Find The Relative Maximum And Minimum Values. F(x,y) = 3x2 + 7y2 + 10 Select The Correct Choice Below

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Understanding how to find relative maximum and minimum values of a multivariable function is a fundamental concept in calculus, particularly in optimization problems. When dealing with functions of two variables, such as $F(x, y) = 3x^2 + 7y^2 + 10$, the process involves critical point analysis, second derivative tests, and examining the behavior of the function in the vicinity of these points. This article provides a comprehensive guide on how to identify the relative extrema of such functions, focusing on the example function provided, and offers insights into selecting the correct choices based on mathematical analysis.

Introduction to Relative Maxima and Minima in Multivariable Functions

Before diving into the specific example, it's essential to understand what relative maxima and minima are in the context of functions with multiple variables.

What Are Relative Maxima and Minima?

- Relative Maximum: A point (x_0, y_0) where the function's value is greater than or equal to all nearby points. In other words, within some neighborhood around (x_0, y_0), $F(x, y) \leq F(x_0, y_0)$.

- Relative Minimum: A point (x_0, y_0) where the function's value is less than or equal to all nearby points. That is, in some neighborhood around (x_0, y_0), $F(x, y) \geq F(x_0, y_0)$.

Importance of Finding Relative Extrema

Identifying these points is crucial in various fields such as economics, engineering, and physics, where optimizing a quantity within certain constraints is necessary. For example, minimizing cost or maximizing efficiency relies on locating these extrema.

Analyzing the Function $F(x, y) = 3x^2 + 7y^2 + 10$

Let's analyze the specific function:

$F(x, y) = 3x^2 + 7y^2 + 10$

This quadratic function is a paraboloid opening upward because the coefficients of x^2 and y^2 are positive.

Step 1: Find Critical Points

Critical points occur where the gradient of the function equals zero.

The gradient vector (∇F) is:

$\nabla F(x, y) = (\partial F/\partial x, \partial F/\partial y)$

Calculate the partial derivatives:

- $\partial F/\partial x = 6x$
- $\partial F/\partial y = 14y$

Set these derivatives equal to zero to find critical points:

$6x = 0 \rightarrow x = 0$

$14y = 0 \rightarrow y = 0$

Critical point: (0, 0)

Step 2: Classify Critical Points Using Second Derivative Test

For functions of two variables, the second derivative test involves the Hessian matrix:

H = | F_xx F_xy |
| F_yx F_yy |

Calculate second derivatives:

- $F_{xx} = \partial^2 F/\partial x^2 = 6$
- $F_{yy} = \partial^2 F/\partial y^2 = 14$
- $F_{xy} = F_{yx} = 0$ (since the mixed derivatives are zero for this quadratic function)

The discriminant D is:

$D = F_{xx} F_{yy} - (F_{xy})^2 = (6)(14) - 0^2 = 84$

Classification:

- Since $D > 0$ and $F_{xx} > 0$, the critical point at (0, 0) is a relative minimum.

Key Point:
Because the function is quadratic with positive coefficients, the critical point at (0, 0) is, in fact, the global minimum.

Understanding the Nature of the Function: Paraboloid Shape

The function $F(x, y) = 3x^2 + 7y^2 + 10$ can be visualized as a paraboloid opening upward.

- The minimum value occurs at the critical point (0, 0), where $F(0, 0) = 10$.
- As x or y move away from zero, the function value increases indefinitely.

Implications for Optimization

- The absolute minimum value of $F(x, y)$ is 10 at (0, 0).
- There are no relative maxima because the paraboloid opens upward and the function tends to infinity as $|x|$ or $|y|$ increase.

Choosing the Correct Answer: Key Takeaways

Given the analysis, the options for the maximum and minimum values are:

- Minimum value: 10 at (0, 0)
- Maximum value: The function has no maximum as it extends infinitely upward.

Therefore, the correct choice typically highlights the minimum value being 10.

Further Considerations and Applications

Understanding the process of finding extrema applies to more complex functions and constrained optimization problems.

Constrained Optimization

Sometimes, the optimization must occur under specific constraints, such as:

- Equations like $g(x, y) = 0$
- Inequalities defining feasible regions

In such cases, methods like Lagrange multipliers are used.

Example: Applying Lagrange Multipliers

Suppose you want to find the minima of $F(x, y) = 3x^2 + 7y^2 + 10$ subject to a constraint $g(x, y) = x + y - 1 = 0$.

The steps involve:

1. Setting up the Lagrangian: $L(x, y, \lambda) = F(x, y) - \lambda(g(x, y))$
2. Taking partial derivatives and solving simultaneously.
3. Classifying the points to find the constrained extrema.

Summary and Key Takeaways

- The critical point (0, 0) for the function $F(x, y) = 3x^2 + 7y^2 + 10$ is a relative and absolute minimum.
- Since the paraboloid opens upward, there is no relative maximum.
- The minimum value of the function is 10 at (0, 0).
- The behavior of quadratic functions helps in understanding extrema in higher dimensions.
- Tools such as the gradient, second derivative test, and Hessian matrix are essential for classification.

Conclusion

In summary, when analyzing the function $F(x, y) = 3x^2 + 7y^2 + 10$, the critical point at (0, 0) is identified as the global minimum with a value of 10. There are no relative maxima because the function tends to infinity as the variables grow large. Recognizing these properties enables mathematicians and professionals to make informed decisions about optimization problems involving multivariable functions.

In the context of selecting the correct choice below for the problem, the appropriate answer is that the minimum value of F(x, y) is 10 at (0, 0), and there is no maximum value.

Frequently Asked Questions

How do you find the critical points of the function $F(x, y) = 3x^2 + 7y^2 + 10$?

To find critical points, compute the partial derivatives with respect to x and y, set them equal to zero, and solve for x and y. That is, $\partial F/\partial x = 6x = 0$ and $\partial F/\partial y = 14y = 0$, which gives the critical point at (0, 0).

What is the significance of the second derivative test in determining the relative maximum or minimum for F(x, y)?

The second derivative test involves calculating the second partial derivatives and the discriminant $D = F_{xx} F_{yy} - (F_{xy})^2$. If $D > 0$ and $F_{xx} > 0$, there's a relative minimum; if $D > 0$ and $F_{xx} < 0$, there's a relative maximum; if $D < 0$, the critical point is a saddle point.

What are the second partial derivatives of $F(x, y) = 3x^2 + 7y^2 + 10$?

$F_{xx} = 6$, $F_{yy} = 14$, and $F_{xy} = 0$.

Based on the second derivative test, is the critical point at (0, 0) a relative maximum, minimum, or saddle point?

Since $F_{xx} = 6 > 0$, $F_{yy} = 14 > 0$, and $D = (6)(14) - 0^2 = 84 > 0$, the point at (0, 0) is a relative minimum.

What is the minimum value of the function $F(x, y) = 3x^2 + 7y^2 + 10$?

The minimum value occurs at (0, 0), and is $F(0, 0) = 3(0)^2 + 7(0)^2 + 10 = 10$.

Are there any maximum values for $F(x, y) = 3x^2 + 7y^2 + 10$?

No, since the quadratic terms are positive and unbounded, the function has no maximum value; it tends to infinity as $|x|$ or $|y|$ increase.

What is the correct choice for the relative extrema of $F(x,y) = 3x^2 + 7y^2 + 10$?

The function has a relative minimum at (0, 0) with a minimum value of 10; there is no relative maximum.

Additional Resources

Find The Relative Maximum And Minimum Values is a fundamental concept in multivariable calculus, essential for understanding the behavior of functions involving multiple variables. In particular, analyzing the function $F(x, y) = 3x^2 + 7y^2 + 10$ provides a clear example of how to identify critical points, classify them as relative maxima, minima, or saddle points, and interpret their significance within the context of the function's graph. This process involves several key steps, including calculating partial derivatives, solving for critical points, and applying second derivative tests. In this article, we will explore these steps in detail, break down the concepts involved, and examine the implications of the function’s structure.

Understanding the Function $F(x, y) = 3x^2 + 7y^2 + 10$

Before diving into the process of finding relative extrema, it's important to analyze the form and properties of the function itself.

Function Characteristics

- Quadratic Nature: The given function is a quadratic form in two variables, x and y. The presence of squared terms indicates the function's curvature and the potential for extremal points.
- Positive Coefficients: Both coefficients of x^2 and y^2 (i.e., 3 and 7) are positive, which suggests that the paraboloid opens upward in both directions.
- Constant Term: The "+10" shifts the graph vertically upward, ensuring the function is always at least 10, regardless of x and y.

Graphical Interpretation

- The surface is an elliptic paraboloid centered at the origin.
- Since the quadratic coefficients are positive, the graph has a "bowl" shape opening upward.
- The minimum value occurs at the lowest point of the paraboloid, which is at the vertex—the point where the surface reaches its minimum.

Finding Critical Points of $F(x, y)$

Critical points are locations where the function's gradient (vector of partial derivatives) equals zero. These points are candidates for local maxima, minima, or saddle points.

Calculating Partial Derivatives

The first step involves computing the partial derivatives of F with respect to x and y:

$$
\frac{\partial F}{\partial x} = 6x
$$

$$
\frac{\partial F}{\partial y} = 14y
$$

Setting Derivatives Equal to Zero

To find critical points:

$$
6x = 0 \Rightarrow x = 0
$$

$$
14y = 0 \Rightarrow y = 0
$$

Thus, the only critical point of F is at (0, 0).

Classifying Critical Points Using Second Derivative Test

Once the critical point is identified, the next step is to classify it.

Second Derivatives and the Hessian Determinant

Compute the second derivatives:

$$
\frac{\partial^2 F}{\partial x^2} = 6
$$
$$
\frac{\partial^2 F}{\partial y^2} = 14
$$
$$
\frac{\partial^2 F}{\partial x \partial y} = 0
$$

Form the Hessian matrix:

$$
H = \begin{bmatrix}
6 & 0 \\
0 & 14
\end{bmatrix}
$$

Calculate the determinant:

$$
D = (6)(14) - (0)^2 = 84
$$

Since $D > 0$ and $\frac{\partial^2 F}{\partial x^2} = 6 > 0$, the critical point at (0,0) is a relative minimum.

Determining the Nature of the Critical Point

Given the positive definiteness of the Hessian matrix, the critical point at (0, 0) is a global minimum because the quadratic form is upward-opening in all directions.

Key features:

- Global Minimum: The function attains its lowest value at (0, 0).
- Function Value at Critical Point: $F(0, 0) = 3(0)^2 + 7(0)^2 + 10 = 10$.

This means the minimum value of the function is 10, achieved at the point (0, 0).

Summary of Findings and Implications

The analysis confirms that:

- The only critical point is at (0, 0).
- The critical point is a relative and global minimum.
- The minimum value of F is 10.

This understanding allows us to interpret the function's behavior comprehensively. Since the paraboloid opens upward, the point (0, 0) is the lowest point on the surface, and the function increases without bound as x or y move away from zero.

Practical Applications of Finding Relative Extrema

Identifying relative maxima and minima is vital in various real-world contexts:

- Optimization Problems: In engineering, economics, and physics, minimizing or maximizing a function corresponds to optimizing resources, costs, or performance.
- Design and Safety: Ensuring systems operate within safe parameters often involves understanding the maximum or minimum stress points.
- Data Analysis: In machine learning, loss functions' extrema indicate optimal model parameters.

Pros and Cons of the Methodology

Pros:

- Systematic Approach: The process of calculating derivatives and applying the second derivative test provides a clear, step-by-step method to classify critical points.
- Applicability: This method works well for quadratic and many smooth functions.
- Insightful: Offers both local and global understanding of the function’s behavior.

Cons:

- Limited to Smooth Functions: Functions with discontinuities or sharp corners require different approaches.
- Higher Dimensions Complexity: For functions involving more variables, the Hessian matrix becomes larger, increasing computational complexity.
- Saddle Points: The second derivative test can sometimes be inconclusive if the Hessian is indefinite.

Conclusion

The process of finding the relative maximum and minimum values of functions like $F(x, y) = 3x^2 + 7y^2 + 10$ exemplifies core principles in multivariable calculus. Through calculating partial derivatives, identifying critical points, and applying the second derivative test, we establish that the function has a single critical point at (0, 0), which is a global minimum with a value of 10. The quadratic form's positive coefficients ensure the surface is bowl-shaped, and the function increases outward from the minimum point. This analysis not only enhances our understanding of the specific function but also illustrates a general approach for analyzing similar functions in optimization, modeling, and scientific research.

Choosing the Correct Option

Given the analysis, the correct choice regarding the extremal points of $F(x, y)$ is that:

- The function has a relative and global minimum at (0, 0) with a minimum value of 10.
- There are no relative maxima or saddle points for this function.

This comprehensive understanding aids in solving practical problems involving multivariable functions and underscores the importance of calculus techniques in various scientific fields.

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