Find The Price Of A Bond Whose Face Value Is 100, It Pays Semi-annual Coupons At A Rate Of 1.5% P.a.,

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Introduction to Bond Pricing

Understanding how to determine the price of a bond is fundamental in fixed-income investing. Bonds are debt instruments issued by corporations, governments, or other entities to raise capital. When investors purchase bonds, they are essentially lending money to the issuer in exchange for periodic interest payments and the return of the principal amount at maturity. The price of a bond today depends on various factors, including the coupon rate, face value, time to maturity, prevailing market interest rates, and the risk profile of the issuer.

This article aims to guide you through the process of calculating the current price of a specific bond with a face value of 100, paying semi-annual coupons at a rate of 1.5% per annum. We will explore the concepts involved, the formulas used, and the step-by-step process to arrive at the bond’s present value.

Understanding Bond Features and Terminology

Bond Face Value (Par Value)

- The amount paid back to the bondholder at maturity.
- In this case: 100 units of currency.

Coupon Rate

- The annual interest rate paid by the bond, expressed as a percentage of face value.
- Here: 1.5% per annum.

Coupon Payment Frequency

- How often interest payments are made.
- Semi-annual means twice per year.

Number of Periods

- Total number of interest payment periods until maturity.
- Depends on the bond’s term to maturity.

Market Interest Rate (Yield to Maturity, YTM)

- The rate of return required by investors based on current market conditions.
- Determines the present value of future cash flows.

Step 1: Calculating the Semi-annual Coupon Payment

Since the bond pays interest semi-annually, the annual coupon rate must be adjusted accordingly.

- **Annual Coupon Rate:** 1.5%
- **Face Value:** 100
- **Semi-annual Coupon Rate:** 1.5% / 2 = 0.75%
- **Semi-annual Coupon Payment:** 100 × 0.75% = 100 × 0.0075 = 0.75

This means the bondholder receives 0.75 units of currency every six months until maturity.

Step 2: Determining the Number of Periods

The total number of periods depends on the bond’s maturity date. For illustration, let's assume:

- Maturity Period: 5 years

Since payments are semi-annual:

- Total Number of Periods (n): 5 years × 2 = 10 periods

If the maturity is different, adjust accordingly.

Step 3: Estimating the Market Yield (YTM) per Period

To calculate the bond’s present value, we need the market interest rate per period (semi-annual YTM).

Note: The market rate (YTM) is often obtained from market data or by comparison with similar bonds. For this example, suppose the current market YTM is 1.2% annually, compounded semi-annually.

- Annual Market Rate: 1.2%
- Semi-annual Market Rate (i): 1.2% / 2 = 0.6% = 0.006 in decimal form

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Step 4: Calculating the Present Value of Coupon Payments

The present value of the annuity of coupon payments is calculated using the formula:

$$
PV_{coupons} = C \times \left(1 - (1 + i)^{-n}\right) / i
$$

Where:
- C = semi-annual coupon payment = 0.75
- i = semi-annual market yield = 0.006
- n = total periods = 10

Calculating:

$$
PV_{coupons} = 0.75 \times \left(1 - (1 + 0.006)^{-10}\right) / 0.006
$$

First, compute $(1 + 0.006)^{-10}$:

$$
(1.006)^{-10} \approx 1 / (1.006)^{10} \approx 1 / 1.0617 \approx 0.9418
$$

Then:

$$
PV_{coupons} = 0.75 \times \left(1 - 0.9418\right) / 0.006 = 0.75 \times 0.0582 / 0.006
$$

$$
PV_{coupons} = 0.75 \times 9.7 \approx 7.275
$$

Thus, the present value of all coupon payments is approximately 7.275 units.

Step 5: Calculating the Present Value of the Face Value (Lump Sum at Maturity)

The face value is repaid at the end of the maturity period:

$$
PV_{face} = F / (1 + i)^n
$$

Where:
- F = face value = 100
- i = 0.006
- n = 10

Calculating:

$$
PV_{face} = 100 / (1.006)^{10}
$$

Previously, we found $(1.006)^{10} \approx 1.0617$, so:

$$
PV_{face} = 100 / 1.0617 \approx 94.2
$$

Step 6: Summing the Present Values to Find the Bond Price

The total bond price is the sum of the present value of coupon payments and the present value of the face value:

$$
Price = PV_{coupons} + PV_{face} \approx 7.275 + 94.2 = 101.475
$$

Therefore, the approximate price of the bond is 101.48 units.

Additional Considerations

Impact of Market Rate Changes

- If the market yield rises above the coupon rate, the bond’s price will fall below face value.
- If market yield falls below the coupon rate, the bond will trade at a premium.

Effect of Maturity Duration

- Longer maturity bonds are more sensitive to interest rate changes.
- The present value calculations would adjust accordingly for different maturities.

Coupon Rate vs. Market Rate

- The bond’s attractiveness depends on how its coupon rate compares to prevailing rates.
- Bonds paying lower coupons than market rates tend to trade at discounts.

Conclusion

Calculating the price of a bond involves understanding its cash flows, discounting those flows at the current market yield, and summing their present values. In this example, with a face value of 100, semi-annual coupon

payments at a rate of 1.5% per annum, and assuming a market YTM of 1.2%, the bond’s price is approximately 101.48 units. This process highlights the fundamental principles of bond valuation, including the impact of interest rates, time to maturity, and payment frequency. Investors and analysts can apply similar calculations with different parameters to assess bond prices in various market scenarios, aiding in informed investment decisions.

Frequently Asked Questions

How do you calculate the price of a bond with semi-annual coupons and a face value of 100 at an annual coupon rate of 1.5%?

The bond price is calculated by discounting the future coupon payments and the face value at the appropriate market interest rate, adjusting for semi-annual periods. Specifically, you find the present value of all semi-annual coupons (each = 1.5% of 100 divided by 2) and the present value of the face value, both discounted at the bond's yield per period.

What is the semi-annual coupon payment for a bond with a 1.5% annual rate and face value of 100?

The semi-annual coupon payment is 0.75 (which is 1.5% of 100 divided by 2), equaling 0.75 per period.

If the market yield is 1.5% per annum, what is the approximate price of the bond?

If the market yield matches the coupon rate (1.5% per annum), the bond's price will be close to its face value of 100, assuming no other market factors. Exact price can be computed by discounting cash flows at the same rate.

How does changing the market interest rate affect the bond's price?

If the market interest rate rises above the coupon rate, the bond's price will fall below its face value. Conversely, if the market rate drops below the coupon rate, the bond's price will rise above face value.

What is the impact of semi-annual coupon payments on bond valuation compared to annual payments?

Semi-annual payments increase the frequency of cash flows, leading to a slightly higher present value due to compounding effects, and make the bond's

duration shorter, affecting its sensitivity to interest rate changes.

Can you provide a formula to find the bond price given the face value, coupon rate, and market yield?

Yes. The bond price P is calculated as: P = (C × [1 - (1 + r)^-n] / r) + (F / (1 + r)^n), where C is the semi-annual coupon, F is the face value, r is the semi-annual market yield, and n is the total number of periods.

Additional Resources

Find The Price Of A Bond Whose Face Value Is 100, It Pays Semi-annual Coupons At A Rate Of 1.5% P.a.

In the realm of fixed-income securities, bonds serve as a cornerstone investment vehicle, providing investors with predictable income streams and capital preservation. A fundamental task for investors, financial analysts, and students alike is to determine the fair price of a bond based on its characteristics and prevailing market conditions. In this comprehensive investigation, we delve into the process of calculating the price of a bond with a specified face value, coupon rate, and payment frequency, using the example: a bond whose face value is 100, paying semi-annual coupons at a rate of 1.5% per annum.

Understanding the Basics of Bond Valuation

Before we embark on the detailed calculation, it’s essential to understand the core concepts involved in bond valuation.

What Is a Bond?

A bond is a debt instrument issued by entities such as governments, corporations, or municipalities to raise capital. The issuer promises to pay the bondholder a series of interest payments, known as coupons, over the life of the bond, and to return the principal (face value) at maturity.

Components of a Bond

- Face Value (Par Value): The amount paid back at maturity, typically 100 or 1,000 units of currency.
- Coupon Rate: The annual interest rate paid on the face value.
- Coupon Payment Frequency: How often interest payments are made (annual, semi-annual, quarterly, etc.)
- Maturity Date: When the principal is repaid.
- Market Yield (Yield to Maturity): The rate of return demanded by investors, reflecting market conditions and risk.

The Importance of Discounting

The bond’s present value (price) is determined by discounting the future cash flows—coupon payments and face value—back to the present using an appropriate discount rate (market yield). This process aligns with the time value of money principle.

The Scenario: Bond with Known Parameters

Given the specifics:

- Face Value (F): 100
- Coupon Rate (C): 1.5% per annum
- Coupon Payment Frequency: Semi-annual
- Coupon Payment (PMT): Calculated based on the annual rate and face value
- Market Yield (i): Not specified explicitly in the problem; which is critical for valuation
- Time to Maturity: Not specified, the duration is necessary for a complete valuation

For the purpose of this detailed analysis, we will assume a maturity period of 10 years and a market yield (YTM) of, say, 1.75% per annum—a common scenario where the yield slightly exceeds the coupon rate, leading to a bond trading at a discount. These assumptions help illustrate the valuation process; in real scenarios, these parameters are derived from current market data.

Step-by-Step Approach to Bond Pricing

1. Calculate the Coupon Payment

Since the coupon rate is annual, and payments are semi-annual:

- Annual Coupon Rate: 1.5%
- Semi-annual Coupon Rate: 1.5% / 2 = 0.75%
- Coupon Payment (PMT): Face value × semi-annual rate = 100 × 0.0075 = 0.75

2. Determine the Number of Payments

- Total periods (n): 10 years × 2 = 20 semi-annual periods

3. Determine the Discount Rate per Period

- Annual Market Yield (YTM): 1.75%
- Semi-annual Yield (i): 1.75% / 2 = 0.875% or 0.00875 in decimal form

4. Calculate Present Value of Coupon Payments

The coupons are an annuity—constant payments over multiple periods. The present value (PV) of an annuity is:

$$
PV_{\text{coupons}} = PMT \times \frac{1 - (1 + i)^{-n}}{i}
$$

Where:

- PMT = 0.75
- i = 0.00875
- n = 20

5. Calculate Present Value of Face Value

The face value is paid at maturity:

$$
PV_{\text{face}} = F \times (1 + i)^{-n}
$$

Where:

- F = 100
- n = 20
- i = 0.00875

Numerical Illustration: Calculating the Bond Price

Let’s perform these calculations step-by-step.

A. Present Value of Coupons

$$
PV_{\text{coupons}} = 0.75 \times \frac{1 - (1 + 0.00875)^{-20}}{0.00875}
$$

First, compute $(1 + 0.00875)^{-20}$:

$$
(1 + 0.00875)^{20} \approx e^{20 \times \ln(1.00875)}
$$

Using the approximation:

$$
\ln(1.00875) \approx 0.008722
$$

So,

$$
20 \times 0.008722 \approx 0.17444
$$

Then,

$$
e^{0.17444} \approx 1.1904
$$

Therefore,

$$
(1 + 0.00875)^{-20} \approx 1 / 1.1904 \approx 0.8409
$$

Now, compute the numerator:

$$
1 - 0.8409 = 0.1591
$$

Finally,

$$
PV_{\text{coupons}} = 0.75 \times \frac{0.1591}{0.00875} \approx 0.75 \times 18.169 \approx 13.6268
$$

B. Present Value of Face Value

$$
PV_{\text{face}} = 100 \times 0.8409 = 84.09
$$

C. Total Bond Price

Adding both components:

$$
\text{Bond Price} = PV_{\text{coupons}} + PV_{\text{face}} \approx 13.627 + 84.09 = 97.717
$$

Therefore, under these assumptions, the bond's fair price is approximately 97.72.

Factors Influencing Bond Price

While this example provides a numerical estimate, in real-world scenarios, several factors can significantly influence the bond's price:

Market Yield Variations

- A higher market yield than the coupon rate results in a lower bond price (discount).
- A lower market yield than the coupon rate results in a higher bond price (premium).

Time to Maturity

- Longer maturities generally increase interest rate risk, affecting bond pricing.

Credit Risk

- Higher risk of default can lead to higher yields and lower bond prices.

Coupon Rate vs. Market Rate

- Bonds with coupons above market yields tend to trade at premiums.
- Bonds with coupons below market yields tend to trade at discounts.

Sensitivity Analysis: How Changes in Yield Affect Price

Given the importance of market yield, investors often perform sensitivity analyses:

- Duration: Measures the sensitivity of bond price to interest rate changes.
- Convexity: Accounts for curvature in price-yield relationships.

For example, if the market yield increases from 1.75% to 2.00%, the bond price will decrease, and vice versa.

Practical Considerations in Bond Valuation

Data Accuracy

- Accurate market yields and maturity dates are essential for precise valuation.

Assumptions and Limitations

- The example assumes a fixed yield and no transaction costs, taxes, or call

features.

Software and Tools

- Financial calculators and spreadsheet software like Excel are commonly used for bond valuation, employing functions such as PV() and RATE().

Conclusion: The Art and Science of Bond Pricing

Determining the price of a bond with a face value of 100, paying semi-annual coupons at a rate of 1.5% per annum, is a straightforward application of present value concepts, provided the market yield and maturity are known. The example above illustrates that, given a market yield of 1.75% and a 10-year maturity, the bond would be valued at approximately 97.72—slightly below par—reflecting the market's expectations and risk environment.

This process underscores the importance of understanding the interplay between coupon rates, yields, time horizons, and payment frequencies. For investors and analysts, mastering these calculations is essential for making informed decisions, managing portfolios, and assessing relative value in fixed-income markets.

In practice, continuous market monitoring and dynamic valuation models are necessary to adapt to changing interest rates, economic conditions, and issuer-specific factors. As such, bond valuation remains both an art and a science—requiring rigorous analysis, attention to detail, and an understanding of broader market forces.

In summary, calculating the price of a bond involves identifying its cash flows, discounting them at an appropriate rate, and summing these present values. Whether for investment decision-making or risk assessment, this fundamental skill provides critical insights into the true worth of fixed-income securities in today's financial landscape.

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