation. Practice with various problems enhances your proficiency in these techniques, which are foundational in coordinate geometry and essential for solving many real-world mathematical problems.

Additional Resources

For further learning, consider exploring:

- Video tutorials on line equations and parallel lines
- Interactive graphing tools to visualize lines and their relationships
- Practice worksheets with varying difficulty levels
- Math textbooks covering algebra and coordinate geometry fundamentals

Understanding these core concepts equips you with the necessary skills to approach a wide range of problems involving lines, slopes, and equations, making you more confident in tackling related mathematical challenges.

Frequently Asked Questions

What is the slope of the line parallel to $2x + 5y = 8$?

First, rewrite the equation in slope-intercept form: $5y = -2x + 8$, so $y = (-2/5)x + 8/5$. The slope is $-2/5$. Therefore, the slope of the parallel line is also $-2/5$.

How do I find the equation of a line parallel to $2x + 5y = 8$ passing through the point $(-1, 0)$?

Use the point-slope form: $y - y_1 = m(x - x_1)$. With slope $m = -2/5$ and point $(-1, 0)$, we get $y - 0 = -2/5(x + 1)$. Simplify to find the equation.

What is the equation of the line passing through $(-1, 0)$ with slope $-2/5$?

Using point-slope form: $y = -2/5(x + 1)$. Simplify: $y = -2/5x - 2/5$.

How can I verify that the line passes through the point $(-1, 0)$?

Substitute $x = -1$ into the line's equation: $y = -2/5(-1) - 2/5 = 2/5 - 2/5 = 0$. Since $y = 0$, the point $(-1, 0)$ lies on the line.

What is the final equation of the line parallel to $2x + 5y = 8$ passing through $(-1, 0)$?

The equation is $y = -2/5x - 2/5$.

Additional Resources

Find The Equation Of The Line That Is Parallel To $2x + 5y = 8$, And Passes Through The Point $(-1, 0)$. 2

Understanding the geometric and algebraic principles behind lines in coordinate geometry is essential for solving problems involving parallel lines and given points. In this detailed exploration, we will dissect the problem statement, understand the fundamental concepts involved, and methodically derive the required equation of the line. This comprehensive review aims to enhance your grasp of the topic, provide clarity on the processes involved, and reinforce your problem-solving skills.

Understanding the Basic Equation of a Line

Before diving into the specifics of the problem, it's crucial to revisit the general forms and properties of linear equations in two variables.

The Standard Form of a Line

- The most common form of a line's equation in Cartesian coordinates is the standard form:

$$Ax + By = C$$

where A , B , and C are constants, and x and y are variables representing any point on the line.

- For example, in the given equation $(2x + 5y = 8)$, the coefficients $(A=2)$, $(B=5)$, and $(C=8)$.

Slope-Intercept Form

- Alternatively, the slope-intercept form is:

$$y = mx + c$$

where:

- m is the slope of the line.
- c is the y-intercept (the point where the line crosses the y-axis).

- Converting the given equation into slope-intercept form helps us understand its slope, which is vital for establishing parallelism.

Analyzing the Given Line: $(2x + 5y = 8)$

To find a line parallel to the given line, we must first determine its slope. This process involves converting the line's equation into slope-intercept form.

Converting to Slope-Intercept Form

Starting with:

$$\begin{aligned} & \backslash[\\ & 2x + 5y = 8 \\ & \backslash] \end{aligned}$$

Solve for y :

$$\begin{aligned} & \backslash[\\ & 5y = -2x + 8 \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & y = -\frac{2}{5}x + \frac{8}{5} \\ & \backslash] \end{aligned}$$

Key insight:

- The slope of the line $(2x + 5y = 8)$ is:

$$\begin{aligned} & \backslash[\\ & m = -\frac{2}{5} \\ & \backslash] \end{aligned}$$

This is a critical step because parallel lines in a plane share the same slope. Therefore, the line we are asked to find must also have a slope of $(-\frac{2}{5})$.

Understanding Parallel Lines in Coordinate Geometry

Definition of Parallel Lines

- Two lines are parallel if they never intersect, regardless of how far they are extended.
- Geometrically, parallel lines have the same slope but different y-intercepts.

Implication for Our Problem

- Since the required line is parallel to $(2x + 5y = 8)$, it must have:

$$m = -\frac{2}{5}$$

- The line passes through the point $(-1, 0)$, i.e., when $(x = -1)$, $(y = 0)$.

Deriving the Equation of the Parallel Line Passing Through $(-1, 0)$

Given the slope $(m = -\frac{2}{5})$ and the point $(-1, 0)$, we can use the point-slope form to find the desired line's equation.

Point-Slope Form of a Line

$$y - y_1 = m(x - x_1)$$

where:

- (x_1, y_1) is the point on the line: here, $(-1, 0)$.
- (m) is the slope: here, $(-\frac{2}{5})$.

Applying the Point-Slope Formula

Substitute the values:

$$y - 0 = -\frac{2}{5}(x - (-1))$$

$$y = -\frac{2}{5}(x + 1)$$

Expanding to Slope-Intercept or Standard Form

Distribute:

$$y = -\frac{2}{5}x - \frac{2}{5}$$

This is the slope-intercept form:

$$\boxed{y = -\frac{2}{5}x - \frac{2}{5}}$$

Alternatively, converting to standard form:

$$5y = -2x - 2$$

$$2x + 5y = -2$$

Thus, the equation of the required line in standard form is:

$$\boxed{2x + 5y = -2}$$

Verification of the Solution

Verification ensures that the derived line indeed passes through $((-1, 0))$ and remains parallel to the original line.

Checking the Point $((-1, 0))$

Plug into the final equation:

$$2x + 5y = -2$$

$$2(-1) + 5(0) = -2$$

$$-2 + 0 = -2$$

which confirms the point lies on the line.

Confirming Parallelism

- The original line's slope: $-\frac{2}{5}$
- The new line's slope: derived from the slope-intercept form, $-\frac{2}{5}$

Since both slopes are identical, the lines are parallel.

Additional Insights and Variations

Beyond the primary solution, it's instructive to consider alternate approaches, implications, and related concepts.

Using the Standard Form Directly

- When lines are given in standard form, the slope can be obtained by:

$$m = -\frac{A}{B}$$

- For the original line $(2x + 5y = 8)$:

$$m = -\frac{2}{5}$$

- For the parallel line passing through $(-1, 0)$, the standard form:

$$2x + 5y = C$$

- To find (C) , substitute the point $(-1, 0)$:

$$2(-1) + 5(0) = C$$

$$-2 + 0 = C$$

$$C = -2$$

- Therefore, the equation:

$$2x + 5y = -2$$

matches our previous derivation.

Graphical Representation

- Graphing both lines reveals their parallelism.
- The original line passes through points such as $((0, \frac{8}{5}))$ and $((4, 0))$.
- The new line passes through $((-1, 0))$ and is shifted vertically, maintaining the same slope.

Real-World Applications

- Parallel lines are fundamental in fields like architecture, engineering, and physics.
- For example, designing parallel beams, roads, or rails requires understanding lines' slopes and positions.

Common Mistakes and Clarifications

Engaging with problems involving parallel lines can lead to typical errors. Recognizing and clarifying these pitfalls enhances problem-solving accuracy.

Potential Mistakes:

1. Confusing the slope with the coefficients in the standard form:

- Remember, the slope $(m = -A/B)$ for lines in standard form $(Ax + By = C)$.

2. Incorrectly converting the given equation:

- Ensure proper algebraic manipulation when converting to slope-intercept form.

3. Misapplying the point-slope formula:

- Always substitute correctly; verify the point lies on the derived line.

4. Mixing up parallel and perpendicular lines:

- Parallel lines share the same slope; perpendicular lines have slopes that are negative reciprocals.

Summary and Final Remarks

This comprehensive review emphasizes the systematic approach to finding the equation of a line parallel to a given line and passing through a specific point. The key steps include:

- Recognizing the importance of the slope in defining line relationships.
- Converting the given line into slope form to identify its slope.
- Applying the point-slope form with the given point and the known slope.
- Converting the equation into standard or slope-intercept form for clarity and verification.
- Confirming the solution by substitution and understanding the geometric implications.

Final Answer:

```
\[
\boxed{
\text{The equation of the line parallel to } 2x + 5y = 8 \text{ passing}
\text{ through } (-1, 0) \text{ is } 2x + 5y = -2
}
\]
```

This line maintains the same slope $(-\frac{2}{5})$ as the original line and passes through the specified point, fulfilling all the problem's criteria

Find The Equation Of The Line That Is Parallel To $2x - 5y = 8$ And Passes Through The Point $(1, 0)$

Find The Equation Of The Line That Is Parallel To $2x - 5y = 8$ And Passes Through The Point $(1, 0)$

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