Find The Standard Form Of The Equation Of The Circle With Endpoints Of A Diameter At The Points (3,4)

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Understanding how to derive the equation of a circle when given specific points, such as the endpoints of its diameter, is an essential skill in coordinate geometry. This process involves several steps including identifying the center and radius of the circle, which are fundamental to formulating its standard equation. This article will guide you through the detailed procedure to find the standard form of the circle's equation when the endpoints of a diameter are provided—in this case, the points (3, 4). Whether you're a student studying geometry or a math enthusiast looking to deepen your understanding, this comprehensive guide will clarify each step involved.

Understanding the Basic Concepts of a Circle in Coordinate Geometry

Before diving into the calculations, it's important to review some fundamental concepts related to circles in the coordinate plane.

The Standard Equation of a Circle

The standard form of a circle's equation is:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where:
- (h, k) is the center of the circle,
- r is the radius.

Knowing the center and radius allows you to write the equation directly.

Endpoints of a Diameter

A diameter of a circle is a straight line passing through the center and touching the circle at two points. The endpoints of a diameter are always on the circle, and the center is the midpoint of that diameter. Therefore, given the endpoints of a diameter, you can find:
- The center by calculating the midpoint,

- The radius by finding the distance from the center to either endpoint.

Step-by-Step Guide to Derive the Equation

Let's proceed step-by-step to derive the standard form of the circle's equation with the given endpoints.

1. Identify the Endpoints of the Diameter

Given points:
- $A = (3, 4)$

Note: The problem states "endpoints of a diameter at the points (3, 4)". Usually, a diameter has two endpoints, but only one point is given. To proceed correctly, we need both endpoints. If the problem as stated implies that the diameter has endpoints at (3, 4) and some other point, we need that second point.

Assumption: Since only one point is provided, and the problem refers to the endpoints of a diameter at (3, 4), we will assume that the endpoints are:

- $A = (3, 4)$
- $B = (x_b, y_b)$

If the original problem intended for both endpoints to be specified, please provide the second point. Otherwise, perhaps the problem is asking for the standard form of the circle with a diameter passing through (3, 4) and another point.

Clarification:
Let’s assume that the diameter's endpoints are at points (3, 4) and (x, y). Since only (3, 4) is provided, perhaps the intended problem was to find the circle with a diameter where one endpoint is (3, 4) and the other is known or to be determined.

Alternative interpretation:
Suppose the problem is to find the circle with a diameter whose endpoints are at points (x_1, y_1) and (x_2, y_2), with one endpoint at (3, 4). If the second endpoint is not specified, then perhaps the problem is missing information.

Assumption to Proceed:
Let's assume the diameter endpoints are at:
- $A = (3, 4)$
- $B = (7, 8)$

This is just an example; you can replace these with actual points if provided.

For the purpose of this guide, let's assume the second endpoint is at (7, 8).

2. Find the Midpoint (Center of the Circle)

The center (h, k) of the circle is the midpoint of the diameter segment.

The midpoint (h, k) is calculated as:

$$
h = \frac{x_1 + x_2}{2}
$$
$$
k = \frac{y_1 + y_2}{2}
$$

Using points $A = (3, 4)$ and $B = (7, 8)$:

$$
h = \frac{3 + 7}{2} = \frac{10}{2} = 5
$$
$$
k = \frac{4 + 8}{2} = \frac{12}{2} = 6
$$

So, the center of the circle is at $(5, 6)$.

3. Calculate the Radius

The radius r is half the length of the diameter, or equivalently, the distance from the center to either endpoint:

$$
r = \text{distance between center and endpoint}
$$

Using point $A = (3, 4)$:

$$
r = \sqrt{(x_1 - h)^2 + (y_1 - k)^2}
$$

$$
r = \sqrt{(3 - 5)^2 + (4 - 6)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}
$$

Alternatively, using point $B = (7, 8)$, the calculation yields the same radius, confirming consistency.

4. Write the Equation in Standard Form

Now, substitute the center $(h, k) = (5, 6)$ and radius $r = 2\sqrt{2}$ into the standard circle equation:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

$$
(x - 5)^2 + (y - 6)^2 = (2\sqrt{2})^2 = 8
$$

Final Equation:

$$
(x - 5)^2 + (y - 6)^2 = 8
$$

Additional Considerations

If the second endpoint is different
Replace $(7, 8)$ with the actual second endpoint to find the correct circle equation.

When only one point is known
If only one endpoint of the diameter is given, the problem cannot be uniquely solved unless additional information (like the radius or another point on the circle) is provided.

Summary

To find the standard form of the circle's equation with endpoints of a diameter:

1. Obtain both endpoints of the diameter: Denote as (x_1, y_1) and (x_2, y_2).
2. Find the midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, which is the circle's center (h, k).
3. Calculate the radius: Distance from the center to either endpoint, $r = \sqrt{(x_1 - h)^2 + (y_1 - k)^2}$.
4. Write the equation: $(x - h)^2 + (y - k)^2 = r^2$.

This systematic approach allows you to derive the standard form of any circle given the endpoints of its diameter.

Conclusion

Finding the standard form of a circle's equation given the endpoints of a diameter involves straightforward coordinate geometry techniques: midpoints and distance formulas. Remember, the key steps are to locate the center via the midpoint of the diameter and determine the radius from the distance to an endpoint. With practice, this process becomes intuitive, enabling you to quickly formulate the circle's equation in standard form for any given endpoints.

Frequently Asked Questions

What is the first step to find the standard form of the circle's equation given endpoints of a diameter at (3,4)?

Calculate the midpoint of the endpoints to find the center of the circle, which will be the average of the x-coordinates and y-coordinates.

How do you find the center of the circle with endpoints (3,4) and another point?

Since only one endpoint is given, you need both endpoints of the diameter to find the center. If the second endpoint isn't provided, the problem may be incomplete. Assuming both endpoints are known, the center is the midpoint of the diameter.

If the endpoints of the diameter are (3,4) and (x2, y2), how do you find the coordinates of the center?

The center's coordinates are ((3 + x2)/2, (4 + y2)/2).

How do you determine the radius of the circle once the endpoints of the diameter are known?

Calculate the distance between the endpoints, then divide by 2 to get the radius: radius = (distance between endpoints) / 2.

What is the formula for the standard equation of a

circle?

The standard form is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

Given endpoints (3,4) and (x2, y2), how do you write the equation of the circle?

First, find the midpoint to get the center (h, k), then calculate the radius using the distance formula, and finally substitute into $(x - h)^2 + (y - k)^2 = r^2$.

If only one endpoint (3,4) is given, can we find the equation of the circle?

No, because the other endpoint of the diameter is needed to determine the circle's center and radius. Additional information is required.

How do you verify that the point (x, y) lies on the circle once the equation is found?

Substitute the point's coordinates into the equation; if both sides are equal, the point lies on the circle.

What assumptions are made if only one endpoint of a diameter is provided in the problem?

The assumption is that the other endpoint is known or can be determined; otherwise, the problem cannot be completed without both endpoints.

Additional Resources

Find The Standard Form Of The Equation Of The Circle With Endpoints Of A Diameter At The Points (3,4)

Introduction

The quest to determine the standard form of a circle’s equation given specific geometric constraints is a fundamental problem in coordinate geometry. In this investigative exploration, we focus on a particular scenario: finding the standard form of the equation of a circle when the endpoints of its diameter are known. Specifically, we analyze the case where the diameter’s endpoints are at the points (3, 4). This problem not only exemplifies key geometric principles but also demonstrates the systematic approach to deriving a circle's equation from given data.

Understanding the Geometric Context

Before delving into calculations, it’s essential to establish a clear understanding of the problem's geometric foundation.

The Significance of a Diameter

A diameter of a circle is a straight line segment passing through the circle's center, with endpoints lying on the circle itself. Notably, the diameter:

- Is the longest chord of the circle.
- Has a length twice the radius.
- Passes through the circle's center, which is the midpoint of the diameter.

Given the endpoints of a diameter, the circle's center and radius can be determined directly, which paves the way for formulating its equation in standard form.

Step 1: Determining the Center of the Circle

The first critical step involves calculating the circle's center, denoted as (h, k). Since the endpoints of the diameter are at points $A(3, 4)$ and $B(x_2, y_2)$, the center C is located at the midpoint of AB.

Midpoint Formula

The midpoint $C(h, k)$ of a segment with endpoints (x_1, y_1) and (x_2, y_2) is given by:

$$
h = \frac{x_1 + x_2}{2}
$$
$$
k = \frac{y_1 + y_2}{2}
$$

In our case, with only one endpoint specified and no second endpoint provided, it’s implied that the problem considers the diameter’s endpoints as known, or perhaps the other endpoint is missing or needs to be determined.

However, based on the problem statement — "Find the standard form of the circle with endpoints of a diameter at (3,4)" — it suggests that only one endpoint is provided. But this could be a misinterpretation; typically, a diameter is defined by two points. For the purpose of this investigation, let's assume the other endpoint is at (x_2, y_2) which must be determined or provided.

Clarification: Since only one point $(3, 4)$ is given, and the problem refers to "endpoints of a diameter at the points (3, 4)," we need to clarify whether the endpoints are both given or only one.

If only a single endpoint is given, then the problem is incomplete.
If both endpoints are at (3, 4), then the circle degenerates to a point (not a circle).

Most reasonable assumption: The problem intends to specify that the diameter's endpoints are at $(3, 4)$ and some other point, which is missing. Alternatively, perhaps the problem wants us to find the circle with a diameter that has one endpoint at $(3, 4)$, and the other endpoint is unknown.

Step 2: Clarifying the Question

Given the ambiguity, the most logical interpretation is:

- The endpoints of the diameter are at (3, 4) and another point.
- The goal is to find the equation of the circle with these endpoints.

Assumption for clarity: Let's assume the other endpoint of the diameter is at (x, y). The task then becomes to express the circle's standard form in terms of these points, or perhaps to find the circle if an additional condition is provided.

Alternatively, it is possible the problem refers to the diameter being at endpoints at $(3, 4)$ and some other point, or possibly the problem is simplified as: the diameter is between $(3, 4)$ and another point, which is yet to be identified.

Step 3: Proceeding with General Solution

To proceed meaningfully, let's consider the case where the diameter's endpoints are at $(3, 4)$ and (x_2, y_2). We aim to find the circle's standard form in terms of these points.

Step 4: Calculating the Center and Radius

Suppose the endpoints are $A(3, 4)$ and $B(x_2, y_2)$.

- The center $C(h, k)$ is at:

$$
h = \frac{3 + x_2}{2}
$$
$$
k = \frac{4 + y_2}{2}
$$

- The radius r is half the length of the diameter, so:

$$
r = \frac{\text{distance between } A \text{ and } B}{2}
$$

where the distance d between the two endpoints is:

$$
d = \sqrt{(x_2 - 3)^2 + (y_2 - 4)^2}
$$

Thus,

$$
r = \frac{1}{2} \times d = \frac{1}{2} \sqrt{(x_2 - 3)^2 + (y_2 - 4)^2}
$$

Step 5: Deriving the Equation of the Circle

The standard form of a circle's equation is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Substituting the expressions for h, k, and r:

$$
\left(x - \frac{3 + x_2}{2} \right)^2 + \left(y - \frac{4 + y_2}{2} \right)^2 = \left(\frac{1}{2} \sqrt{(x_2 - 3)^2 + (y_2 - 4)^2} \right)^2
$$

Simplify the right side:

$$
= \frac{1}{4} \times [(x_2 - 3)^2 + (y_2 - 4)^2]
$$

Thus, the general form of the circle's equation in terms of the endpoints is:

\[

$$
\left(x - \frac{3 + x_2}{2} \right)^2 + \left(y - \frac{4 + y_2}{2} \right)^2 = \frac{(x_2 - 3)^2 + (y_2 - 4)^2}{4}
$$

Step 6: Specific Example

To ground this in concrete terms, let's consider a specific example where the second endpoint is at $(7, 8)$.

- Endpoints: $A(3, 4)$ and $B(7, 8)$.

- Center:

$$
h = \frac{3 + 7}{2} = \frac{10}{2} = 5
$$

$$
k = \frac{4 + 8}{2} = \frac{12}{2} = 6
$$

- Distance between endpoints:

$$
d = \sqrt{(7 - 3)^2 + (8 - 4)^2} = \sqrt{4^2 + 4^2} = \sqrt{16 + 16} = \sqrt{32} = 4 \sqrt{2}
$$

- Radius:

$$
r = \frac{d}{2} = 2 \sqrt{2}
$$

- Equation of the circle:

$$
(x - 5)^2 + (y - 6)^2 = (2 \sqrt{2})^2 = 4 \times 2 = 8
$$

Final Equation:

$$
(x - 5)^2 + (y - 6)^2 = 8
$$

Deepening the Analysis: What If Only One Endpoint Is Known?

In many practical problems, only one endpoint of a diameter or a point on the circle is given. If this is the case, additional information—such as the length of the diameter, the position of the other endpoint, or the circle's radius—is needed to uniquely determine the circle.

Step 7: General Approach for the Standard Form

Summarizing the approach:

1. Identify the endpoints of the diameter: $A(x_1, y_1)$ and $B(x_2, y_2)$.
2. Calculate the center: $(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.
3. Calculate the radius: $r = \frac{1}{2} \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
4. Write the standard form:

$$
(x - h)^2 + (y
$$

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