For A Consumer With Demand Function Q=1005p 1/2, Find: A) Consumer Surplus(CS), At Price P 0=9 B) CS,

For A Consumer With Demand Function Q=1005p^{1/2}, Find: A) Consumer Surplus(CS), At Price P_0=9 B) CS,

---

Introduction

Understanding consumer surplus is fundamental in microeconomics, as it measures the benefit consumers receive when they purchase a product at a price lower than the maximum they are willing to pay. In this article, we analyze a specific demand function, Q = 1005p^{1/2}, and calculate the consumer surplus at a given market price P_0=9. We will also explore the derivation of the demand function, the concept of consumer surplus, and the implications of the results.

Understanding the Demand Function Q = 1005p^{1/2}

Interpreting the Demand Function

The demand function provided, Q = 1005p^{1/2}, expresses the relationship between the quantity demanded (Q) and the price (p). It indicates that:

- The quantity demanded varies directly with the square root of the price.
- As price increases, demand increases, but at a decreasing rate—reflecting typical demand behavior.

Implications of the Demand Function

- The demand function suggests a nonlinear relationship between price and quantity.
- The demand is positively related to the square root of price, which implies diminishing marginal demand

as price increases.
- Consumers' maximum willingness to pay at different quantities can be derived from the inverse demand function.

Deriving the Inverse Demand Function

To analyze consumer surplus, it is essential to express the maximum willingness to pay for each quantity demanded—that is, the inverse demand function $p = f(Q)$.

Step-by-Step Derivation

Given:

$$
Q = 1005 p^{1/2}
$$

Solve for p:

1. Divide both sides by 1005:

$$
\frac{Q}{1005} = p^{1/2}
$$

2. Square both sides to eliminate the square root:

$$
p = \left(\frac{Q}{1005}\right)^2
$$

Thus, the inverse demand function is:

$$
p(Q) = \left(\frac{Q}{1005}\right)^2
$$

This function indicates the maximum price a consumer is willing to pay for a given quantity Q.

Calculating Consumer Surplus (CS)

Consumer surplus represents the difference between what consumers are willing to pay and what they actually pay.

General Formula for Consumer Surplus

$$
CS = \int_{0}^{Q_0} p(Q) \, dQ - P_0 \times Q_0
$$

Where:

- Q_0 = quantity demanded at the market price P_0.
- $p(Q)$ = inverse demand function.

Part A: Consumer Surplus at Price $P_0=9$

Step 1: Find the Quantity Demanded Q_0 at Price $P_0=9$

Using the original demand function:

$$
Q = 1005 p^{1/2}
$$

Plug in $p = 9$:

$$
Q_0 = 1005 \times \sqrt{9} = 1005 \times 3 = 3015
$$

So, at price 9, the consumer demands 3015 units.

Step 2: Compute the Consumer Surplus

Using the inverse demand function:

$$
p(Q) = \left(\frac{Q}{1005}\right)^2
$$

The consumer surplus is:

$$
CS = \int_{0}^{Q_0} p(Q) \, dQ - P_0 \times Q_0
$$

Calculate the integral:

$$
CS = \int_{0}^{3015} \left(\frac{Q}{1005}\right)^2 dQ - 9 \times 3015
$$

Simplify the integral:

$$
CS = \frac{1}{(1005)^2} \int_{0}^{3015} Q^2 dQ - 9 \times 3015
$$

Calculate the integral:

$$
\int_{0}^{Q_0} Q^2 dQ = \frac{Q_0^3}{3}
$$

Plugging in values:

$$
CS = \frac{1}{(1005)^2} \times \frac{(3015)^3}{3} - 9 \times 3015
$$

Calculate numerator:

$$
(3015)^3 = 3015 \times 3015 \times 3015
$$

Since $3015 = 3 \times 1005$, then:

$$
(3015)^3 = (3 \times 1005)^3 = 3^3 \times 1005^3 = 27 \times 1005^3
$$

Now, substitute back:

$$
CS = \frac{1}{1005^2} \times \frac{27 \times 1005^3}{3} - 9 \times 3015
$$

Simplify numerator:

$$
\frac{27 \times 1005^3}{3} = 9 \times 1005^3
$$

Thus,

$$
CS = \frac{1}{1005^2} \times 9 \times 1005^3 - 9 \times 3015
$$

Simplify:

$$
CS = 9 \times 1005^3 / 1005^2 - 9 \times 3015
$$

$$
CS = 9 \times 1005 - 9 \times 3015
$$

Calculate:

$$
9 \times 1005 = 9045
$$

\[

$$
9 \times 3015 = 27135
$$

Finally,

$$
CS = 9045 - 27135 = -18090
$$

Since consumer surplus cannot be negative, check the calculations for consistency.

Note: The negative result indicates an error in the calculation—likely due to misinterpretation of the integral limits or the substitution process. Let's revisit the integral calculation carefully.

Revised Calculation of Consumer Surplus

Integral:

$$
CS = \int_{0}^{Q_0} p(Q) \, dQ - P_0 \times Q_0
$$

with:

$$
p(Q) = \left(\frac{Q}{1005}\right)^2
$$

So,

$$
CS = \int_{0}^{Q_0} \left(\frac{Q}{1005}\right)^2 dQ - P_0 \times Q_0
$$

Expressed explicitly:

$$
CS = \frac{1}{(1005)^2} \int_{0}^{Q_0} Q^2 dQ - P_0 \times Q_0
$$

Calculate the integral:

$$
\int_{0}^{Q_0} Q^2 dQ = \frac{Q_0^3}{3}
$$

Plug in $Q_0=3015$:

$$
CS = \frac{1}{(1005)^2} \times \frac{(3015)^3}{3} - 9 \times 3015
$$

Express $(3015)^3$:

$$
(3015)^3 = (3 \times 1005)^3 = 27 \times 1005^3
$$

Substitute:

$$
CS = \frac{1}{1005^2} \times \frac{27 \times 1005^3}{3} - 9 \times 3015
$$

Simplify numerator:

$$
\frac{27 \times 1005^3}{3} = 9 \times 1005^3
$$

So,

$$
CS = \frac{1}{1005^2} \times 9 \times 1005^3 - 9 \times 3015
$$

Simplify the fraction:

$$
\frac{9 \times 1005^3}{1005^2} = 9 \times 1005
$$

Therefore:

$$
CS = 9 \times 1005 - 9 \times 3015
$$

Calculate:

$$
9 \times 1005 = 9045
$$

$$
9 \times 3015 = 27135
$$

Finally:

$$
CS = 9045 - 27135 = -18090
$$

Again, negative consumer surplus indicates an inconsistency, because consumer surplus should be positive.

Correcting the approach:

The error stems from the incorrect interpretation of the integral bounds. Consumer surplus is the area between the demand curve and the price line, from zero quantity up to Q_0.

But because the demand function is positive and decreasing in price, the integral should be:

$$
CS = \int_{p=0}^{p=P_0} Q(p) \, dp
$$

Alternatively, the consumer surplus can be calculated as:

$$
CS = \int_{p=P_0}^{p_{max}} Q(p) \, dp
$$

Frequently Asked Questions

What is the demand function given in the problem?

The demand function provided is Q = 1005 p^{1/2}, where Q is the quantity demanded and p is the price.

How do you find the consumer surplus when the price P_0 is 9?

To find the consumer surplus at P_0=9, first determine the maximum willingness to pay (the choke price), then compute the area of the triangle between the demand curve and P_0 up to the maximum price.

What is the first step in calculating consumer surplus for this demand function?

The first step is to find the maximum price consumers are willing to pay when the quantity demanded drops to zero, which involves setting Q=0 and solving for p.

How do you compute the choke price for the demand function Q=1005p^{1/2}?

Since demand is zero when Q=0, and Q=1005p^{1/2}, setting Q=0 gives p=0. Alternatively, to find the maximum price consumers are willing to pay for a positive demand, we analyze the demand at Q approaching zero.

What is the formula for consumer surplus in this context?

Consumer surplus (CS) is the area of the triangle between the demand curve and the price level, calculated as CS = ∫ from P_0 to P_max of (demand price - P) dQ, or more straightforwardly as CS = (1/2) (base) (height).

How do you calculate the consumer surplus at P_0=9 for this demand function?

Calculate the maximum quantity demanded at P_0=9, find the corresponding maximum price (or choke price), then compute CS as (1/2) (Q at P=0 - Q at P=9) (P_0 - P at Q=0).

What is the significance of the half-power in the demand function for calculating consumer surplus?

The half-power (square root) indicates a nonlinear demand curve, requiring integration or algebraic methods to find consumer surplus, rather than simple linear calculations.

Can you provide a step-by-step summary to find consumer surplus in this problem?

Yes. First, find the demand quantity at P=9: Q=1005 9^{1/2}. Next, determine the choke price where Q=0 (which is p=0). Then, calculate the maximum willingness to pay (the intercept). Finally, compute the area of the triangle between the demand curve and P=9 to find the consumer surplus.

Additional Resources

For A Consumer With Demand Function Q=1005p^{1/2}, Find: A) Consumer Surplus (CS) at Price P_0=9

Unveiling Consumer Surplus: An In-Depth Analysis of the Demand Function Q=1005p^{1/2}

Understanding consumer surplus is central to microeconomic analysis, offering insights into the benefits consumers derive from market transactions beyond what they pay. This article delves into the intricacies of calculating consumer surplus for a specific demand function, Q = 1005p^{1/2}, with a focus on the scenario where the market price is P_0 = 9. We will explore the derivation of consumer surplus, interpret the economic implications, and discuss the broader significance of such calculations in economic theory and policymaking.

Introduction

Consumer surplus (CS) is a measure of the welfare or benefit that consumers receive when they purchase a good or service at a price lower than the maximum price they are willing to pay. It is graphically represented as the area between the demand curve and the market price, up to the quantity purchased.

Given a demand function:

$$Q = 1005 p^{1/2}$$

where:

- Q = Quantity demanded
- p = Price

Our goal is to compute the consumer surplus at a specific market price $P_0 = 9$.

The Demand Function and Its Characteristics

Understanding $Q = 1005 p^{1/2}$

This demand function exhibits a nonlinear, square root relationship between price and quantity demanded. As p increases, Q increases at a decreasing rate, consistent with typical demand behavior.

- Inverse demand function: To analyze consumer surplus, it's often more straightforward to work with the inverse demand function $p(Q)$, expressing price as a function of quantity demanded.

- Implication for consumer surplus: The area under the demand curve from zero to the equilibrium quantity, minus the total expenditure (price times quantity), yields consumer surplus.

Deriving the Inverse Demand Function

Given:

$$Q = 1005 p^{1/2}$$

Solve for p:

$$p^{1/2} = \frac{Q}{1005}$$

$$p = \left(\frac{Q}{1005}\right)^2$$

Thus, the inverse demand function is:

$$p(Q) = \left(\frac{Q}{1005}\right)^2$$

Calculating Consumer Surplus at Price $P_0=9$

Step 1: Find the Quantity Demanded at $P_0=9$

Using the original demand function:

\[

Q_{0} = 1005 \times (9)^{1/2}
\]

Calculate:

\[
\sqrt{9} = 3
\]

Therefore:

\[
Q_{0} = 1005 \times 3 = 3015
\]

This is the quantity demanded at the market price of 9.

Step 2: Determine the Maximum Willingness to Pay

The consumer's maximum willingness to pay corresponds to the demand curve's intercept, i.e., the price at which quantity demanded would be zero.

Set \(Q=0 \):

\[
0 = 1005 p^{1/2}
\]
\[
p^{1/2} = 0 \Rightarrow p=0
\]

This indicates that as the price approaches zero, the quantity demanded approaches zero. The demand curve starts at the highest price consumers are willing to pay for an infinitesimal quantity, but since the demand function is only defined for positive prices, the maximum willingness to pay can be approximated as approaching infinity for very small quantities or analyzed at the highest feasible price.

In this case, since the demand function is defined for positive prices, the maximum price consumers are willing to pay corresponds to the intercept at the highest quantity of interest, which is theoretically unbounded, but for practical purposes, the maximum willingness to pay is the price at zero quantity demand.

However, because the demand function is unbounded at \(p \to 0 \), but for the purposes of calculating consumer surplus at \(P_0=9 \), we consider the highest price consumers would pay for the initial quantity demanded.

Step 3: Calculate Consumer Surplus (CS)

The consumer surplus is the area between the demand curve and the market price from zero up to the quantity $Q_0=3015$:

$$
CS = \int_{0}^{Q_0} p(Q) \, dQ - P_0 \times Q_0
$$

Plugging in the inverse demand function:

$$
CS = \int_{0}^{Q_0} \left(\frac{Q}{1005}\right)^2 dQ - 9 \times 3015
$$

Simplify the integral:

$$
CS = \frac{1}{(1005)^2} \int_{0}^{Q_0} Q^2 \, dQ - 9 \times 3015
$$

Calculate the integral:

$$
\int_{0}^{Q_0} Q^2 \, dQ = \frac{Q_0^3}{3}
$$

Thus:

$$
CS = \frac{1}{(1005)^2} \times \frac{Q_0^3}{3} - 9 \times 3015
$$

Substitute $Q_0=3015$:

$$
CS = \frac{1}{(1005)^2} \times \frac{(3015)^3}{3} - 9 \times 3015
$$

Numerical Computation and Interpretation

Computing the Components

- $(1005)^2$:

$$
1005^2 = (1000 + 5)^2 = 1,000,000 + 2 \times 1000 \times 5 + 25 = 1,000,000 + 10,000 + 25 = 1,010,025
$$

- $Q_0^3 = 3015^3$:

Calculate stepwise:

$$
3015^2 = (3000 + 15)^2 = 9,000,000 + 2 \times 3000 \times 15 + 225 = 9,000,000 + 90,000 + 225 = 9,090,225
$$

Then,

$$
Q_0^3 = 3015 \times 9,090,225
$$

Compute:

$$
3015 \times 9,090,225 \approx 3015 \times 9,090,225
$$

This is a large number; approximate:

$$
3015 \times 9,090,225 \approx (3000 + 15) \times 9,090,225 = 3000 \times 9,090,225 + 15 \times 9,090,225
$$

Calculate:

$$
3000 \times 9,090,225 = 27,270,675,000
$$

$$
15 \times 9,090,225 = 136,353,375

\]

Adding:

\[
Q_0^3 \approx 27,270,675,000 + 136,353,375 = 27,407,028,375
\]

Final Calculation

Now, compute the consumer surplus:

\[
CS = \frac{1}{1,010,025} \times \frac{27,407,028,375}{3} - 9 \times 3015
\]

\[
CS = \frac{1}{1,010,025} \times 9,135,676,125 - 27,135
\]

Calculate:

\[
\frac{9,135,676,125}{1,010,025} \approx 9,045
\]

(approximate division)

Finally:

\[
CS \approx 9,045 - 27,135 = -18,090
\]

This negative value indicates an inconsistency, likely due to the rough approximations or the unbounded nature of the demand at high prices. In practice, consumer surplus should be positive; the negative result suggests that the demand function's structure or domain assumptions need careful reconsideration.

Interpretative Insights and Broader Implications

Theoretical Significance

The calculation illustrates the importance of precise mathematical treatment when assessing consumer welfare. The demand function's nonlinearity implies that consumer surplus is sensitive to the specific parameters and the price level considered.

Practical Considerations

In real-world scenarios, demand functions are often bounded or adjusted to reflect realistic consumer behavior. The unbounded nature of the demand function at zero quantity demands careful interpretation, especially when integrating to find consumer surplus.

Policy and Market Implications

- Market Efficiency: Accurate estimates of consumer surplus help policymakers evaluate the efficiency of markets and the impact of price interventions.

- Welfare Analysis: Understanding how consumer surplus varies with price changes informs debates on taxation, subsidy policies, and market regulation.

- Demand Function Specification: The form of the demand function critically influences welfare calculations; nonlinear functions require careful analytical and numerical methods.

Conclusion

Calculating consumer surplus for the demand function $Q = 1005 p^{1/2}$ at $P_0 = 9$ reveals both the power and the complexity of microeconomic analysis. While

For A Consumer With Demand Function Q 1005p 1 2 Find A Consumer Surplus Cs At Price P 0 9 B Cs

For A Consumer With Demand Function Q 1005p 1 2 Find A Consumer Surplus Cs At Price P 0 9 B Cs

Related Articles

- Green Power Company Is Considering Buying A New Machine That Will Last For 11 Years. The Machine Cost
- Jane Welcomes New Ideas And Change. Janes Supervisor Tends To Resist Any Kind Of Change. Jane And Her
- In An Experiment To Determine The Empirical Formula Of Magnesium Chloride, 0.50 G Of Magnesium Ribbon