Find The Function With Derivative F'(x)=e^x That Passes Through The Point P= (0,4/3). F(x)= 1 /3e^3x-

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Understanding how to determine an unknown function based on its derivative and a specific point is a fundamental aspect of calculus. In this guide, we will explore the process of finding the original function $F(x)$ given its derivative $F'(x) = e^x$ and a point $P = (0, \frac{4}{3})$ that the function passes through. We will also analyze the proposed form $F(x) = \frac{1}{3} e^{3x} -$ and clarify whether it aligns with the given conditions. This comprehensive explanation will help students and enthusiasts grasp the concepts of integration, initial conditions, and function reconstruction in calculus.

Understanding the Problem: Derivative and Point Conditions

Given Information

- Derivative of the function: $F'(x) = e^x$
- Point through which the function passes: $P = (0, \frac{4}{3})$
- Proposed form of the function: $F(x) = \frac{1}{3} e^{3x} -$

Objective

1. Determine the original function $F(x)$ that satisfies the derivative and passing point conditions.
2. Verify if the proposed form $F(x) = \frac{1}{3} e^{3x} -$ matches these conditions.

Step 1: Find the General Antiderivative of $F'(x) = e^x$

Integration Process

The derivative $F'(x) = e^x$ suggests that the original function $F(x)$ is an antiderivative of e^x. Recall that the indefinite integral of e^x is straightforward:

$$
F(x) = \int e^x \, dx
$$

Since the integral of e^x with respect to x is $e^x + C$, where C is the constant of integration, we have:

$$
F(x) = e^x + C
$$

This is the general form of the function satisfying the derivative condition.

Implication of the General Form

- The family of functions with derivative e^x is $F(x) = e^x + C$, where C can be any real number.
- To determine the specific C, we need to utilize the point $P = (0, \frac{4}{3})$.

Step 2: Use the Point to Find the Constant of Integration

Applying the Point $P = (0, \frac{4}{3})$

- Substitute $x = 0$ and $F(0) = \frac{4}{3}$ into the general form:

$$
F(0) = e^0 + C = 1 + C
$$

- Set this equal to the y-coordinate of the point:

$$
1 + C = \frac{4}{3}
$$

- Solve for C:

$$
C = \frac{4}{3} - 1 = \frac{4}{3} - \frac{3}{3} = \frac{1}{3}
$$

Result:

$$
F(x) = e^x + \frac{1}{3}
$$

This is the unique function with derivative e^x passing through the point $(0, \frac{4}{3})$.

Step 3: Verify the Proposed Function $F(x) = \frac{1}{3} e^{3x} - $

Now, let's examine whether the provided form:

$$
F(x) = \frac{1}{3} e^{3x} - \text{(something)}
$$

aligns with the derivative $F'(x) = e^x$.

Calculating the Derivative of the Proposed Form

- Differentiate:

$$
F(x) = \frac{1}{3} e^{3x} + K
$$

(where K is a constant representing the subtraction term)

- Derivative:

$$
F'(x) = \frac{1}{3} \times 3 e^{3x} = e^{3x}
$$

- Since $F'(x) = e^{3x}$, this does not match the given $F'(x) = e^x$. Therefore, unless the form is adjusted, it's incompatible.

Adjusting the Proposed Form

Suppose the function was intended as:

$$
F(x) = \frac{1}{3} e^{3x} + C
$$

- Its derivative is $F'(x) = e^{3x}$, which is not equal to e^x.

- To get $F'(x) = e^x$, the antiderivative must be:

$$
F(x) = e^x + C
$$

- Hence, the initial proposal $F(x) = \frac{1}{3} e^{3x} -$ does not align with the derivative e^x. It seems to be a different function with a different derivative.

Conclusion:

The correct function, based on the derivative and point provided, is:

$$
F(x) = e^x + \frac{1}{3}
$$

and the proposed form appears inconsistent with the initial derivative condition.

Additional Insights and Applications

Understanding Derivatives and Antiderivatives

- Derivatives measure the rate of change of a function.
- Antiderivatives are functions whose derivatives give the original function.
- Knowing the derivative and a point allows for the precise determination of the original function.

Practical Applications

- Physics: Calculating displacement from velocity.
- Economics: Finding cost functions from marginal costs.
- Engineering: Signal processing and system responses.

Summary and Key Takeaways

- The derivative \(F'(x) = e^x\) integrates to \(F(x) = e^x + C\).
- The point \(P = (0, \frac{4}{3})\) helps determine the constant \(C = \frac{1}{3}\).
- The specific function satisfying all conditions is:

  \[
  \boxed{
  F(x) = e^x + \frac{1}{3}
  }
  \]

  - The proposed form \(F(x) = \frac{1}{3} e^{3x} - \) does not match the derivative \(e^x\), indicating a different function altogether.

Final Remarks

Understanding how to reconstruct a function from its derivative and a point is a crucial skill in calculus that underpins many real-world applications. Always verify that the derivative of your candidate function matches the given derivative, and use initial points to determine any constants of integration. Whether you're solving academic problems or applying calculus in practical scenarios, these principles form the foundation for accurate and meaningful analysis.

If you need further assistance with similar calculus problems or concepts, consider exploring topics like definite integrals, differential equations, and applications of derivatives in various fields.

Frequently Asked Questions

What is the general form of the function F(x) given its derivative F'(x) = e^x?

The general form of F(x) is F(x) = e^x + C, where C is an arbitrary constant.

How do we find the specific constant C for the function passing through P = (0, 4/3)?

Substitute x=0 and F(0)=4/3 into F(x) = e^x + C, giving 4/3 = e^0 + C, so C = 4/3 - 1 = 1/3.

What is the explicit form of the function F(x) that passes through P=(0, 4/3)?

F(x) = e^x + 1/3.

Is the function F(x) = 1/3 e^x + 4/3 consistent with the given point P=(0, 4/3)?

No, because plugging in x=0 gives F(0) = 1/3 + 4/3 = 5/3, which does not match the point (0, 4/3). The correct function is F(x) = e^x + 1/3.

What is the significance of the constant C in the function's general solution?

The constant C shifts the graph vertically; in this case, C=1/3 ensures the function passes through the specified point.

How does the given derivative F'(x)=e^x help in verifying the function's correctness?

Since the derivative of F(x) = e^x + 1/3 is F'(x) = e^x, it matches the given derivative, confirming the function's correctness.

Additional Resources

Find The Function With Derivative F'(x) = e^x That Passes Through the Point P = (0, 4/3). F(x) = 1 / 3 e^{3x} - ...

In this comprehensive guide, we will explore finding the function with derivative F'(x) = e^x that passes through the point P = (0, 4/3). This problem combines fundamental concepts of calculus, specifically integration and initial conditions, to determine an unknown function. Understanding how to work backward from a derivative to find the original function is a core skill in calculus, and this example provides a clear illustration of that process.

Understanding the Problem Setup

Before jumping into calculations, it's essential to understand what the problem is asking:

- Given the derivative: F'(x) = e^x
- Point the function passes through: P = (0, 4/3)
- Goal: Find the explicit form of the function F(x)

The problem involves two main steps:

1. Integrating the derivative to find the general form of $F(x)$
2. Using the point $P = (0, 4/3)$ to determine the constant of integration

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Step 1: Integrate the Derivative $F'(x) = e^x$

The first step is straightforward: since $F'(x) = e^x$, integrating both sides with respect to x gives us the general form of $F(x)$:

$$
F(x) = \int e^x \, dx
$$

The integral of e^x is well-known:

$$
F(x) = e^x + C
$$

where C is the constant of integration. This is the general indefinite integral, representing all functions whose derivative is e^x.

Step 2: Apply the Initial Condition to Find C

To find the particular function that passes through the point $P = (0, 4/3)$, substitute $x = 0$ and $F(0) = 4/3$ into the general form:

$$
F(0) = e^0 + C = 1 + C
$$

Set this equal to 4/3:

$$
1 + C = \frac{4}{3}
$$

Solve for C:

$$
C = \frac{4}{3} - 1 = \frac{4}{3} - \frac{3}{3} = \frac{1}{3}
$$

Final Function: Explicit Form of $F(x)$

Putting it all together, the function is:

$$
F(x) = e^x + \frac{1}{3}
$$

This function has a derivative of e^x and passes through the point (0, 4/3).

Additional Insights and Variations

Confirming the Derivative

To double-check, differentiate $F(x)$:

$$
F'(x) = \frac{d}{dx} \left(e^x + \frac{1}{3} \right) = e^x
$$

which matches the given derivative exactly.

Visualizing the Function

Graphing $F(x) = e^x + 1/3$ reveals an exponential curve shifted vertically upward by 1/3 units. The point (0, 4/3) lies on this curve, confirming the correctness.

General Approach to Similar Problems

This problem exemplifies the typical approach to inverse problems in calculus involving derivatives:

1. Identify the indefinite integral of the derivative to find the general form.
2. Use initial or boundary conditions to determine the constant of integration.
3. Verify the solution by differentiation or substitution into the original conditions.

Common Pitfalls and Tips

- Forgetting the constant of integration: Always include the constant C when integrating.
- Misapplying initial conditions: Ensure you substitute the correct x-value and corresponding F(x)-value.
- Confusing derivative and original function: Remember, integration is the reverse process of differentiation.

Summary

To summarize, the process of finding the function with derivative $F'(x) = e^x$ that passes through the point $P = (0, 4/3)$ involves:

- Integrating the derivative to obtain the general solution: $F(x) = e^x + C$
- Applying the initial condition to solve for C: $C = 1/3$
- Writing the particular solution: $F(x) = e^x + 1/3$

This method underscores the fundamental link between derivatives and original functions in calculus, highlighting how initial conditions allow us to pinpoint a specific solution from a family of functions.

In conclusion, understanding how to find the original function from its derivative, especially with initial conditions, is a vital skill in calculus. This example demonstrates a straightforward application of integration and substitution, reinforcing key concepts that form the foundation for more advanced analysis and problem-solving in mathematics.

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