Find The First Partial Derivatives And Evaluate Each At The Given Point. Function Point W = 3x2y 7xyz

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Understanding how to compute partial derivatives is fundamental in multivariable calculus, especially when analyzing how a function changes with respect to each variable independently. In this article, we will explore the process of finding the first partial derivatives of a given function and then evaluate these derivatives at a specified point. The function under consideration is:

$$ W = 3x^2 y + 7xyz $$

This function involves multiple variables (x, y, z), and calculating its partial derivatives provides insight into the rate of change of W concerning each variable independently. Let's begin by understanding the structure of the function and then proceed step-by-step through the derivatives and their evaluations.

Understanding the Function

Before diving into derivatives, it's helpful to analyze the structure of the function:

$$ W = 3x^2 y + 7xyz $$

- The first term, $3x^2 y$, is a product of x^2 and y, scaled by 3.
- The second term, $7xyz$, is a product of all three variables, scaled by 7.

This function combines polynomial terms involving different variables, making it a good candidate for applying basic rules of differentiation such as the power rule and the product rule.

Finding the First Partial Derivatives

Partial derivatives measure how the function W changes as one variable varies, keeping the others constant. We will compute:

- $\frac{\partial W}{\partial x}$
- $\frac{\partial W}{\partial y}$
- $\frac{\partial W}{\partial z}$

Let's analyze each one carefully.

Partial Derivative with respect to x: $\frac{\partial W}{\partial x}$

To find $\frac{\partial W}{\partial x}$:

1. Differentiate the first term, $3x^2 y$:
- Treat y as a constant.
- The derivative of x^2 with respect to x is $2x$.

- So, $\frac{\partial}{\partial x}(3x^2 y) = 3 \times 2x \times y = 6xy$.

2. Differentiate the second term, $7xyz$:
- Treat y and z as constants.
- The derivative of x is 1.
- So, $\frac{\partial}{\partial x}(7xyz) = 7 y z$.

Combining these results:

$$
\boxed{
\frac{\partial W}{\partial x} = 6xy + 7 y z
}
$$

Partial Derivative with respect to y: $\frac{\partial W}{\partial y}$

To find $\frac{\partial W}{\partial y}$:

1. Differentiate the first term, $3x^2 y$:
- Treat x as a constant.
- The derivative of y with respect to y is 1.
- So, $\frac{\partial}{\partial y}(3x^2 y) = 3x^2$.

2. Differentiate the second term, $7xyz$:
- Treat x and z as constants.
- The derivative of y is 1.
- So, $\frac{\partial}{\partial y}(7xyz) = 7 x z$.

Combined result:

$$
\boxed{
\frac{\partial W}{\partial y} = 3x^2 + 7 x z
}
$$

Partial Derivative with respect to z: $\frac{\partial W}{\partial z}$

To find $\frac{\partial W}{\partial z}$:

1. Differentiate the first term, $3x^2 y$:
- Since there is no z in this term, its derivative with respect to z is 0.

2. Differentiate the second term, $7xyz$:
- Treat x and y as constants.
- The derivative of z is 1.
- So, $\frac{\partial}{\partial z}(7xyz) = 7 x y$.

Result:

$$
\boxed{
\frac{\partial W}{\partial z} = 7 x y
}
$$

Evaluating the Partial Derivatives at a Given Point

Suppose we are asked to evaluate these derivatives at the point $(x, y, z) = (x_0, y_0, z_0)$. For illustrative purposes, let's choose:

$$ (x, y, z) = (1, 2, 3) $$

Now, substitute these values into each derivative.

Evaluating $\frac{\partial W}{\partial x}$ at $(1, 2, 3)$:

$$
\frac{\partial W}{\partial x} = 6xy + 7 y z
$$

Substitute:

$$
6 \times 1 \times 2 + 7 \times 2 \times 3 = 12 + 42 = 54
$$

Result:

$$
\boxed{
\left.\frac{\partial W}{\partial x}\right|_{(1,2,3)} = 54
}
$$

Evaluating $\frac{\partial W}{\partial y}$ at $(1, 2, 3)$:

$$
\frac{\partial W}{\partial y} = 3x^2 + 7 x z
$$

Substitute:

$$
3 \times 1^2 + 7 \times 1 \times 3 = 3 + 21 = 24
$$

Result:

$$
\boxed{
\left.\frac{\partial W}{\partial y}\right|_{(1,2,3)} = 24
}
$$

Evaluating $\frac{\partial W}{\partial z}$ at $(1, 2, 3)$:

$$
\frac{\partial W}{\partial z} = 7 x y
$$

Substitute:

$$
7 \times 1 \times 2 = 14
$$

Result:

$$
\boxed{
\left.\frac{\partial W}{\partial z}\right|_{(1,2,3)} = 14
}
$$

Summary of Results

Partial Derivative	Expression	Evaluated at (1, 2, 3)
$\frac{\partial W}{\partial x}$	$6xy + 7 y z$	54
$\frac{\partial W}{\partial y}$	$3x^2 + 7 x z$	24
$\frac{\partial W}{\partial z}$	$7 x y$	14

These calculations demonstrate how partial derivatives provide a detailed picture of how a multivariable function behaves locally around a specific point. They are essential in optimization, gradient calculations, and understanding the sensitivity of the function to changes in each variable.

Conclusion

In this comprehensive guide, we have:

- Analyzed the structure of the function $W = 3x^2 y + 7xyz$.
- Computed the first partial derivatives with respect to each variable using rules of differentiation.
- Evaluated these derivatives at a specific point $(x, y, z) = (1, 2, 3)$.

Mastering the process of finding and evaluating partial derivatives is crucial in multivariable calculus, with applications spanning physics, engineering, economics, and data science. By practicing these steps with different functions and points, you can enhance your understanding of how multivariable functions behave and how to apply derivatives effectively in various contexts.

Frequently Asked Questions

What is the first partial derivative of $W = 3x^2y + 7xyz$ with respect to x?

The partial derivative of W with respect to x is $\partial W/\partial x = 6xy + 7yz$.

What is the first partial derivative of $W = 3x^2y + 7xyz$ with respect to y?

The partial derivative of W with respect to y is $\partial W/\partial y = 3x^2 + 7xz$.

What is the first partial derivative of $W = 3x^2y + 7xyz$ with respect to z?

The partial derivative of W with respect to z is $\partial W/\partial z = 7xy$.

How do you evaluate $\partial W/\partial x$ at the point $(x, y, z) = (1, 2, 3)$?

Substitute $x=1$, $y=2$, $z=3$ into $\partial W/\partial x = 6xy + 7yz$: $\partial W/\partial x = 612 + 723 = 12 + 42 = 54$.

How do you evaluate $\partial W/\partial y$ at the point $(x, y, z) = (1, 2, 3)$?

Substitute into $\partial W/\partial y = 3x^2 + 7xz$: $\partial W/\partial y = 31^2 + 713 = 3 + 21 = 24$.

How do you evaluate $\partial W/\partial z$ at the point $(x, y, z) = (1, 2, 3)$?

Substitute into $\partial W/\partial z = 7xy$: $\partial W/\partial z = 712 = 14$.

Why are partial derivatives important in analyzing the function $W = 3x^2y + 7xyz$?

Partial derivatives measure how W changes with respect to each variable independently, helping to understand the function's behavior and optimize it.

Can you explain the process of finding the first partial derivatives for a multivariable function?

Yes, to find the first partial derivatives, differentiate the function with respect to one variable while treating all other variables as constants.

What is the significance of evaluating partial derivatives at a specific point?

Evaluating partial derivatives at a specific point provides the rate of change of the function in each variable's direction at that point, useful for tangent plane and gradient calculations.

Additional Resources

Find The First Partial Derivatives And Evaluate Each At The Given Point. Function Point $W = 3x^2y + 7xyz$

In the realm of multivariable calculus, understanding how a function behaves with respect to each of its variables is crucial for analyzing complex systems. Whether in physics, engineering, or economics, the ability to compute and interpret partial derivatives provides insight into how a change in one variable influences the entire system. Today, we delve into a specific function: $W = 3x^2 y + 7xyz$. Our goal is to find the first partial derivatives of this function with respect to x, y, and z, and then evaluate each derivative at a specified point. This process not only sharpens our calculus skills but also illuminates the nuanced ways in which the variables interact within the function's structure.

Understanding the Function: $W = 3x^2 y + 7xyz$

Before embarking on derivative calculations, it's essential to grasp the nature of the function itself. The function W is a multivariable expression involving three variables—x, y, and z. It combines quadratic, linear, and multiplicative terms:

- The term $3x^2 y$ indicates a quadratic dependence on x, scaled by y.
- The term $7xyz$ is a product of all three variables, scaled by 7.

This structure signifies that W responds differently to changes in each variable, depending on their current values and the interaction terms. To analyze how W changes with each variable independently, we compute its partial derivatives.

The Concept of Partial Derivatives

Partial derivatives measure the rate at which a multivariable function changes as one variable varies, with all other variables held constant. For a function $W(x, y, z)$:

- $\frac{\partial W}{\partial x}$ assesses how W changes when x changes, with y and z fixed.
- $\frac{\partial W}{\partial y}$ measures the sensitivity of W to changes in y.
- $\frac{\partial W}{\partial z}$ examines how W responds to variations in z.

Calculating these derivatives involves applying standard differentiation rules—product rule, chain rule, power rule—while treating other variables as constants.

Step-by-Step Calculation of the Partial Derivatives

1. Partial Derivative with Respect to x: $\frac{\partial W}{\partial x}$

Given:

$$
W = 3x^2 y + 7xyz
$$

Treat y and z as constants:

- For $3x^2 y$, the derivative with respect to x is:

\[

$$
\frac{\partial}{\partial x} (3x^2 y) = 3 y \times \frac{\partial}{\partial x} (x^2) = 3 y \times 2 x = 6 x y
$$

- For $7xyz$, the derivative with respect to x (with y, z constants):

$$
\frac{\partial}{\partial x} (7xyz) = 7 y z \times \frac{\partial}{\partial x} (x) = 7 y z
$$

Combining these:

$$
\boxed{
\frac{\partial W}{\partial x} = 6 x y + 7 y z
}
$$

---

2. Partial Derivative with Respect to y: $\frac{\partial W}{\partial y}$

Treat x and z as constants:

- $3x^2 y$:

$$
\frac{\partial}{\partial y} (3x^2 y) = 3 x^2
$$

- $7xyz$:

$$
\frac{\partial}{\partial y} (7xyz) = 7 x z
$$

Adding these together:

$$
\boxed{
\frac{\partial W}{\partial y} = 3 x^2 + 7 x z
}
$$

---

3. Partial Derivative with Respect to z: $\frac{\partial W}{\partial z}$

Treat x and y as constants:

- $3x^2 y$:

$$
\frac{\partial}{\partial z} (3x^2 y) = 0
$$
(since it's independent of z)

- $7xyz$:

$$

$$
\frac{\partial}{\partial z} (7xyz) = 7 x y
$$

Thus:

$$
\boxed{
\frac{\partial W}{\partial z} = 7 x y
}
$$

Evaluating the Partial Derivatives at the Given Point

Suppose the point of evaluation is $(x, y, z) = (1, 2, 3)$. Substituting these into the derivatives:

- $\frac{\partial W}{\partial x}$:

$$
6 \times 1 \times 2 + 7 \times 2 \times 3 = 12 + 42 = 54
$$

- $\frac{\partial W}{\partial y}$:

$$
3 \times 1^2 + 7 \times 1 \times 3 = 3 + 21 = 24
$$

- $\frac{\partial W}{\partial z}$:

$$
7 \times 1 \times 2 = 14
$$

Thus, at the point $(1, 2, 3)$:

$$
\boxed{
\frac{\partial W}{\partial x} = 54,\quad
\frac{\partial W}{\partial y} = 24,\quad
\frac{\partial W}{\partial z} = 14
}
$$

Significance of the Results

Understanding these derivatives at a specific point provides valuable insight into the function's local behavior:

- The high value of $\frac{\partial W}{\partial x}$ (54) indicates that at $(1, 2, 3)$, a small increase in x would significantly increase W. The term $6 x y$ dominates, meaning W is highly sensitive to changes in x owing to the quadratic dependence.

- The derivative $\frac{\partial W}{\partial y} = 24$ suggests that y also influences W notably, especially because of the linear term $3x^2$ and the interaction $7 x z$.

- The derivative $\frac{\partial W}{\partial z} = 14$ shows a moderate sensitivity to changes in z, primarily through the $7xy$ term.

Knowing these sensitivities is essential in fields like optimization, where understanding the gradient vector—composed of these partial derivatives—guides decision-making or system adjustments.

Broader Applications and Implications

Calculating partial derivatives and evaluating them at specific points is foundational across various disciplines. Here are some real-world scenarios where such calculations are vital:

- Engineering Design: Engineers optimize parameters in systems with multiple variables, such as minimizing energy consumption or maximizing structural stability by analyzing how each variable influences the overall system.

- Economics: Economists assess how changing one factor, like labor or capital, impacts output or profit, often represented through multivariable functions.

- Physics: In thermodynamics or electromagnetism, understanding the rate of change of physical quantities with respect to multiple parameters aids in modeling behaviors and predicting outcomes.

- Data Science: Gradient-based algorithms, like gradient descent, rely on partial derivatives to iteratively improve models and find optimal solutions.

Mastering the computation and interpretation of partial derivatives empowers professionals in these fields to make informed decisions based on quantitative insights.

Conclusion

The process of finding the first partial derivatives of a multivariable function and evaluating them at a specific point is a fundamental skill in calculus with extensive applications. For the function $W = 3x^2 y + 7xyz$, we've demonstrated how to systematically compute each derivative, interpret their significance, and evaluate them at a given point. These derivatives reveal how sensitive the function is to changes in each variable, providing a snapshot of its local behavior.

As systems grow more complex, the ability to dissect their mathematical models through partial derivatives becomes increasingly vital. Whether optimizing a design, predicting economic trends, or understanding physical phenomena, these tools offer clarity and precision. By mastering these calculations, students and professionals alike gain a powerful means to analyze and influence the multifaceted systems that shape our world.

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