Find The Work Required To Move An Object In The Force Field F=x,2 Along The Straight Line From A(0,0)

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Introduction

In physics and engineering, understanding how to calculate the work done by a force field in moving an object from one point to another is fundamental. When dealing with vector fields, especially force fields, the work depends on the path taken between the initial and final points. This article provides a comprehensive guide on calculating the work required to move an object within a specific force field, $\mathbf{F} = (x, 2)$, along a straight line from point A(0,0). We will explore the underlying concepts, step-by-step procedures, and relevant mathematical tools necessary to solve this type of problem efficiently.

Understanding the Force Field $\mathbf{F} = (x, 2)$

Nature of the Force Field

The given force field is:

$$
\mathbf{F}(x, y) = (x, 2)
$$

This indicates that:

- The force in the x-direction varies linearly with the x-coordinate.
- The force in the y-direction is constant and equals 2 everywhere in the field.

Visualizing the Force Field

Visualizing the force field helps in understanding how the object experiences force at different points:

- At any point (x, y), the force points rightward with magnitude proportional to x and upward with a constant magnitude of 2.
- The field is non-uniform in the x-direction but uniform in the y-direction.

Implications for Work Calculation

Since the force field is position-dependent, the work done depends on the path taken, which makes the calculation an application of line integrals in vector calculus.

Path Specification: Straight Line from A(0,0)

Defining the Path

The problem specifies moving the object along a straight line from point A(0,0). To compute work, we need:

- The equation of the line from $(0,0)$ to the final point (x_f, y_f).
- A parameterization of this line that allows us to evaluate the line integral.

General Form of the Path

Suppose the endpoint B is at (x_f, y_f). The straight line from A to B can be parameterized as:

$$
\mathbf{r}(t) = (x(t), y(t))
$$

with

$$
x(t) = x_f t, \quad y(t) = y_f t, \quad t \in [0, 1]
$$

This parameterization satisfies:

- $t=0$ corresponds to the start point $(0,0)$.
- $t=1$ corresponds to the endpoint (x_f, y_f).

Calculating the Work Done: Mathematical Framework

Line Integral of a Vector Field

The work W done by a force $\mathbf{F}$ in moving an object along a path C from point A to point B is given by the line integral:

$$
W = \int_{C} \mathbf{F} \cdot d\mathbf{r}
$$

where:

- $d\mathbf{r}$ is the differential displacement vector along the path.
- $\mathbf{F}$ is evaluated at each point along the path.

Expressing the Line Integral

Using the parameterization $\mathbf{r}(t)$:

$$
d\mathbf{r} = \frac{d\mathbf{r}}{dt} dt = (x'(t), y'(t)) dt
$$

and the work becomes:

$$
W = \int_{0}^{1} \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt
$$

Step-by-Step Calculation

1. Parameterize the Path

Assuming the endpoint B is at (x_f, y_f):

$$
x(t) = x_f t, \quad y(t) = y_f t
$$

then,

$$
x'(t) = x_f, \quad y'(t) = y_f
$$

2. Evaluate the Force Field Along the Path

Since $\mathbf{F} = (x, 2)$:

$$
\mathbf{F}(\mathbf{r}(t)) = (x(t), 2) = (x_f t, 2)
$$

3. Compute the Dot Product

$$
\mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) = (x_f t, 2) \cdot (x_f, y_f) = x_f t \cdot x_f + 2 \cdot y_f = x_f^2 t + 2 y_f
$$

4. Set Up the Integral

The work done is:

$$
W = \int_0^1 (x_f^2 t + 2 y_f) dt
$$

5. Integrate

$$
W = \int_0^1 x_f^2 t dt + \int_0^1 2 y_f dt = x_f^2 \left[\frac{t^2}{2} \right]_0^1 + 2 y_f [t]_0^1
$$

which simplifies to:

$$
W = x_f^2 \cdot \frac{1}{2} + 2 y_f \cdot 1 = \frac{x_f^2}{2} + 2 y_f
$$

Final Expression for Work

The total work required to move an object along a straight line from $A(0,0)$ to $B(x_f, y_f)$ in the force field $\mathbf{F} = (x, 2)$ is:

$$
\boxed{
W = \frac{x_f^2}{2} + 2 y_f
}
$$

This result emphasizes that the work depends on both the endpoint coordinates and the specific path taken, which, in this case, is a straight line.

Special Cases and Applications

Moving Along the x-axis

- When $y_f = 0$, the work simplifies to:

$$
W = \frac{x_f^2}{2}
$$

- This indicates that the work depends quadratically on the x-coordinate.

Moving Along the y-axis

- When $x_f = 0$, the work simplifies to:

$$
W = 2 y_f
$$

- The work is directly proportional to the displacement in the y-direction.

Practical Applications

- Designing robotic movement paths where forces vary with position.
- Calculating work in electrostatic or gravitational fields with position-dependent forces.
- Engineering applications involving force fields in fluid dynamics or magnetic fields.

Additional Considerations

Path Independence and Conservative Fields

- The force field $\mathbf{F} = (x, 2)$ is not conservative because its curl is non-zero.
- Therefore, the work depends on the path taken, and the straight line is just one example.

- For path-independent fields, the work can be calculated using potential functions; however, since this field is non-conservative, the integral's value depends on the specific path.

Extending to Other Paths

- To evaluate work along a different path, parameterize that path and repeat the integral calculation.
- For curved paths, the same method applies; only the parameterization changes.

Conclusion

Calculating the work required to move an object within a force field is a fundamental problem in physics that involves applying vector calculus principles, particularly line integrals. For the force field $\mathbf{F} = (x, 2)$, moving an object along a straight line from the origin to a point (x_f, y_f), the work done is:

$$
\boxed{
W = \frac{x_f^2}{2} + 2 y_f
}
$$

This expression provides a direct way to evaluate work based on the endpoint coordinates and helps in analyzing energy transfer in various physical systems. Understanding these calculations enhances our ability to design efficient systems and analyze forces in complex environments.

References

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Keywords for SEO Optimization

- Work calculation in force fields
- Line integrals in physics
- Force field $\mathbf{F} = (x, 2)$
- Moving objects in vector fields
- Path-dependent work in physics
- Calculus applications in physics
- Energy transfer in force fields
- Vector calculus in engineering

Frequently Asked Questions

What is the main goal when calculating the work required to move an object in a force field like F = (x, 2)?

The main goal is to determine the total work done by the force field in moving the object along a specified path, which involves integrating the force along that path.

How do you parametrize the straight line from point A(0,0) to point B(x, y) in this problem?

You can parametrize the line using a parameter t such that $r(t) = (x(t), y(t))$, where t ranges from 0 to 1, with $x(t) = at$ and $y(t) = bt$ for some constants a and b depending on the endpoint.

What is the general formula for work done by a force field along a curve?

The work W is given by the line integral $W = \int_C F \cdot dr$, where F is the force vector and dr is the differential displacement along the curve C.

How do you evaluate the line integral for the force field F = (x, 2) along a straight line?

Substitute the parametric equations into the force components, compute the dot product $F \cdot dr/dt$, and integrate with respect to t over the interval [0, 1].

What specific steps are involved in calculating the work required to move the object from (0,0) to (x, y) in this force field?

1. Parametrize the line segment from (0,0) to (x, y). 2. Compute F at each point along the path. 3. Find dr/dt. 4. Calculate the dot product $F \cdot dr/dt$. 5. Integrate this expression over t from 0 to 1.

What is the value of the work done in moving the object from (0,0) to (x, y) in the force field F = (x, 2)?

The work is given by the integral $W = \int_0^1 F(r(t)) \cdot r'(t)\, dt$. Performing the integration yields $W = (1/2)\, x^2 + 2\, y$, assuming the straight line from (0,0) to (x,y) is parametrized linearly.

How does the direction of the force field influence the work calculation?

The force field's components determine the integrand in the line integral.

Since F = (x, 2), the force depends on x, affecting the work calculation depending on the path taken.

Can the work done depend on the path chosen between two points in this force field?

In this case, because the force field is not conservative, the work may depend on the path taken. However, for a straight line, the calculation is straightforward; for other paths, the work may differ.

What are common applications or physical interpretations of calculating work in such force fields?

This calculation models real-world scenarios like moving objects in gravitational or electromagnetic fields, helping understand energy requirements, efficiency, and system behavior in physics and engineering contexts.

Additional Resources

Find The Work Required To Move An Object In The Force Field F = xi + 2j Along The Straight Line From A(0,0)

When analyzing the movement of an object within a force field, a fundamental question often arises: How much work is required to move an object from one point to another? Specifically, in the context of a force field expressed as F = xi + 2j, determining the work involved in moving an object along a specified path—from the origin A(0,0) to some endpoint—requires understanding the principles of work, path integrals, and the nature of the force field itself. This article provides a comprehensive guide on finding the work required to move an object in the force field F = xi + 2j along the straight line from A(0,0), breaking down the problem step-by-step, exploring the concepts involved, and illustrating the calculations with clarity.

Understanding the Problem Context

Before diving into calculations, it's essential to restate the problem in precise terms:

- Force Field: F = xi + 2j, where i and j are the unit vectors in the x and y directions, respectively.
- Initial Point: A(0,0)
- Final Point: B, which must be specified or determined.
- Path: The straight line from A(0,0) to B.
- Objective: Find the total work done in moving an object along this path under the influence of the force field.

The first step in solving such a problem is to define the endpoint B precisely, as this determines the path. For the sake of completeness, let's assume the endpoint is at B(x_f, y_f), where x_f and y_f are specified. If the problem states a particular endpoint, substitute accordingly.

Step 1: Clarify the Path and Parameters

Defining the Path

Since the movement is along a straight line from $A(0,0)$ to $B(x_f, y_f)$, we can parametrize this path using a parameter t that varies from 0 to 1:

- When $t=0$: position is at $A(0,0)$.
- When $t=1$: position is at $B(x_f, y_f)$.

Parametric Equations

The parametric equations of the path are:

- $x(t) = x_f t$
- $y(t) = y_f t$

where $t \in [0,1]$.

Derivatives of the Path

The derivatives are:

- $dx/dt = x_f$
- $dy/dt = y_f$

which are constants because the path is a straight line.

Step 2: Express the Force Field Along the Path

The force field $F(x,y) = xi + 2j$ depends on the position (x, y). Along the path:

- $F(t) = x(t)\, i + 2\, j = (x_f t)\, i + 2\, j$

Thus, at any point t, the force vector becomes:

- $F(t) = (x_f t)\, i + 2\, j$

Step 3: Calculate the Differential Work Element

The work dW done in moving the object an infinitesimal distance ds along the path is:

$dW = F \cdot dr$

where:

- F is the force vector at the point,
- dr is the differential displacement vector along the path.

Using the parametric representation:

- $dr/dt = (dx/dt)\, i + (dy/dt)\, j = x_f\, i + y_f\, j$

So,

$$dr = (x_f\, i + y_f\, j)\, dt$$

and the dot product:

$$F \cdot dr = [(x_f\, t)\, i + 2\, j] \cdot [x_f\, i + y_f\, j]\, dt$$

which simplifies to:

$$F \cdot dr = [(x_f\, t)\, x_f + 2\, y_f]\, dt$$

Step 4: Set Up the Integral for Work

The total work W required to move the object from A to B along the straight line is obtained by integrating dW over t from 0 to 1:

$$W = \int_0^1 F \cdot dr = \int_0^1 [(x_f\, t)\, x_f + 2\, y_f]\, dt$$

Expressed explicitly:

$$W = \int_0^1 [x_f^2\, t + 2\, y_f]\, dt$$

Step 5: Perform the Integration

The integral becomes:

$$W = \int_0^1 x_f^2\, t\, dt + \int_0^1 2\, y_f\, dt$$

Calculating each term separately:

- First integral:

$$\int_0^1 x_f^2\, t\, dt = x_f^2\, (1/2)\, t^2 \,|_0^1 = x_f^2\, (1/2)\, 1 = (1/2)\, x_f^2$$

- Second integral:

$$\int_0^1 2\, y_f\, dt = 2\, y_f\, t \,|_0^1 = 2\, y_f\, 1 = 2\, y_f$$

Adding these results, the total work is:

$$W = (1/2)\, x_f^2 + 2\, y_f$$

Final Expression for Work

$$W = (1/2)\, x_f^2 + 2\, y_f$$

This formula provides the work required to move an object from $A(0,0)$ to $B(x_f, y_f)$ along the straight line in the force field $F = xi + 2j$.

Additional Considerations

1. Nature of the Force Field

The force field F = xi + 2j is a conservative vector field because it can be derived from a potential function. To verify this, we can check whether F is conservative:

- The curl of F:

$\nabla \times F = (\partial/\partial x)(2) - (\partial/\partial y)(x) = 0 - 0 = 0$

Since the curl is zero, the field is conservative, and the work done is path-independent. Therefore, the work depends only on the initial and final points, not on the specific path taken.

2. Potential Function

Given that F is conservative, the potential function V(x, y) can be found:

- $\nabla V = -F$

So:

- $\partial V/\partial x = -x$
- $\partial V/\partial y = -2$

Integrating:

- $V(x, y) = -(1/2)\, x^2 - 2\, y + C$

The work done in moving from A to B:

$W = V(A) - V(B) = [V(0,0)] - [V(x_f, y_f)]$

Calculating:

- $V(0,0) = 0 - 0 + C = C$

- $V(x_f, y_f) = -(1/2)\, x_f^2 - 2\, y_f + C$

Therefore:

$W = [C] - [-(1/2)\, x_f^2 - 2\, y_f + C] = (1/2)\, x_f^2 + 2\, y_f$

which matches the previous result, confirming the calculation.

Practical Example

Suppose you are to move an object from A(0,0) to B(3,4) in the force field F = xi + 2j.

Plugging into the formula:

$W = (1/2)\, 3^2 + 2\; 4 = (1/2)\; 9 + 8 = 4.5 + 8 = 12.5$ units of work

This quantifies the energy required to perform this task under the specified force field.

Summary of Key Points

- Path parametrization simplifies the integral calculation.
- The force field is linear in x, with a constant component in y.
- The work is evaluated using the line integral ∫ F · dr.
- The force field F = xi + 2j is conservative, making the work path-independent.
- The potential function simplifies calculations and confirms the path independence.

Conclusion

Understanding how to find the work required to move an object in a force field involves a few fundamental steps: parametrizing the path, expressing the force along that path, setting up the line integral, and performing the integration. For the specific force field F = xi + 2j, moving along a straight line from A(0,0) to B(x_f, y_f) yields a straightforward formula for the work:

$W = (1/2)\ x_f^2 + 2\ y_f$

This approach exemplifies the power of vector calculus in solving physics and engineering problems, providing both a theoretical framework and practical tools for analyzing work in force fields.

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