Find The Equation Of The Plane Through The Point P=(4,4,2) And Parallel To The Plane 2 Y-4 X-3 Z=-9 .

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Understanding how to find the equation of a plane that passes through a specific point and is parallel to another given plane is a fundamental concept in three-dimensional geometry. This process involves leveraging the properties of planes, such as their normal vectors, and applying algebraic methods to derive the desired equation. In this article, we will walk through the step-by-step procedure to find the equation of a plane passing through the point P=(4,4,2) and parallel to the plane 2Y - 4X - 3Z = -9, ensuring clarity and depth for learners and enthusiasts alike.

Understanding the Basics of Plane Equations

Before diving into the solution, it’s essential to revisit the fundamental form of a plane's equation and the properties that relate to parallel planes.

The General Equation of a Plane

The most common form of a plane's equation in three-dimensional space is:

$$
A x + B y + C z + D = 0
$$

where:

- A, B, C are the coefficients that form the normal vector $\vec{n} = (A, B, C)$ to the plane.
- D is a constant that shifts the plane relative to the origin.

Normal Vectors and Parallel Planes

Two planes are parallel if and only if their normal vectors are parallel or scalar multiples of each other. This means that if the normal vector of the first plane is $\vec{n}_1$ and that of the second plane is $\vec{n}_2$, then:

$$
\vec{n}_1 \parallel \vec{n}_2
$$

or

$$
\vec{n}_1 = k \vec{n}_2 \quad \text{for some scalar } k
$$

In our problem, since the second plane is given, we can extract its normal vector directly from its coefficients.

Step-by-Step Solution to Find the Required Plane Equation

The process involves identifying the normal vector of the given plane, then using the point and normal vector to formulate the plane's equation.

Step 1: Extract the Normal Vector of the Given Plane

The given plane equation is:

$$
2Y - 4X - 3Z = -9
$$

Rearranged into the standard form:

$$
-4X + 2Y - 3Z = -9
$$

or:

$$
-4x + 2y - 3z + 9 = 0
$$

From this, the coefficients are:

$$
A = -4, \quad B = 2, \quad C = -3
$$

Thus, the normal vector of the given plane is:

$$
\vec{n} = (-4, 2, -3)
$$

Step 2: Confirm the Normal Vector for the Parallel Plane

Since the new plane is parallel to the given one, it will have the same normal vector:

$$
\vec{n}_{new} = (-4, 2, -3)
$$

The goal is to find the equation of the plane passing through point $P=(4,4,2)$ with this normal vector.

Step 3: Use the Point-Normal Form of a Plane Equation

The point-normal form of a plane's equation is:

$$
A (x - x_0) + B (y - y_0) + C (z - z_0) = 0
$$

where (x_0, y_0, z_0) is a point on the plane (here, $P=(4,4,2)$) and (A, B, C) is the normal vector.

Substituting the known values:

$$
-4(x - 4) + 2(y - 4) - 3(z - 2) = 0
$$

Step 4: Simplify the Equation

Expanding the terms:

$$
-4x + 16 + 2y - 8 - 3z + 6 = 0
$$

Combine like terms:

$$
-4x + 2y - 3z + (16 - 8 + 6) = 0
$$

$$
-4x + 2y - 3z + 14 = 0
$$

To write the equation in standard form:

$$
-4x + 2y - 3z = -14
$$

Alternatively, multiply both sides by -1 to make the coefficient of x positive:

$$
4x - 2y + 3z = 14
$$

This is the equation of the plane passing through $P=(4,4,2)$ and parallel to the given plane.

Final Equation of the Plane

The complete answer is:

$$
\boxed{
4x - 2y + 3z = 14
}
$$

This plane is parallel to the original plane $2Y - 4X - 3Z = -9$ and passes through the point $P=(4,4,2)$.

Additional Insights and Tips

Verifying Parallelism

To confirm that the two planes are parallel, ensure their normal vectors are scalar multiples:

- Normal vector of original plane: $\vec{n}_1 = (-4, 2, -3)$

- Normal vector of new plane: $\vec{n}_2 = (4, -2, 3)$

Notice:

$$
(4, -2, 3) = -1 \times (-4, 2, -3)
$$

They are scalar multiples, confirming the planes are parallel.

Understanding the Role of the Constant Term

The constant term (here, 14) determines the position of the plane relative to the origin. Changing this constant shifts the plane without altering its orientation, as long as the normal vector remains the same.

Summary of Key Steps

- Identify the normal vector from the given plane coefficients.
- Use the point-normal form of a plane equation with the known point and normal vector.
- Simplify and write the final equation in standard form.
- Verify parallelism by comparing normal vectors.

Conclusion

Finding the equation of a plane through a specific point and parallel to a given plane involves understanding the relationship between planes and their normal vectors. By extracting the normal vector from the given plane, applying the point-normal form, and simplifying, you can derive the exact equation of the desired plane. This process is crucial in various applications across geometry, physics, and engineering, where spatial relationships and plane equations are fundamental.

Remember, mastering this method enhances spatial visualization skills and prepares you for more complex three-dimensional geometric problems. Whether you're a student, educator, or professional, these concepts form the backbone of understanding the geometry of our three-dimensional world.

Frequently Asked Questions

How do you find the equation of a plane passing through a point and parallel to a given plane?

To find the equation, identify the normal vector of the given plane, which remains the same for the parallel plane. Then, use the point to write the equation in the form $n \cdot (r - r_0) = 0$, where n is the normal vector and r_0 is the given point.

What is the normal vector of the plane 2Y - 4X - 3Z = -9?

Rearranging the plane equation as -4X + 2Y - 3Z = -9, the normal vector is derived from the coefficients of X, Y, Z, which is n = (-4, 2, -3).

How do you write the equation of a plane passing through point P=(4,4,2) with a known normal vector?

Use the point-normal form: $(x - x_0, y - y_0, z - z_0) \cdot n = 0$. Substituting P and n, the equation becomes (-4)(x - 4) + 2(y - 4) - 3(z - 2) = 0.

What is the equation of the plane through P=(4,4,2) parallel to 2Y - 4X - 3Z = -9?

The equation is -4(x - 4) + 2(y - 4) - 3(z - 2) = 0, which simplifies to -4x + 16 + 2y - 8 - 3z + 6 = 0, or -4x + 2y - 3z + 14 = 0.

How do you verify that the new plane is parallel to the given plane?

Check that both planes have the same normal vector. Since both equations have coefficients (-4, 2, -3), their normal vectors are identical, confirming parallelism.

Can you provide the final simplified equation of the plane passing through P=(4,4,2) and parallel to 2Y - 4X - 3Z = -9?

Yes, the simplified equation is -4x + 2y - 3z + 14 = 0.

Additional Resources

Geometry

Understanding the Problem: Finding the Equation of a Plane Through a Point and Parallel to a Given Plane

In the realm of three-dimensional geometry, the task of determining the equation of a plane based on certain conditions is both fundamental and widely applicable. Here, we are presented with a classic problem: Find the equation of a plane that passes through a specific point, P = (4, 4, 2), and is parallel to a given plane, 2Y - 4X - 3Z = -9.

This problem highlights core concepts in vector geometry and plane equations, and solving it involves understanding the relationship between planes, their normal vectors, and how to leverage this understanding to formulate the required plane's equation.

Key Concepts and Foundations

Before delving into the step-by-step solution, it's essential to review some fundamental concepts that underpin the process:

1. Equation of a Plane

The general form of a plane's equation in three-dimensional space is:

$$ A x + B y + C z + D = 0 $$

where:

- (A, B, C) is the normal vector to the plane, perpendicular to every line lying on the plane.
- D is a constant that shifts the plane's position in space.

2. Parallel Planes

- Two planes are parallel if their normal vectors are parallel (i.e., scalar multiples of each other).
- Parallel planes have the same normal vector (or proportional normal vectors), but different D values.

3. Finding a Plane Through a Point

- To find the specific plane passing through a point P, once the normal vector is known, substitute P into the plane equation to solve for D.

Step 1: Extracting the Given Plane's Normal Vector

The given plane's equation is:

$$ 2 y - 4 x - 3 z = -9 $$

Rearranged into standard form:

$$ -4 x + 2 y - 3 z = -9 $$

From this, the coefficients of x, y, z give the normal vector:

$$ \mathbf{n} = \langle -4, 2, -3 \rangle $$

This vector is perpendicular to the plane and is crucial because the plane we seek must be parallel to this plane, and thus will have the same normal vector (or a scalar multiple).

Step 2: Establishing the Normal Vector for the New Plane

Since the new plane is parallel to the given plane, their normal vectors are parallel. Therefore:

$$ \mathbf{n}_{new} = k \times \mathbf{n}_{original} $$

for some scalar k. However, for simplicity, and because the problem does not specify any scaling, we can use the same normal vector:

$$ \mathbf{n} = \langle -4, 2, -3 \rangle $$

This choice simplifies calculations and aligns with the general property of parallel planes having identical or proportional normal vectors.

Step 3: Formulating the Equation of the New Plane

The general form of the plane is:

$$ -4 x + 2 y - 3 z + D = 0 $$

where D is to be determined.

Step 4: Using the Point P = (4, 4, 2) to Find D

Since the plane passes through point P, substitute $(x, y, z) = (4, 4, 2)$:

$$
-4(4) + 2(4) - 3(2) + D = 0
$$

Calculating:

$$
-16 + 8 - 6 + D = 0
$$
$$
(-16 + 8 - 6) + D = 0
$$
$$
(-14) + D = 0
$$

Solving for D:

$$
D = 14
$$

Final Equation of the Plane

Putting everything together, the equation of the plane is:

$$
-4 x + 2 y - 3 z + 14 = 0
$$

or, equivalently,

$$
4 x - 2 y + 3 z - 14 = 0
$$

This is the equation of the plane passing through the point $P = (4, 4, 2)$ and parallel to the plane $2 y - 4 x - 3 z = -9$.

Additional Insights and Variations

1. Understanding the Role of Scalar Multiples

While we used the original normal vector directly, it's worth noting:

- If desired, the normal vector could be scaled by any non-zero scalar k, which would change D accordingly, but the geometric plane remains the same.
- For example, using $2 \times \mathbf{n} = \langle -8, 4, -6 \rangle$, the D value would double, leading to a different constant term, but the same plane.

2. Verifying Parallelism

To verify the planes are parallel:

- Check if their normal vectors are scalar multiples.
- Since both have normal vectors proportional to $\langle -4, 2, -3 \rangle$, the planes are indeed parallel.

3. Visualizing the Plane

- The plane intersects the coordinate axes at points where the other variables are zero, which can be calculated for a better geometric understanding.
- For example, setting $y = 0, z = 0$:

$$4x - 14 = 0 \Rightarrow x = \frac{14}{4} = 3.5$$

- Similarly, other intercepts can be found for a complete picture.

Practical Applications and Relevance

Understanding how to find a plane's equation given a point and a parallel plane has numerous applications across engineering, computer graphics, physics, and more. For instance:

- Design and Manufacturing: Creating components that need to be aligned with existing surfaces.
- Computer Graphics: Calculating planes for rendering scenes.
- Physics: Analyzing fields or wavefronts constrained to certain surfaces.

Summary of the Process

To encapsulate, the method involves:

1. Identifying the normal vector of the reference plane.
2. Using the same normal vector for the parallel plane.
3. Substituting the known point into the plane equation to solve for the constant D.
4. Formulating the final equation accordingly.

By following these steps, the problem becomes straightforward, illustrating the elegance and power of vector geometry in solving three-dimensional problems.

Conclusion

The journey to determine the equation of a plane through a point and parallel to a given plane exemplifies the synergy of algebraic and geometric reasoning. It underscores the importance of understanding normal vectors, the properties of parallelism, and how points influence the positioning of planes in space.

The derived plane:

$$
\boxed{4 x - 2 y + 3 z - 14 = 0}
$$

not only satisfies the conditions but also reinforces fundamental principles of three-dimensional geometry, serving as a reliable template for tackling similar problems with confidence and clarity.

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