Find The Solution Of Xy" + 5xy' + (4 + 1x)y = 0, X > 0 Of The Form Y1 = Xr Cnxn Where C = 1. Enter

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Understanding and solving differential equations is a fundamental aspect of mathematical analysis, especially in fields such as physics, engineering, and applied mathematics. The given differential equation:

\[X y'' + 5x y' + (4 + 1x) y = 0 \quad \text{where} \quad X > 0 \]

is a second-order linear differential equation with variable coefficients. Our goal is to find solutions of the form:

\[Y_1 = X^r C_n x^n \quad \text{where} \quad C = 1 \]

and determine the values of \(r \) and \(n \) that satisfy this equation.

In this comprehensive article, we will explore the step-by-step process of solving this differential equation using substitution methods, the Frobenius method, and related techniques. We will also analyze the behavior of solutions and their applications in real-world scenarios.

Understanding the Differential Equation

Before delving into solution strategies, it's essential to understand the structure of the differential equation:

\[X y'' + 5x y' + (4 + 1x) y = 0 \]

This equation is a second-order linear ordinary differential equation (ODE) with variable coefficients because of the presence of terms like \(X y'' \) and \(5x y' \). The coefficients are polynomial functions of \(x \), making the equation a candidate for methods such as power series solutions or Frobenius method.

Why Choose a Solution of the Form \(Y_1 = X^r C_n

x^n \)?

The proposed form:

$$ Y_1 = X^r C_n x^n $$

suggests a solution that is a product of powers of X and x, which is common when solving differential equations with variable coefficients. Specifically:

- The exponent r relates to the behavior near $X = 0$ (or as $X \to 0$), often associated with indicial equations.
- The coefficient C_n (with $C = 1$) represents the constant multiple.
- The power n represents the degree of the polynomial or series expansion.

This form is inspired by the Frobenius method, which seeks solutions as power series with indicial roots determining the leading powers.

Step-by-Step Solution Approach

To find the solution, we proceed through the following steps:

1. Substituting the Trial Solution

Assuming:

$$ y = X^r x^n $$

we differentiate y to obtain y' and y'':

$$
\begin{aligned}
y &= X^r x^n \\
y' &= \frac{dy}{dx} = r X^{r-1} x^n + n X^r x^{n-1} \\
y'' &= r(r-1) X^{r-2} x^n + 2 r n X^{r-1} x^{n-1} + n(n-1) X^r x^{n-2}
\end{aligned}
$$

Note: Since y is a function involving both X and x, and the equation involves derivatives with respect to x, it is important to clarify whether the derivatives are with respect to x or X. Given the notation, it's typical to assume derivatives are with respect to x. We treat X as x for simplicity, or consider that X is a parameter. For this problem, since $X > 0$ and the derivatives involve y' and y'' w.r.t. x, we treat X as x.

Thus, the derivatives become:

$$
\begin{aligned}
y &= x^r x^n = x^{r + n} \\
y' &= (r + n) x^{r + n - 1} \\
y'' &= (r + n)(r + n - 1) x^{r + n - 2}
\end{aligned}
$$

But the original problem seems to involve differential equations with y'', y', and y in terms of x, with coefficients involving x, so this simplifies our substitution.

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2. Simplifying the Equation

Substitute y, y', and y'' into the original differential equation:

$$ x y'' + 5 x y' + (4 + x) y = 0 $$

which simplifies to:

$$
x \cdot (r + n)(r + n - 1) x^{r + n - 2} + 5 x \cdot (r + n) x^{r + n - 1} + (4 + x) x^{r + n} = 0
$$

Dividing through by $x^{r + n}$:

$$
(r + n)(r + n - 1) + 5 (r + n) + (4 + x) = 0
$$

However, this step indicates the need for a more precise approach, perhaps involving power series expansion rather than assuming a direct substitution.

Applying the Frobenius Method

Since the differential equation involves variable coefficients, the Frobenius method is a suitable technique for finding solutions around regular singular points such as $x = 0$.

1. Formulating the Series Solution

Assume a solution of the form:

\[y(x) = x^{r} \sum_{n=0}^{\infty} a_n x^{n} \]

where:

- \(r \) is the indicial exponent determined by the lowest power of \(x \) in the series.
- \(a_0 \neq 0 \).

2. Deriving the Indicial Equation

Substitute the series into the differential equation, and equate coefficients for the lowest power of \(x \) to obtain the indicial equation, which determines possible values of \(r \).

Determining the Indicial Equation and Roots

The indicial equation is derived from the lowest power term after substitution. For this, identify the powers of \(x \) in each term:

- \(y \sim x^r \)
- \(y' \sim r x^{r-1} \)
- \(y'' \sim r(r-1) x^{r-2} \)

Substituting into the original differential equation:

\[
x \cdot r(r-1) x^{r-2} + 5x \cdot r x^{r-1} + (4 + x) x^{r} = 0
\]

Simplify each term:

\[
r(r-1) x^{r-1} + 5 r x^{r} + 4 x^{r} + x^{r+1} = 0
\]

At the lowest power, \(x^{r-1} \), the terms are:

\[
r(r-1) x^{r-1}
\]

and the remaining are of higher order (\(x^{r} \), \(x^{r+1} \)), which are neglected in the indicial equation.

Therefore, the indicial equation is:

\[

$$
r(r-1) = 0
$$

which yields roots:

$$
r = 0 \quad \text{or} \quad r = 1
$$

These roots indicate two linearly independent solutions with different behaviors near $x = 0$.

Finding the Series Coefficients

After determining r, we proceed to find the coefficients a_n by substituting the series back into the differential equation and equating coefficients of like powers.

This process involves:

- Setting up recurrence relations for a_n.
- Solving these relations to generate the series solution.

Given that $C = 1$, the general solutions take the form:

$$ y = x^{r} \sum_{n=0}^{\infty} a_n x^{n} $$

where a_0 is arbitrary (often set to 1 for simplicity).

Constructing the Solutions

Based on the roots $r = 0$ and $r = 1$, the two solutions are:

Solution 1: $r = 0$

$$
y_1(x) = \sum_{n=0}^{\infty} a_n x^{n}
$$

This represents a Frobenius series solution regular at $x=0$.

Solution 2: $r = 1$

$$
y_2(x) = x \sum_{n=0}^{\infty} b_n x^{n}
$$

which vanishes at $x=0$ but behaves differently due to the multiplicative factor x.

---

0 is a classic second-order linear differential equation that presents an intriguing challenge for students and practitioners of differential equations. The problem asks us to find a solution of a specific form, namely $Y_1 = X^r C x^n$, where $C = 1$. This particular form suggests the use of power series solutions or substitution methods that exploit the structure of the differential equation. In this article, we will explore the process of solving this differential equation in depth, breaking down each step, and providing insights into the techniques used and their applicability.

Understanding the Differential Equation

Before diving into the solution strategy, it’s essential to understand the nature of the differential equation:

$$ X y'' + 5 X y' + (4 + 1X) y = 0 $$

which can be rewritten as:

$$ X y'' + 5 X y' + (4 + X) y = 0 $$

for $X > 0$.

This is a second-order linear differential equation with variable coefficients, involving terms proportional to y'', y', and y, each multiplied by functions of X. The presence of the term $(4 + X) y$ indicates a non-constant coefficient, which typically suggests that standard methods like constant coefficient techniques are not directly applicable.

Identifying the Solution Form

The problem specifies seeking a solution of the form:

$$ Y_1 = X^r C x^n $$

where $C = 1$. The notation appears to suggest that the solution is of a power-law form, possibly multiplied by a constant. Since $C = 1$, the solution simplifies to:

$$Y_1 = X^r x^n$$

but note the notation inconsistency: the variables are both X and x, which likely refer to the same independent variable. Assuming they do, the form reduces to:

$$y = X^r \cdot X^n = X^{r + n}$$

or perhaps the problem intends to incorporate a multiplicative constant C with the power term.

Given the form, the key idea is to assume a solution of the form:

$$y = X^m$$

where m is to be determined. This is a typical approach when the differential equation has variable coefficients that are polynomial or power functions, as in the Frobenius method.

Transforming the Equation for Power Solutions

Suppose $y = X^m$. Then:

- The first derivative:

$$
y' = \frac{dy}{dX} = m X^{m-1}
$$

- The second derivative:

$$
y'' = m (m - 1) X^{m - 2}
$$

Substituting these into the differential equation:

$$
X \cdot y'' + 5 X \cdot y' + (4 + X) y = 0
$$

we get:

$$
X \cdot m (m - 1) X^{m - 2} + 5 X \cdot m X^{m - 1} + (4 + X) X^m = 0
$$

Simplify each term:

$$
m (m - 1) X^{m - 1} + 5 m X^{m} + 4 X^{m} + X^{m + 1} = 0
$$

Divide through by $X^{m - 1}$:

$$
m (m - 1) + 5 m X + 4 X + X^2 = 0
$$

This expression involves powers of X, indicating that assuming a simple power solution may not lead to a straightforward algebraic characteristic equation unless X is a constant, which it is not in this case.

Thus, the direct substitution method suggests that the solution is more complex, and we might need to consider a Frobenius series expansion or a substitution that simplifies the variable coefficients.

Applying the Frobenius Method or Substitution

Since the differential equation involves variable coefficients with X terms, the Frobenius method is a suitable approach. This method involves assuming a solution in the form:

$$
y = X^r \sum_{n=0}^\infty a_n X^n
$$

where r is the indicial exponent to be determined, and a_n are coefficients.

Alternatively, recognizing the structure of the equation, we can attempt a substitution to reduce it to a more familiar form.

Change of Variable: Transforming the Equation

Let’s attempt the substitution:

$$
t = X
$$

which preserves the variable but allows us to consider solution techniques based on known forms.

Alternatively, transforming the original equation into a standard form such as the Bessel or Kummer equations might be possible if we can manipulate the coefficients accordingly.

Deriving the Indicial Equation

Assuming a Frobenius series solution:

$$
y = X^r \sum_{n=0}^\infty a_n X^n
$$

we can compute derivatives:

$$
y' = r X^{r-1} \sum_{n=0}^\infty a_n X^n + X^r \sum_{n=1}^\infty n a_n X^{n-1}
$$

$$
y'' = r (r-1) X^{r-2} \sum_{n=0}^\infty a_n X^n + 2 r X^{r-1} \sum_{n=1}^\infty n a_n X^{n-1} + X^r \sum_{n=2}^\infty n(n-1) a_n X^{n-2}
$$

Substituting y, y', and y'' into the differential equation, and collecting like powers of X, leads to an indicial equation for r, which determines the dominant behavior of solutions near $X = 0$.

This process is lengthy but systematic, involving equating coefficients of the lowest power of X to zero to find r. The resulting indicial equation often takes the form:

$$
a r(r-1) + b r + c = 0
$$

for some constants a, b, c.

Solving the Indicial Equation

The specific form of the indicial equation depends on the coefficients and the structure of the differential equation. Once r is found, subsequent coefficients a_n can be determined recursively.

In this particular problem, the form of the differential equation and the initial assumption suggest that the solution will involve special functions, such as Bessel functions or confluent hypergeometric functions, depending on the nature of the coefficients.

Features and Characteristics of the Solution

When solving differential equations of this type, several features and considerations are noteworthy:

- Power Series Expansion: The Frobenius method allows us to find solutions expressed as power series, which are valid near regular singular points.

- Special Functions: The solutions often involve special functions, especially when the equation resembles standard forms like Bessel, Legendre, or hypergeometric equations.

- Indicial Equation: Determining the roots of the indicial equation guides the behavior of solutions near singular points and whether solutions are polynomial, logarithmic, or involve other special functions.

- Parameter Constraints: The values of parameters such as r influence the convergence and form of the solutions, and certain parameter choices may lead to polynomial solutions.

Pros and Cons of the Frobenius Method in This Context

Pros:

- Provides a systematic way to find solutions near singular points.
- Can generate power series solutions that approximate the behavior of the solution.
- Facilitates the identification of special functions involved.

Cons:

- Often involves lengthy algebra and recursive calculations.
- Convergence of the series may be limited to certain regions.
- May not yield closed-form solutions unless the differential equation matches known forms.

Conclusion and Final Remarks

The differential equation $X y'' + 5 X y' + (4 + X) y = 0$ presents a challenging but instructive problem in the theory of differential equations. Although directly assuming a solution of the form $y = X^r$ leads to complex algebra, the Frobenius method offers a structured approach to find solutions in power series form. The key steps involve:

- Identifying potential singular points and the nature of the solution at those points.
- Deriving the indicial equation to determine the dominant behavior.
- Computing recurrence relations for series coefficients.
- Recognizing the potential involvement of special functions.

While the explicit closed-form solution may involve advanced functions, the process outlined provides a comprehensive framework for tackling similar equations. The approach emphasizes the importance of understanding the structure of differential equations, judiciously choosing solution forms, and leveraging power series methods to uncover solutions that might otherwise remain elusive.

Features Summary:

- Suitable for equations with variable coefficients.
- Can produce approximate solutions near singular points.
- Essential for understanding the qualitative behavior of solutions.

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