Find Two Solutions Of The Equation. Give Your Answers In Degrees (0 S 0 < 360) And In Radians (0 5

Find Two Solutions Of The Equation. Give Your Answers In Degrees (0 ≤ θ < 360) And In Radians (0 ≤ θ ≤ 2π)

When solving trigonometric equations, one of the fundamental tasks is to determine all possible solutions within a specified interval. This process involves understanding the periodic nature of trig functions and applying algebraic techniques to isolate the variable. Whether you're working with sine, cosine, or tangent, finding two solutions of an equation and expressing them in both degrees and radians is essential for comprehensive understanding and application. In this article, we will explore methods to find solutions to various types of trigonometric equations, with detailed examples and step-by-step solutions, optimized for clarity and SEO.

Understanding the Basics of Trigonometric Equations

Before diving into specific examples, it's important to grasp the foundational concepts related to solving trigonometric equations.

Key Concepts and Definitions

- **Unit Circle:** A circle with radius 1 centered at the origin, crucial for understanding sine and cosine values.
- **Periodicity:** Sine and cosine functions repeat every 360° (2π radians), while tangent repeats every 180° (π radians).
- **Inverse Functions:** Used to find angles given a sine, cosine, or tangent value.
- **Principal Values:** The main solutions within a specific interval, typically [0°, 360°) or [0, 2π) radians.

Methodology for Finding Solutions

When solving a trigonometric equation, follow a systematic approach:

Step-by-Step Process

1. **Isolate the trigonometric function:** Get the expression into a form like sin θ = a, cos θ = b, or tan θ = c.
2. **Determine the basic solutions:** Find the reference angle(s) where the function equals the given value.
3. **Account for the periodicity:** Add the period (360° or 2π radians) as needed to find all solutions within the interval.
4. **Express solutions in degrees and radians:** Convert angles appropriately, ensuring they fall within the

specified ranges.

Example 1: Solving a Basic Sine Equation

Suppose we want to find solutions for the equation:

$\sin \theta = 0.5$

within the interval 0°, 360° and in radians 0, 2π.

Step 1: Find the Reference Angle

- The reference angle θ_0 for $\sin \theta = 0.5$ is 30°, since $\sin 30° = 0.5$.
- In radians, 30° = π/6.

Step 2: Determine All Solutions in the Interval

- Sine is positive in the first and second quadrants, so solutions are:

1. $\theta = 30°$ (π/6 radians)
2. $\theta = 180° - 30° = 150°$ (5π/6 radians)

Step 3: Express Solutions in Degrees and Radians

- Degrees: 30°, 150°
- Radians: π/6, 5π/6

Summary of Solutions

- Degrees: 30°, 150°
- Radians: π/6, 5π/6

Example 2: Solving a Cosine Equation

Find solutions for:

$\cos \theta = -\sqrt{2}/2$

within $0° \le \theta < 360°$, and $0 \le \theta \le 2\pi$.

Step 1: Find the Reference Angle

- The value $\cos \theta = -\sqrt{2}/2$ corresponds to an angle where cosine is negative.
- The reference angle is 45°, since $\cos 45° = \sqrt{2}/2$.
- In radians, 45° = π/4.

2: Determine All Solutions in the Interval

- Cosine is negative in the second and third quadrants.
- Solutions:

1. $\theta = 180° - 45° = 135°$ ($3\pi/4$ radians)
2. $\theta = 180° + 45° = 225°$ ($5\pi/4$ radians)

3: Express in Degrees and Radians

- Degrees: 135°, 225°
- Radians: $3\pi/4$, $5\pi/4$

Example 3: Solving a Tangent Equation

Solve:

$\tan \theta = 1$

within the interval $0° \leq \theta < 360°$, and $0 \leq \theta \leq 2\pi$.

Step 1: Find the Reference Angle

- $\tan \theta = 1$ occurs at $\theta = 45°$, which is $\pi/4$ radians.

2: Find All Solutions

- Tangent is positive in the first and third quadrants.
- Solutions:

1. $\theta = 45°$ ($\pi/4$ radians)
2. $\theta = 180° + 45° = 225°$ ($5\pi/4$ radians)

3: Express in Degrees and Radians

- Degrees: 45°, 225°
- Radians: $\pi/4$, $5\pi/4$

Key Points to Remember When Solving Trigonometric Equations

- Always consider the periodicity of the function involved.
- Check the sign of the function to determine the correct quadrants.
- Use inverse functions to find reference angles.
- Convert angles between degrees and radians accurately.
- Ensure solutions fall within the specified interval.

Conclusion: Practice and Application

Mastering how to find two solutions of a trigonometric equation in both degrees and radians is a crucial skill in mathematics, physics, engineering, and other sciences. Practicing different types of equations enhances understanding of unit circle properties, periodicity, and algebraic manipulation. Whether dealing with sine, cosine, or tangent, the core approach remains consistent: isolate, find reference angles, consider quadrants, and convert solutions into the desired units. By following these steps and understanding the key concepts, you'll be well-equipped to solve a wide range of trigonometric equations confidently and accurately.

Optimized for SEO Keywords:
- Find solutions of trigonometric equations
- Solutions in degrees and radians
- How to solve sine, cosine, tangent equations
- Trigonometric equations with solutions in 0 to 360 degrees
- Converting angles between degrees and radians
- Periodicity of sine, cosine, tangent functions
- Reference angles in trigonometry
- Solving trig equations step-by-step

Frequently Asked Questions

Find two solutions of the equation sin(x) = 0.5 in the interval 0° to 360° and in radians.

Solutions in degrees: x = 30°, 150°. In radians: x = π/6, 5π/6.

Determine two solutions of the equation cos(x) = -☐ 2/2 within 0° to 360° and in radians.

Solutions in degrees: x = 135°, 225°. In radians: x = 3π/4, 5π/4.

Find two solutions of the equation tan(x) = 1 in the interval 0° to 360° and in radians.

Solutions in degrees: x = 45°, 225°. In radians: x = π/4, 5π/4.

Solve the equation 2sin(x) - 1 = 0 for x in 0° to 360° and radians.

Solutions in degrees: x = 30°, 150°. In radians: x = π/6, 5π/6.

Find two solutions of the equation sec(x) = 2 in the interval 0° to 360° and in radians.

Solutions in degrees: x = 60°, 300°. In radians: x = π/3, 5π/3.

Determine two solutions of the equation cot(x) = 1 in 0° to 360° and

radians.

Solutions in degrees: x = 45°, 225°. In radians: x = π/4, 5π/4.

Find solutions of the equation sin(2x) = 0 in the interval 0° to 360° and in radians.

Solutions in degrees: x = 0°, 90°, 180°, 270°. In radians: x = 0, π/2, π, 3π/2.

Solve the equation cos(2x) = 0.5 for x between 0° and 360° and in radians.

Solutions in degrees: x = 15°, 75°, 105°, 165°, 195°, 255°, 285°, 345°. In radians: x = π/12, 5π/12, 7π/12, 11π/12, 13π/12, 17π/12, 19π/12, 23π/12.

Find two solutions of the equation tan(3x) = 0 in 0° to 360° and radians.

Solutions in degrees: x = 0°, 60°, 120°, 180°, 240°, 300°. In radians: x = 0, π/3, 2π/3, π, 4π/3, 5π/3.

Additional Resources

Solving Trigonometric Equations: A Comprehensive Guide to Finding Two Solutions in Degrees and Radians

In the world of mathematics, trigonometry is a vital branch that deals with the relationships between the angles and sides of triangles. Whether you're a student preparing for exams, an educator designing curriculum, or a professional applying trigonometry in engineering or physics, understanding how to find solutions to trigonometric equations is essential. Today, we'll explore a common yet crucial task: finding two solutions of a trigonometric equation within specified ranges — specifically in degrees (0° < θ < 360°) and radians (0 < θ < 2π). Think of this as a detailed product review, where we analyze the features, applications, and best practices for solving such equations.

Understanding the Core Concept: Why Find Solutions in Both Degrees and Radians?

Before diving into the methods, it's important to understand why solutions are often expressed both in degrees and radians. Degrees are more intuitive for many, especially in everyday contexts, while radians are the standard measure in higher mathematics, calculus, and many scientific applications because of their natural relationship with the properties of circles.

Key points:

- Degrees: Range from 0° to 360°, representing a full circle in 1° increments.
- Radians: Range from 0 to 2π, with 1 radian ≈ 57.2958°, corresponding to the length of an arc in a unit circle.

Knowing how to convert between these units and interpret solutions in both forms enhances versatility in problem-solving.

Fundamental Strategies for Solving Trigonometric Equations

The process of solving trigonometric equations involves a combination of algebraic manipulation, understanding of unit circle properties, and sometimes iterative or graphical methods. Here's a step-by-step overview:

1. Identify the Type of Equation

Common forms include:

- Basic functions: $\sin \theta = a$, $\cos \theta = a$, $\tan \theta = a$
- Equations involving multiple functions or identities: $\sin^2 \theta + \cos^2 \theta = 1$, or equations involving combined functions.

Understanding the form helps determine the best approach.

2. Isolate the Trigonometric Function

For example, if you have an equation like:

$$\sin \theta = \frac{1}{2}$$

try to isolate the sine function to solve for θ.

3. Use Inverse Trigonometric Functions

Apply inverse functions to find principal solutions:

- $\theta = \sin^{-1}(a)$
- $\theta = \cos^{-1}(a)$
- $\theta = \tan^{-1}(a)$

Ensure your calculator or software is set to the correct mode (degrees or radians) before computing.

4. Determine All Solutions within the Given Range

Since inverse functions give only principal solutions, you need to consider the periodicity of the functions:

- Sine and cosine have a period of 360° (2π radians).
- Tangent has a period of 180° (π radians).

Use symmetry and periodicity properties to find all solutions within the specified range.

5. Express Solutions Clearly

Present solutions explicitly, considering the periodic nature:

- In degrees:

$$\theta = \text{principal solution} + 360^\circ \times n$$

- In radians:

$$\theta = \text{principal solution} + 2\pi \times n$$

where n is an integer chosen so that the solutions lie within the specified range.

Example Problem: Find Two Solutions of the Equation in Degrees and Radians

Let's consider a specific example to illustrate these strategies:

$$
\sin \theta = \frac{\sqrt{3}}{2}
$$

Given: $0^\circ < \theta < 360^\circ$ and $0 < \theta < 2\pi$

Step 1: Find the Principal Solution(s) in Degrees and Radians

Using inverse sine:

$$
\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)
$$

Recall the exact value:

$$
\sin 60^\circ = \frac{\sqrt{3}}{2}
$$

and

$$
\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}
$$

So, the principal solutions are:

- In degrees: $\theta = 60^\circ$
- In radians: $\theta = \frac{\pi}{3}$

Step 2: Find All Solutions within the Range

Since sine is positive in the first and second quadrants, the second solution corresponds to:

- In degrees: $180^\circ - 60^\circ = 120^\circ$
- In radians: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Therefore, the two solutions are:

Degrees	Radians	Description
60°	$\frac{\pi}{3}$	First solution (quadrant I)
120°	$\frac{2\pi}{3}$	Second solution (quadrant II)

Step 3: Verify the Solutions are within the Given Ranges

- Degrees: Both solutions are within $0^\circ < \theta < 360^\circ$.
- Radians: Both solutions are within $0 < \theta < 2\pi$.

Additional Solutions and Periodicity

While the problem asks for two solutions, understanding the periodicity of sine reveals that more solutions exist beyond the initial range. For example:

- In degrees:

$$
\theta = 60^\circ + 360^\circ \times n, \quad n \in \mathbb{Z}
$$

- In radians:

$$
\theta = \frac{\pi}{3} + 2\pi \times n, \quad n \in \mathbb{Z}
$$

To find solutions within a different range, adjust n accordingly.

Applying This Approach to Other Trigonometric Equations

This methodology extends to other equations, such as:

- $\cos \theta = a$
- $\tan \theta = a$
- More complex equations involving multiple functions

Key considerations:

- For cosine, solutions are symmetric about the y-axis, so solutions often occur at:

$$
\theta = \pm \cos^{-1}(a) + 360^\circ \times n
$$

- For tangent, which has a period of 180°, solutions are:

$$
\theta = \tan^{-1}(a) + 180^\circ \times n
$$

In degrees and radians, always remember to verify that solutions fall within the specified ranges.

Practical Tips for Accurate Solutions

- Use a calculator or software: Confirm your inverse trigonometric calculations are in the correct mode.
- Check the solutions: Substitute back into the original equation to verify.
- Be mindful of quadrants: Use reference angles and symmetry properties to find all solutions.
- Express solutions clearly: Show the general solution form along with particular solutions within the specified range.
- Convert units when necessary: To switch between degrees and radians, use:

\[
\text{Radians} = \frac{\pi}{180} \times \text{Degrees}
\]

\[
\text{Degrees} = \frac{180}{\pi} \times \text{Radians}
\]

Conclusion: Mastering Solutions in Degrees and Radians

Finding two solutions of a trigonometric equation within the range \(0^\circ < θ < 360^\circ\) or \(0 < θ < 2π\) is a fundamental skill that combines algebra, inverse functions, and a deep understanding of the unit circle. By systematically identifying principal solutions, leveraging symmetry and periodicity, and carefully converting units, you can confidently solve a wide array of trigonometric problems.

Whether you're tackling basic equations or complex identities, these strategies serve as your toolkit for precise, reliable solutions. Remember, practice makes perfect — so use a variety of equations to sharpen your skills and become proficient in both degrees and radians. With this comprehensive understanding, you'll navigate the fascinating landscape of trigonometry with confidence and clarity.

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