Find The Value Of The Constant K That Makes The Function Continuous. 22 5x - 12 If X #4 G(x) = X-4 Kx

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Introduction

In calculus and advanced mathematics, the concept of continuity plays a vital role in understanding the behavior of functions. A function is continuous at a point if there are no abrupt jumps or breaks in its graph at that point. Ensuring the continuity of functions is essential in various applications, from engineering to physics, and it forms the foundation for many calculus concepts such as derivatives and integrals.

One common problem faced by students and professionals alike involves determining the value of a constant that makes a piecewise function continuous across its domain. This problem not only tests your understanding of the definition of continuity but also your ability to manipulate functions algebraically and apply limits effectively.

In this article, we will explore a specific problem: Find the value of the constant K that makes the function continuous given by

\[
G(x) =
\begin{cases}
22 \cdot 5x - 12 & \text{if } x \neq 4 \\
X - 4 Kx & \text{if } x = 4
\end{cases}
\]

Understanding this problem requires a detailed look at piecewise functions, limits, and the conditions necessary for continuity. We will break down the problem step-by-step, providing clear explanations and calculations to help you grasp the underlying principles and arrive at the correct value of K.

Understanding Piecewise Functions and Continuity

What Is a Piecewise Function?

A piecewise function is a function defined by different expressions based on the value of the input variable, x. These functions are used to model situations where the behavior of the function changes at certain points.

For example, the function G(x) in our problem is defined differently at x = 4 and elsewhere:

- For all $x \neq 4$, $G(x) = 22 \cdot 5x - 12$
- At x = 4, G(x) = X - 4 Kx

The goal is to determine the value of K such that G(x) is continuous at x = 4.

What Does Continuity Mean?

A function G(x) is continuous at a point x = a if the following three conditions are satisfied:

1. G(a) is defined: The function has a value at x = a.
2. Limit exists at x = a: The limit of G(x) as x approaches a exists.
3. Limit equals the function value: $\lim_{x \to a} G(x) = G(a)$.

If any of these conditions are violated, the function has a discontinuity at that point. In our case, the critical point is x = 4, and we need to ensure the function is continuous at this point by appropriately choosing K.

Analyzing the Function G(x)

Given Function Definition

The piecewise function is given as:

$$
G(x) =
\begin{cases}
22 \cdot 5x - 12, & x \neq 4 \\
X - 4 Kx, & x = 4
\end{cases}
$$

Note: The notation appears to have some inconsistencies, especially with "X - 4 Kx" at x=4. It's important to interpret this correctly. Likely, it means:

$$
G(4) = 4 - 4K \times 4
$$

which simplifies to:

$$
G(4) = 4 - 16K
$$

Similarly, the expression for $x \neq 4$ is:

$$
G(x) = 22 \times 5x - 12
$$

which simplifies to:

$$
G(x) = 110x - 12
$$

Rewriting the Function for Clarity

- For $x \neq 4$: $G(x) = 110x - 12$
- At $x=4$: $G(4) = 4 - 16K$

Our task is to find the value of K such that the function is continuous at $x=4$. Therefore, the key step is to ensure:

$$
\lim_{x \to 4} G(x) = G(4)
$$

Calculating the Limit as x Approaches 4

Limit from the Left and Right

Since the function is defined differently at $x=4$, but the same for all $x \neq 4$, the limit as x approaches 4 is simply the limit of $110x - 12$ as x approaches 4.

$$
\lim_{x \to 4} G(x) = \lim_{x \to 4} (110x - 12)
$$

This is a standard limit, which can be directly evaluated:

$$
\lim_{x \to 4} (110x - 12) = 110 \times 4 - 12 = 440 - 12 = 428
$$

Therefore,

$$
\lim_{x \to 4} G(x) = 428
$$

Ensuring Continuity at x=4

Recall that the function must satisfy:

$$
G(4) = \lim_{x \to 4} G(x)
$$

From earlier, we have:

$$
G(4) = 4 - 16K
$$

and

$$
\lim_{x \to 4} G(x) = 428
$$

Set these equal to ensure continuity:

$$
4 - 16K = 428
$$

Solving for K

Now, solve the equation:

$$
4 - 16K = 428
$$

Subtract 4 from both sides:

$$
-16K = 428 - 4
$$
$$
-16K = 424
$$

Divide both sides by -16:

$$
K = \frac{424}{-16} = -\frac{424}{16}
$$

Simplify the fraction:

$$
K = -\frac{424}{16} = -\frac{106}{4} = -\frac{53}{2}
$$

Final Answer and Conclusion

The value of the constant K that makes the piecewise function continuous at x=4 is:

$$
\boxed{K = -\frac{53}{2}}
$$

This means that by setting K to $-\frac{53}{2}$, the function G(x) will have no jumps or breaks at x=4, ensuring a smooth transition between the two pieces.

Summary of Key Steps

- Understand the structure of the piecewise function.
- Calculate the limit of the function as x approaches the critical point.
- Set the function's value at $x=4$ equal to this limit.
- Solve the resulting equation for K.

Additional Tips for Solving Similar Problems

- Always verify the domain and the definition of the function at the point of interest.
- Remember that the limit depends on the function's behavior as x approaches the point, not necessarily the value at the point.
- When dealing with piecewise functions, focus on the limit from both sides if the function is defined differently on either side.
- Simplify algebraic expressions to avoid errors in calculations.
- Practice with different functions to strengthen your understanding of continuity conditions.

Conclusion

Determining the value of a constant that ensures the continuity of a function is a fundamental skill in calculus. It involves understanding the concepts of limits, function definitions, and the conditions for continuity. In our specific problem, by analyzing the limit as x approaches 4 and equating it to the function's value at that point, we found that the constant K must be $-\frac{53}{2}$.

Mastering these techniques will help you analyze more complex functions and ensure their smooth behavior across their entire domain. Whether you're studying calculus for academic purposes or applying these concepts in real-world scenarios, understanding how to manipulate and analyze functions for continuity is invaluable.

Remember: Practice makes perfect. Try creating similar problems with different functions and points of discontinuity to strengthen your skills in analyzing and ensuring function continuity.

Frequently Asked Questions

What is the goal when finding the value of the constant K in

the given function?

The goal is to determine the value of K that makes the function continuous at the point where the function's definition changes, which in this case is at $x = 4$.

What is the given piecewise function in the problem?

The function is $G(x) = 22 + 5x - 12$ if $x \neq 4$, and $G(x) = x - 4Kx$ if $x = 4$.

Why do we need to evaluate the limit of G(x) as x approaches 4?

Because for G(x) to be continuous at $x = 4$, the limit of G(x) as x approaches 4 must equal the value of G(4).

How do you find the limit of G(x) as x approaches 4?

Since for $x \neq 4$, $G(x) = 22 + 5x - 12$, the limit as x approaches 4 is simply $22 + 5(4) - 12$.

What is the value of the limit of G(x) as x approaches 4?

The limit is $22 + 20 - 12 = 30$.

How do you ensure the function is continuous at x = 4?

By setting G(4) equal to the limit of G(x) as x approaches 4, which gives the equation for solving K.

What is the value of G(4) in terms of K?

$G(4) = 4 - 4K\ 4 = 4 - 16K$.

How do you solve for K to make the function continuous?

Set G(4) = limit as x approaches 4: $4 - 16K = 30$, then solve for K: $4 - 16K = 30 \rightarrow -16K = 26 \rightarrow K = -26/16 = -13/8$.

What is the final value of K that makes the function continuous?

The value of K is $-13/8$.

Additional Resources

Find The Value Of The Constant K That Makes The Function Continuous

Understanding the concept of continuity in functions is fundamental in calculus and mathematical analysis. When dealing with piecewise functions, the challenge often lies in determining the specific values of parameters or constants that ensure the function remains continuous across its entire domain. In this article, we will thoroughly analyze the problem: Find the value of the constant K that makes the function continuous, given the piecewise function

$$
G(x) = \begin{cases}
22 + 5x - 12, & \text{if } x \neq 4 \\
Kx, & \text{if } x = 4
\end{cases}
$$

This problem exemplifies the typical scenario where a piecewise function involves an unknown constant, and the goal is to find this constant such that the function is continuous at the point where the pieces meet—in this case, at $x = 4$.

Understanding Continuity in Piecewise Functions

What Is Continuity?

Continuity at a point $x = a$ means that the function’s limit as x approaches a from both sides equals the function’s value at that point. Formally, a function $f(x)$ is continuous at $x = a$ if:

$$
\lim_{x \to a} f(x) = f(a)
$$

In the context of piecewise functions, ensuring continuity at the transition point involves verifying that the limit from the left and right exists and equals the function’s value at that point.

Why Is Continuity Important?

- Mathematical Modeling: Many real-world phenomena require models that are continuous to reflect smooth changes.
- Calculus Operations: Derivatives and integrals rely on the function being continuous.
- Analyzing Behavior: Continuity helps in understanding the function's behavior around specific points.

Analyzing the Given Function

The given function is:

$$
G(x) =
\begin{cases}
22 + 5x - 12, & \text{if } x \neq 4 \\
Kx, & \text{if } x = 4
\end{cases}
$$

First, simplify the expression for $x \neq 4$:

$$
G(x) = 22 + 5x - 12 = (22 - 12) + 5x = 10 + 5x
$$

So, the function simplifies to:

$$
G(x) =
\begin{cases}
10 + 5x, & x \neq 4 \\
Kx, & x = 4
\end{cases}
$$

The key is to find the value of K such that G is continuous at $x = 4$.

Establishing Continuity at $x = 4$

Step 1: Compute the Limit as $x \to 4$

Since the function is defined as $10 + 5x$ for all $x \neq 4$, the limit as x approaches 4 from both sides is:

$$
\lim_{x \to 4} G(x) = \lim_{x \to 4} (10 + 5x) = 10 + 5 \times 4 = 10 + 20 = 30
$$

Step 2: Set the Function Value at $x = 4$

For the function to be continuous at $x = 4$, the value at this point must be equal to the limit:

$$
G(4) = K \times 4 = 4K

\]

Step 3: Equate Limit and Function Value

$$
\lim_{x \to 4} G(x) = G(4) \Rightarrow 30 = 4K
$$

Solving for K:

$$
K = \frac{30}{4} = \frac{15}{2} = 7.5
$$

Thus, the constant K that makes the function continuous at $x = 4$ is 7.5.

Features and Implications of the Solution

Features of the Function with $K = 7.5$

- The function becomes continuous at $x=4$.
- The value at $x=4$ is:

$$
G(4) = 4 \times 7.5 = 30
$$

- The function remains linear elsewhere with a slope of 5, ensuring smoothness at the transition point.

Implications of the Constant K

- The value of K aligns the pointwise value with the limit, eliminating any jump discontinuity.
- The behavior of the function at $x=4$ now seamlessly connects the piecewise parts.

Pros and Cons of the Approach

Pros:
- Simplicity: The problem reduces to solving a straightforward equation.

- Clarity: The step-by-step approach highlights the importance of limits and function values in continuity.
- Educational Value: Reinforces understanding of limits, function evaluation, and piecewise functions.

Cons:
- Assumes familiarity: Requires understanding of limits and continuity concepts.
- Limited scope: Only applies to functions with well-defined limits; more complex functions may require advanced techniques.
- Potential for oversight: Missing the point that the limit must be calculated from both sides, though in this case, the function is continuous everywhere else.

Additional Considerations and Variations

What If the Function Was Different?

Suppose the piecewise function had more complicated expressions or additional points of discontinuity. The same approach could be extended by:
- Computing limits from the left and right separately.
- Ensuring the function values match these limits at the transition points.
- Adjusting constants or parameters to satisfy continuity conditions.

Continuity in More Complex Functions

For functions involving absolute values, roots, or piecewise definitions with different forms, the process involves:
- Breaking down the limit calculations into cases.
- Carefully analyzing the behavior near transition points.
- Ensuring the limit exists and matches the function’s value at those points.

Summary and Final Remarks

To conclude, finding the value of K that makes the given piecewise function continuous at $x=4$ involves analyzing the limit of the function as $x \to 4$ and setting it equal to the function value at that point. In our case, the simplified function led us to the conclusion that:

$$
\boxed{
K = \frac{15}{2} = 7.5
}
$$

This value ensures that the function transitions smoothly at $x=4$, with no discontinuity. Recognizing the importance of limits in this context is crucial for understanding and manipulating piecewise functions in calculus.

By mastering these foundational concepts, students and mathematicians can better handle more complex problems involving continuity, limits, and piecewise functions in their studies and practical applications.

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