Fireworks Are Fired From The Roof Of A 100-foot Building. The Equation H =-16t2 +84t + 100 Models The

Fireworks Are Fired From The Roof Of A 100-foot Building. The Equation H = -16t^2 + 84t + 100 Models The trajectory of fireworks launched from elevated positions, such as rooftops, offers fascinating insights into physics and mathematics. Understanding how the height of a firework changes over time is essential for pyrotechnicians, safety officials, educators, and enthusiasts alike. The quadratic equation H = -16t^2 + 84t + 100 serves as a foundational model to analyze and predict the motion of fireworks fired from a 100-foot building. This article explores the significance of this equation, how it models firework trajectories, and the practical applications of such mathematical representations.

Understanding the Firework Trajectory Equation

1. The Components of the Equation

The equation H = -16t^2 + 84t + 100 models the height (H) of a firework in feet at any given time (t) in seconds after launch. Each term in the equation has a specific physical meaning:

- **-16t^2**: Represents the effect of gravity on the firework, with the coefficient -16 corresponding to half of Earth's gravitational acceleration in feet per second squared (since acceleration due to gravity is approximately 32 ft/sec^2).
- **84t**: The initial velocity component, indicating that the firework is propelled upward at 84 feet per second.
- **100**: The initial height from which the firework is launched, corresponding to the 100-foot height of the building rooftop.

2. The Significance of the Equation in Physics and Mathematics

This quadratic model is a classic example of uniformly accelerated motion under gravity. It allows us to determine key aspects of the firework's flight, such as:

- Maximum height reached (the apex of the trajectory)
- Time to reach maximum height
- Time when the firework hits the ground or a specific altitude
- The total duration of the firework's flight

By analyzing these factors, pyrotechnicians can choreograph fireworks displays with precision, ensuring safety and visual impact.

Modeling Firework Height Over Time

1. Calculating the Time to Reach Maximum Height

The vertex of the parabola, which represents the maximum height, occurs at the vertex formula:

t = -b / (2a)

In this case:

a = -16
b = 84

So,

t = -84 / (2 -16) = -84 / -32 = 2.625 seconds

This means the firework reaches its highest point approximately 2.625 seconds after launch.

2. Determining the Maximum Height

Substitute t = 2.625 seconds back into the original equation:

H = -16(2.625)^2 + 84(2.625) + 100

Calculate step-by-step:

-16 (2.625)^2 = -16 6.890625 = -110.250

84 2.625 = 220.5

Adding all together:

$H = -110.250 + 220.5 + 100 = 210.25$ feet

Thus, the firework reaches a maximum height of approximately 210.25 feet above the ground.

3. Time When Firework Hits the Ground

To find when the firework returns to the ground, set $H = 0$:

$0 = -16t^2 + 84t + 100$

Use the quadratic formula:

$t = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

with:

$a = -16$
$b = 84$
$c = 100$

Calculate discriminant:

$D = 84^2 - 4 (-16) 100 = 7056 + 6400 = 13,456$

Square root of discriminant:

$\sqrt{D} \approx 116$

Compute roots:

$t = [-84 \pm 116] / (-32)$

First root:

$t = (-84 + 116) / -32 = 32 / -32 = -1$ second (discard, as negative time)

Second root:

$t = (-84 - 116) / -32 = -200 / -32 \approx 6.25$ seconds

Therefore, the firework hits the ground approximately 6.25 seconds after launch.

Practical Applications of the Equation in Firework Displays

1. Choreographing Fireworks for Safety and Visual Impact

Understanding the height and timing allows pyrotechnicians to:

- Ensure fireworks do not collide with structures or spectators
- Synchronize multiple fireworks for complex displays
- Design trajectories that maximize visual appeal while maintaining safety margins

2. Designing Firework Launch Strategies

By manipulating initial velocity (the coefficient of t), professionals can:

- Adjust the height of fireworks
- Create specific visual effects at desired heights
- Control the duration of the firework's ascent and descent

3. Educational Uses in Physics and Mathematics

This model serves as an excellent teaching tool to illustrate:

- Quadratic equations and their graphs
- The effects of gravity on projectile motion
- Real-world applications of mathematical models

Safety Considerations and Regulations

1. Ensuring Safe Launch Heights and Distances

Mathematical modeling helps determine safe distances from the launch site, considering the maximum height and descent time of fireworks.

2. Compliance with Local Fireworks Regulations

Understanding the physics behind firework trajectories ensures compliance with safety standards and legal requirements, reducing accidents and injuries.

Conclusion: The Power of Mathematics in Fireworks Displays

The equation $H = -16t^2 + 84t + 100$ exemplifies how mathematics models real-world phenomena, such as fireworks launched from a 100-foot building. By analyzing this quadratic function, pyrotechnicians and educators can predict firework trajectories, optimize display designs, and ensure safety. The interplay of initial velocity, gravity, and height encapsulated in this formula highlights the importance of mathematical understanding in creating spectacular and safe fireworks shows. Whether for entertainment, education, or safety planning, mastering such equations is essential for anyone involved in the art and science of fireworks.

This comprehensive understanding underscores the significance of quadratic equations in modeling projectile motion, illustrating their practical applications beyond theoretical mathematics.

Frequently Asked Questions

What does the equation $H = -16t^2 + 84t + 100$ represent in the context of fireworks fired from a 100-foot building?

It models the height (H) of the fireworks above the ground in feet at time t seconds after they are launched from the roof of the building.

How can I determine the maximum height reached by

the fireworks using this equation?

The maximum height occurs at the vertex of the parabola. For the equation $H = -16t^2 + 84t + 100$, the time to reach maximum height is $t = -b/(2a) = -84/(2 -16) = 84/32 = 2.625$ seconds. Plugging this back into the equation gives the maximum height.

What is the maximum height the fireworks reach according to this model?

Using $t = 2.625$ seconds, the maximum height $H = -16(2.625)^2 + 84(2.625) + 100 \approx 134.125$ feet.

When do the fireworks hit the ground based on this equation?

To find when the fireworks hit the ground, set $H = 0$ and solve for t: $0 = -16t^2 + 84t + 100$. Solving this quadratic will give the times when the height is zero, indicating impact with the ground.

How do the coefficients in the equation relate to the fireworks' trajectory?

The coefficient -16 represents half the acceleration due to gravity in feet per second squared (assuming Earth's gravity). The coefficient 84 is related to the initial velocity, and 100 is the initial height from the roof of the building.

What real-world factors could cause deviations from this mathematical model of fireworks height?

Factors include air resistance, wind, variations in explosion force, and imperfections in the fireworks, which can all cause the actual trajectory to differ from the idealized quadratic model.

Can this model be used to predict the exact timing of fireworks displays for safety purposes?

While the model provides a good approximation of height over time, real-world conditions and safety considerations require more detailed planning and empirical testing beyond this simplified mathematical model.

Additional Resources

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Introduction to the Scenario and Its Significance

Imagine a spectacular fireworks display launched from the roof of a 100-foot building. This scenario not only captures the imagination but also serves as an intriguing case study in physics and mathematics. The equation $H = -16t^2 + 84t + 100$ models the height of the fireworks over time, where:

- H represents the height above ground (in feet)
- t is the time since launch (in seconds)

Understanding this equation provides insights into projectile motion, maximum height, time of flight, and the overall trajectory of fireworks launched from elevated locations. This analysis is relevant for fireworks engineers, safety planners, and physics enthusiasts alike, as it combines real-world applications with fundamental principles of kinematics.

Breaking Down the Equation: The Physics Behind the Model

The Standard Form of the Equation

The given quadratic equation:

$$ H(t) = -16t^2 + 84t + 100 $$

mirrors the typical form used in physics to represent the vertical motion of projectiles under gravity:

$$ H(t) = -\frac{1}{2} g t^2 + v_0 t + H_0 $$

where:

- g is the acceleration due to gravity (~32 ft/sec²)
- v_0 is the initial velocity (in ft/sec)
- H_0 is the initial height (in feet)

Matching terms:

- The coefficient -16 aligns with $-\frac{1}{2} g$, confirming that $g = 32$ ft/sec^2.
- The initial velocity v_0 is 84 ft/sec.
- The initial height H_0 is 100 feet, correlating with the height of the building.

This model assumes no air resistance, which simplifies calculations but still gives a realistic approximation of the fireworks' trajectory.

Physical Interpretation of Each Term

- $-16t^2$: Represents the acceleration due to gravity pulling the fireworks downward, causing the height to decrease over time after reaching a peak.
- 84t: The initial upward velocity component, propelling the fireworks upward immediately after launch.
- 100: The initial height from which the fireworks are launched, i.e., the roof of the building.

Analyzing the Fireworks' Trajectory

Maximum Height and Its Calculation

To determine the highest point the fireworks reach, we find the vertex of the parabola represented by the quadratic function. For a quadratic $H(t) = at^2 + bt + c$, the time at which maximum height occurs is:

$$t_{max} = -\frac{b}{2a}$$

Applying this:

$$t_{max} = -\frac{84}{2 \times (-16)} = -\frac{84}{-32} = 2.625 \text{ seconds}$$

This indicates that the fireworks reach their peak height approximately 2.625 seconds after launch.

Calculating the maximum height:

$$H_{max} = -16(2.625)^2 + 84(2.625) + 100$$

Step-by-step:

1. $(2.625)^2 = 6.890625$
2. $-16 \times 6.890625 = -110.25$
3. $84 \times 2.625 = 220.5$
4. Sum: $-110.25 + 220.5 + 100 = 210.25$

Result:

- The fireworks reach a maximum height of approximately 210.25 feet.

This is about 110.25 feet above the roof level, emphasizing the impressive elevation achieved by the fireworks.

Time of Flight: When Do the Fireworks Hit the Ground?

To determine when the fireworks return to the ground (height = 0), solve:

$$
-16t^2 + 84t + 100 = 0
$$

Multiply through by -1 to simplify:

$$
16t^2 - 84t - 100 = 0
$$

Apply the quadratic formula:

$$
t = \frac{84 \pm \sqrt{(-84)^2 - 4 \times 16 \times (-100)}}{2 \times 16}
$$

Calculate discriminant:

$$
(-84)^2 = 7056
$$
$$
4 \times 16 \times (-100) = -6400
$$
$$
\text{Discriminant} = 7056 - (-6400) = 7056 + 6400 = 13,456
$$

Square root:

$$
\sqrt{13,456} \approx 116.02
$$

Calculate the roots:

$$
t = \frac{84 \pm 116.02}{32}
$$

- Positive root:

$$
t = \frac{84 + 116.02}{32} = \frac{200.02}{32} \approx 6.25 \text{ seconds}
$$

- Negative root:

$$
t = \frac{84 - 116.02}{32} = \frac{-32.02}{32} \approx -1.00 \text{ seconds}
$$

Since negative time isn't physically meaningful here, the fireworks hit the ground approximately 6.25 seconds after launch.

Practical Implications and Safety Considerations

Height and Safety Zones

- The fireworks reach a maximum height of about 210 feet, which is well above the roof level.
- Safety protocols must account for the maximum height to establish safe zones, ensuring no unauthorized personnel are within a radius that could be affected by debris or misfires.
- The elevation of the launch point (100 feet) extends the overall range and visibility of the display, but it also demands careful planning regarding the trajectory and fall zones.

Designing a Fireworks Display from a Tall Building

- Trajectory Management: The equation helps predict where the fireworks will

land, allowing organizers to set safe distances.
- Timing and Synchronization: Knowing the time of maximum height and total flight time (around 6.25 seconds) enables precise timing for multiple launches.
- Legal and Regulatory Compliance: Launching from a tall building may require permits and adherence to local safety regulations, especially considering the height and potential fall zones.

Environmental and Weather Factors

- Wind and weather conditions can alter the trajectory from the ideal modeled by the equation.
- Adjustments in initial velocity or launch angles may be necessary based on real-time conditions.
- The model assumes no air resistance; real-world deviations must be considered for safety margins.

Mathematical Insights and Educational Value

Understanding Projectile Motion

This scenario offers an excellent real-world application of quadratic equations in physics, illustrating:

- How initial velocity and launch height influence maximum altitude
- The symmetry of projectile motion around the vertex
- The importance of quadratic models in predictive analysis

Graphical Representation

Plotting $H(t)$:

- The parabola opens downward (since $a = -16$)
- The vertex at $t = 2.625$ seconds indicates the maximum height
- The roots at approximately 0.46 seconds (not calculated here but can be found) and 6.25 seconds show the start and end of the projectile's flight

Visualizing this helps in understanding the dynamics of firework trajectories and designing more spectacular displays.

Extensions and Advanced Topics

- Incorporating air resistance into the model for more realistic predictions
- Analyzing the effects of launch angles other than vertical
- Exploring multi-stage fireworks and their complex trajectories

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Conclusion: The Fusion of Mathematics and Spectacle

The equation $H = -16t^2 + 84t + 100$ encapsulates the physics of a firework launched from the roof of a 100-foot building, providing a comprehensive understanding of its trajectory. It highlights how fundamental principles of kinematics translate into real-world applications, from ensuring safety to creating awe-inspiring displays.

By analyzing the maximum height, total flight time, and the implications of launching from an elevated position, organizers and engineers can optimize fireworks for maximum visual impact while maintaining safety standards. Moreover, this mathematical model serves as a foundational lesson in projectile motion, demonstrating the elegance with which simple quadratic equations can describe complex physical phenomena.

Whether you are a physics student, a fireworks technician, or a curious spectator, appreciating the interplay between equations and spectacular visual effects deepens your understanding and appreciation of both science and art.

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