Find The Standard Matrices A And A' For T = T2 T1 And T' = T1 T2. T1: R2 R2, T1(x, Y) = (x 5y, 2x 2y)

Find The Standard Matrices A And A' For T = T2 T1 And T' = T1 T2. T1: R2 R2, T1(x, y) = (x + 5y, 2x + 2y)

Understanding how to find the standard matrices for composed linear transformations is fundamental in linear algebra. Specifically, when dealing with transformations such as T = T2 T1 and T' = T1 T2, identifying their matrices A and A' provides insight into their geometric and algebraic properties. In this article, we will explore the process of determining the standard matrices A and A' for these transformations, starting from the given linear map T1: $R^2 \to R^2$, defined by T1(x, y) = (x + 5y, 2x + 2y).

Understanding Linear Transformations and Standard Matrices

What is a Linear Transformation?

A linear transformation is a function T: $R^2 \to R^2$ (or between any vector spaces) that preserves vector addition and scalar multiplication. Formally, for vectors u, v in R^2 and scalar c:

- T(u + v) = T(u) + T(v)
- T(cu) = cT(u)

Given such a transformation, it can be represented by a matrix A such that:

T(x) = A x

where x is a vector in R^2, and A is a 2×2 matrix.

Standard Matrix of a Linear Transformation

The standard matrix of a linear transformation T can be found by observing the images of the basis vectors of R^2, usually $e_1 = (1,0)$ and $e_2 = (0,1)$. The columns of the matrix A are $T(e_1)$ and $T(e_2)$, expressed as vectors.

Given Transformation T1 and Its Matrix Representation

Definition of T1

The transformation T1: $R^2 \to R^2$ is given by:

$T1(x, y) = (x + 5y, 2x + 2y)$

This can be viewed as a matrix multiplication:

$$A_1 = \begin{bmatrix}
1 & 5 \\
2 & 2
\end{bmatrix}$$

since:

$T1(1, 0) = (1 + 0, 2 + 0) = (1, 2)$

$T1(0, 1) = (0 + 5, 0 + 2) = (5, 2)$

which are the columns of the matrix A_1.

Finding Matrices for Compositions of Transformations

Understanding Composition of Linear Transformations

Given two linear transformations T1 and T2 with matrices A and B respectively, their composition T = T2 T1 is represented by the matrix:

$A_T = B A$

Similarly, for T' = T1 T2, the matrix representation is:

$A_{T'} = A B$

where the order of multiplication reflects the order of composition.

Key Point

- For T = T2 T1, the matrix is $A_2 A_1$.
- For T' = T1 T2, the matrix is $A_1 A_2$.

Our goal is to find matrices A and A' corresponding to these compositions, given A_1 and the matrices representing T2 and T1.

Determining the Matrices for T2 and T1

Matrix for T1

As established, the matrix for T1 is:

$$A_1 = \begin{bmatrix} 1 & 5 \\ 2 & 2 \end{bmatrix}$$

Matrix for T2

Suppose T2: $R^2 \to R^2$ is a linear transformation with unknown matrix B:

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Our task is to find B, or at least understand how B interacts with A_1 to form the matrices for T and T'.

Finding the Matrices A and A' for the Compositions

Matrix for T = T2 T1

Since T = T2 T1, its matrix A is:

$A = B A_1$

Similarly, for T' = T1 T2, its matrix A' is:

$A' = A_1 B$

The challenge is that B is unknown unless additional information about T2 is provided.

Example: Assuming a Specific T2 to Illustrate the Process

Suppose T2 is a scaling transformation

Let’s assume T2 scales vectors by a factor of 3 in both directions:

B = \[
\begin{bmatrix}
3 & 0 \\
0 & 3
\end{bmatrix}
\]

This simplifies calculations and illustrates how to find A and A'.

Calculating A for T = T2 T1

$A = B A_1$ = \[
\begin{bmatrix}
3 & 0 \\
0 & 3
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & 5 \\
2 & 2
\end{bmatrix}
\] = \[
\begin{bmatrix}
31 + 02 & 35 + 02 \\

01 + 32 & 05 + 32
\end{bmatrix}
= \[
\begin{bmatrix}
3 & 15 \\
6 & 6
\end{bmatrix}
\]

Thus, the matrix A for T = T2 T1 is:

A = \[
\begin{bmatrix}
3 & 15 \\
6 & 6
\end{bmatrix}
\]

Calculating A' for T' = T1 T2

A' = A_1 B = \[
\begin{bmatrix}
1 & 5 \\
2 & 2
\end{bmatrix}
\] \[
\begin{bmatrix}
3 & 0 \\
0 & 3
\end{bmatrix}
\] = \[
\begin{bmatrix}
13 + 50 & 10 + 53 \\
23 + 20 & 20 + 23
\end{bmatrix}
= \[
\begin{bmatrix}
3 & 15 \\
6 & 6
\end{bmatrix}
\]

Note that in this particular example, both matrices are identical because the scaling transformation is symmetric.

General Process to Find Standard Matrices for Any T2

Step 1: Identify or define T2

- Determine the matrix B of T2, either given or by analyzing how T2 acts on basis vectors.

Step 2: Compute matrix for T = T2 T1

- Multiply B by A_1:

$A = B A_1$

Step 3: Compute matrix for T' = T1 T2

- Multiply A_1 by B:

$A' = A_1 B$

Step 4: Interpret the matrices

- The resulting matrices A and A' reveal how the compositions of transformations affect vectors in R^2.

Key Takeaways and Applications

- Understanding the composition of linear transformations through matrix multiplication is crucial in linear algebra.
- The order of multiplication matters: T = T2 T1 corresponds to $A = B A_1$, while T' = T1 T2 corresponds to $A' = A_1 B$.
- Knowing the matrices allows for easy computation of images of vectors, eigenvalues, eigenvectors, and understanding the geometric effects such as scaling, rotation, or shearing.
- This methodology can be extended to higher-dimensional spaces and more complex transformations.

Conclusion

Finding the standard matrices A and A' for the compositions T = T2 T1 and T' = T1 T2 involves understanding the matrices representing each transformation and performing matrix multiplication in the appropriate order. Starting from the known matrix of T1, and assuming or determining the matrix of T2, we can systematically compute A and A'. This process provides a powerful tool for analyzing the effects of combined linear transformations in various applications, from computer graphics to engineering.

If specific details about T2 are provided, such as its matrix or its action on basis vectors, the matrices for the compositions can be precisely calculated. Mastery of this process enhances your ability to work with complex linear systems and deepens your understanding of the structure and behavior of linear transformations.

Frequently Asked Questions

What is the standard matrix of the transformation T1: $R^2 \to R^2$ defined by T1(x, y) = (x + 5y, 2x - 2y)?

The standard matrix A1 of T1 is obtained by applying T1 to the standard basis vectors. T1(1, 0) = (1 + 0, 21 - 0) = (1, 2), and T1(0, 1) = (0 + 5, 0 - 2) = (5, -2). Therefore, A1 = [[1, 5], [2, -2]].

How do you compute the standard matrix for the combined transformation T = T2 T1?

The standard matrix for T = T2 T1 is found by multiplying the matrix of T2 by that of T1, i.e., A_T = A_T2 A_T1, where A_T1 is the matrix of T1 and A_T2 is the matrix of T2.

Given T1's matrix A1, how would you find the matrix for T' = T1 T2?

The matrix for T' = T1 T2 is obtained by multiplying the matrix of T1 by that of T2, i.e., A_T' = A_T1 A_T2. The order of multiplication is crucial because matrix multiplication is not commutative.

If T2 is an unknown transformation, how can we determine the matrices A_T1 and A_T2 explicitly?

We determine A_T1 from the given T1, which is [[1, 5], [2, -2]]. To find

A_T2, we need the definition of T2 or its effects on basis vectors. Once both matrices are known, we can compute compositions by matrix multiplication.

What is the significance of finding standard matrices A and A' for the transformations T = T2 T1 and T' = T1 T2?

Finding the matrices helps us understand how the transformations act on vectors, allows for easy composition, and facilitates analysis of properties like invertibility and eigenvalues, providing insight into the behavior of T and T'.

Additional Resources

Find The Standard Matrices A And A' For T = T2 T1 And T' = T1 T2. T1: R2 R2, T1(x, y) = (x + 5y, 2x + 2y)

Introduction

The problem of finding the standard matrices for linear transformations is fundamental in linear algebra, providing insights into how these transformations operate within vector spaces. In particular, understanding the composite transformations $T = T_2 T_1$ and $T' = T_1 T_2$, where T_1 is defined on $\mathbb{R}^2$ by $T_1(x, y) = (x + 5y, 2x + 2y)$, offers a comprehensive look at how matrix multiplication encapsulates the composition of linear maps. This article aims to thoroughly analyze and derive the standard matrices A and A' representing T and T', respectively, delving into the properties, calculations, and implications of these transformations.

Understanding the Transformation T_1

Definition and Properties of T_1

Given $T_1: \mathbb{R}^2 \to \mathbb{R}^2$ defined by:

$$T_1(x, y) = (x + 5y, 2x + 2y)$$

This transformation combines a linear operation involving addition and scalar multiplication. It can be viewed as a matrix-vector multiplication:

$$T_1 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

which immediately suggests that the matrix representing T_1 in the standard basis is:

$$ A_1 = \begin{bmatrix} 1 & 5 \\ 2 & 2 \end{bmatrix} $$

Understanding this matrix is crucial because it forms the building block for the composite transformations.

Features of T_1

- Linearity: T_1 is a linear transformation because it can be represented as matrix multiplication, which preserves addition and scalar multiplication.

- Invertibility: To determine whether T_1 is invertible, evaluate its determinant:

$$ \det(A_1) = (1)(2) - (5)(2) = 2 - 10 = -8 \neq 0 $$

Since the determinant is non-zero, T_1 is invertible, meaning it has an inverse transformation.

- Geometric interpretation: T_1 can be viewed as a combination of shear and scaling transformations, given the off-diagonal entries.

Finding the Standard Matrix for $T = T_2 T_1$

Understanding the Composite Transformation $T = T_2 T_1$

The transformation T is the composition of T_2 after T_1, meaning:

$$ T = T_2 \circ T_1 $$

which applies T_1 first, then T_2 to the resulting vector. To find the matrix A representing T in the standard basis, we need:

$$ A = A_2 A_1 $$

where A_2 is the matrix of T_2. However, the problem statement does not explicitly define T_2; typically, in such problems, T_2 is another linear transformation on $\mathbb{R}^2$ with a known matrix. For the sake of completeness, let's assume T_2 is similarly linear and represented by:

$$ T_2(x, y) = (a_{11}x + a_{12}y, a_{21}x + a_{22}y) $$

with matrix:

$$ A_2 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} $$

Note: If specific T_2 is given elsewhere, substitute accordingly.

Deriving the Matrix A for T

Given the composition:

$$ T = T_2 T_1 $$

we compute:

$$ A = A_2 A_1 $$

which involves standard matrix multiplication.

Example Calculation (Hypothetical)

Suppose $A_2 = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, representing a scaling transformation.

Then,

$$ A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} (2)(1)+(0)(2) & (2)(5)+(0)(2) \\ (0)(1)+(3)(2) & (0)(5)+(3)(2) \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ 6 & 6 \end{bmatrix} $$

This matrix A represents the composite transformation T.

Features and Implications

- Matrix multiplication order: Since matrix multiplication is not commutative, the order $T_2 T_1$ matters. Reversing the order yields a different matrix.

- Effect on vectors: The composite transformation applies T_1 first, then T_2, which can be visualized as sequential geometric transformations.

Finding the Standard Matrix for $T' = T_1 T_2$

Understanding $T' = T_1 T_2$

Similarly, T' is the composition:

$$ T' = T_1 \circ T_2 $$

which applies T_2 first, then T_1. Its matrix representation is:

$$ A' = A_1 A_2 $$

using the matrices of T_1 and T_2.

Calculating A' for the same example

Using the previous hypothetical A_2:

$$ A' = A_1 A_2 = \begin{bmatrix} 1 & 5 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} (1)(2)+(5)(0) & (1)(0)+(5)(3) \\ (2)(2)+(2)(0) & (2)(0)+(2)(3) \end{bmatrix} = \begin{bmatrix} 2 & 15 \\ 4 & 6 \end{bmatrix} $$

Again, the order significantly affects the resulting matrix.

Features and Implications

- Order sensitivity: Unlike scalar functions, the sequence of composition impacts the final matrix, emphasizing the importance of operation order.

- Impact on geometric transformations: Different compositions can lead to varied effects such as rotations, scalings, shears, or their combinations.

General Approach to Computing Standard Matrices

Step 1: Identify the individual transformations

- Write each transformation as a matrix based on given definitions.

Step 2: Determine the order of composition

- For $T = T_2 T_1$, compute $A_2 A_1$.

- For $T' = T_1 T_2$, compute $A_1 A_2$.

Step 3: Perform matrix multiplication

- Use standard rules for matrix multiplication to find the composite matrix.

Step 4: Interpret the results

- Analyze determinants to assess invertibility.

- Examine eigenvalues/eigenvectors for understanding transformation properties.

Step 5: Verify the transformations

- Apply the matrices to basis vectors to confirm correctness.

Practical Applications and Significance

Pros of Understanding Composite Matrices

- Predicting transformation effects: Knowing the matrices allows precise predictions of how vectors are transformed.

- Simplifying complex transformations: Decomposing transformations into known matrices aids in understanding and computing complex operations.

- Applications in graphics and engineering: Transformations such as rotations, scalings, and shears are foundational in computer graphics, robotics, and physics.

Cons and Challenges

- Order dependence: Mistakes in the sequence can lead to incorrect matrices.

- Complexity with higher dimensions: In larger vector spaces, calculations become more intricate.

- Interpretability: Matrix representations may not always provide intuitive geometric insights without further analysis.

Conclusion

Finding the standard matrices A and A' for transformations $T = T_2 T_1$ and $T' = T_1 T_2$ involves understanding the individual transformations, their matrix representations, and the non-commutative nature of matrix multiplication. T_1, characterized by a simple linear map, serves as the foundational building block. The composite transformations highlight how sequence impacts the overall effect—a critical concept in linear algebra. Whether in theoretical mathematics, computer graphics, or engineering, mastering the derivation and interpretation of these matrices is essential for analyzing and applying linear transformations effectively.

References

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- Strang, G. (2009). Introduction to Linear Algebra. Wellesley-Cambridge Press.
- Hoffman, K., & Kunze, R. (1971). Linear Algebra. Prentice-Hall.

Note: The specific matrices for T_2 were assumed hypothetically for illustration. To perform precise calculations, actual definitions of T_2 are necessary.

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