For The Arithmetic Sequence Beginning With The Terms {-1, 2, 5, 8, 11, 14...}, What Is The Sum Of The

For The Arithmetic Sequence Beginning With The Terms {-1, 2, 5, 8, 11, 14...}, What Is The Sum Of The this sequence? Understanding how to find the sum of an arithmetic sequence is an essential skill in mathematics, especially in algebra and beyond. This article will explore the detailed process of calculating the sum of an arithmetic progression, using the specific sequence provided as a case study. Whether you're a student, teacher, or math enthusiast, mastering this concept helps in solving various mathematical problems efficiently and accurately.

Understanding the Arithmetic Sequence {-1, 2, 5, 8, 11, 14...}

Before diving into the sum calculations, it's important to analyze the sequence itself. Recognizing the pattern and properties of this sequence will lay the foundation for understanding how to compute its sum.

Identifying the First Term and Common Difference

The sequence provided is: -1, 2, 5, 8, 11, 14...

- The first term, denoted as a_1, is -1.
- To find the common difference d, subtract any term from the subsequent term:

$$
d = 2 - (-1) = 3
$$

or

$$
5 - 2 = 3
$$

This confirms the sequence increases by 3 each time.

Expressing the Sequence in General Terms

An arithmetic sequence can be expressed using the formula:

$$
a_n = a_1 + (n - 1)d
$$

where:

- a_n is the nth term,
- a_1 is the first term (-1),
- d is the common difference (3),
- n is the position of the term in the sequence.

So, for this sequence:

$$
a_n = -1 + (n - 1) \times 3
$$

which simplifies to:

$$
a_n = 3n - 4
$$

Understanding this formula allows us to find any term in the sequence directly.

Calculating the Sum of the Arithmetic Sequence

The core of this article is to determine the sum of the sequence, either for a specific number of terms or in general. Let's explore the methods step by step.

Sum of the First n Terms: The Formula

The sum of the first n terms of an arithmetic sequence, denoted as S_n, can be calculated using the formula:

$$
S_n = \frac{n}{2} \times (a_1 + a_n)
$$

Alternatively, using the general term:

$$
S_n = \frac{n}{2} \times [2a_1 + (n - 1)d]
$$

Both formulas are equivalent and useful depending on the available data.

Applying the Formula to Our Sequence

Suppose we want to find the sum of the first n terms of the sequence $-1, 2, 5, 8, 11, 14, \dots$. Using the formula:

$$
S_n = \frac{n}{2} \times [2a_1 + (n - 1)d]
$$

Plugging in the known values:

$$
a_1 = -1, \quad d = 3
$$

we get:

$$
S_n = \frac{n}{2} \times [2 \times (-1) + (n - 1) \times 3]
$$

which simplifies to:

$$
S_n = \frac{n}{2} \times [-2 + 3(n - 1)]
$$

Further expanding:

$$
S_n = \frac{n}{2} \times [-2 + 3n - 3] = \frac{n}{2} \times (3n - 5)
$$

Thus, the sum of the first n terms is:

$$
\boxed{S_n = \frac{n}{2} \times (3n - 5)}
$$

This formula allows for quick calculation once n is known.

Calculating the Sum for a Specific Number of Terms

Suppose you want the sum of the first 10 terms of the sequence. Using the derived formula:

$$
S_{10} = \frac{10}{2} \times (3 \times 10 - 5) = 5 \times (30 - 5) = 5 \times 25 = 125
$$

Therefore, the sum of the first 10 terms is 125.

Example: Sum of the First 20 Terms

Applying the same method:

$$
S_{20} = \frac{20}{2} \times (3 \times 20 - 5) = 10 \times (60 - 5) = 10 \times 55 = 550
$$

The sum of the first 20 terms is 550.

Finding the Sum of the Sequence Up to a Certain Term

Sometimes, you might be asked to find the sum of the sequence up to the term where a_n reaches a specific value or the sequence ends at a certain point.

Determining the Number of Terms for a Given Last Term

Suppose you want to find the sum of all terms up to the 15th term. From the general term formula:

$$
a_n = 3n - 4
$$

Set $a_n =$ desired last term; for example, if the last term is 41:

$$
41 = 3n - 4
$$
$$
3n = 45
$$
$$
n = 15
$$

This confirms the 15th term is 41, and the sum of the first 15 terms can be calculated as:

$$
S_{15} = \frac{15}{2} \times (3 \times 15 - 5) = 7.5 \times (45 - 5) = 7.5 \times 40 = 300
$$

Hence, the sum of the first 15 terms ending at 41 is 300.

Summing Infinite Arithmetic Sequences

While finite sequences are straightforward to sum using the formulas above, infinite arithmetic sequences require caution.

When Can an Infinite Arithmetic Sequence Have a Finite Sum?

An infinite arithmetic sequence can only have a finite sum if its common difference d is zero, meaning all terms are equal, or if the sequence converges, which is only possible when the terms tend toward zero in a specific way.

In our sequence:

$$
a_n = 3n - 4
$$

since $d = 3 \neq 0$, the sequence diverges to infinity, and the sum does not converge to a finite number as $n \to \infty$.

Therefore, the sum of an infinite sequence with a non-zero common difference is infinite.

Practical Applications of Arithmetic Sequence Sums

Understanding how to compute the sum of an arithmetic sequence has numerous real-world applications:

- Financial calculations, like summing periodic payments or investments.
- Engineering problems involving evenly spaced measurements.
- Algorithm analysis where stepwise increasing sequences are involved.
- Planning and scheduling tasks with regular intervals.

Example: Financial Planning

Suppose you plan to save a fixed amount each month, and the amounts form an arithmetic sequence. Calculating the total savings over a period involves summing an arithmetic sequence, which can be simplified using the formulas discussed.

Summary and Key Takeaways

- The sequence $-1, 2, 5, 8, 11, 14, \ldots$ is an arithmetic sequence with first term $a_1 = -1$ and common difference $d = 3$.

- The nth term is given by $a_n = 3n - 4$.
- The sum of the first n terms is:

$$
S_n = \frac{n}{2} \times (3n - 5)
$$

- To find the sum up to a specific term, first determine n using the general term formula, then apply the sum formula.
- Infinite sums of arithmetic sequences with non-zero common difference do not converge to a finite value.

Conclusion

Mastering the calculation of sums of arithmetic sequences is a fundamental aspect of mathematics. By understanding the sequence's properties, deriving the general term, and applying the sum formulas, you can efficiently compute the total of any finite arithmetic sequence. The sequence beginning with $-1, 2, 5, 8, 11, 14, \ldots$ exemplifies a typical case, demonstrating how these formulas work in practice. Whether for academic purposes, financial planning, or engineering tasks, these skills are invaluable tools in your mathematical toolkit.

Frequently Asked Questions

What is the common difference of the arithmetic sequence {-1, 2, 5, 8, 11, 14...}?

The common difference is 3, since each term increases by 3.

What is the first term of the sequence?

The first term is -1.

How can we find the sum of the first n terms of this sequence?

Use the formula for the sum of an arithmetic series: $S_n = n/2 (2a + (n - 1)d)$, where a is the first term and d is the common difference.

What is the explicit formula for the nth term of the sequence?

The nth term is $a_n = -1 + (n - 1) 3$, which simplifies to $a_n = 3n - 4$.

How do you find the sum of the first 10 terms of the sequence?

Calculate using $S_{10} = 10/2 (2 -1 + (10 - 1) 3) = 5 (-2 + 27) = 5 25 = 125$.

What is the sum of the first 20 terms of this sequence?

Using the formula: S_20 = 20/2 (2 -1 + (20 - 1) 3) = 10 (-2 + 57) = 10 55 = 550.

If the sequence continues indefinitely, what is the sum of all terms?

Since the sequence is infinite and unbounded, the sum diverges and does not have a finite value.

Can we find the sum of the sequence from the 5th to the 15th term?

Yes, sum from term 5 to 15 can be found by calculating S_15 - S_4, where S_n is the sum of the first n terms.

What is the value of the 50th term in the sequence?

Using a_n = 3n - 4, a_50 = 350 - 4 = 150 - 4 = 146.

Additional Resources

For The Arithmetic Sequence Beginning With The Terms {-1, 2, 5, 8, 11, 14...}, What Is The Sum Of The Sequence?

Understanding the sum of an arithmetic sequence is fundamental in mathematics, especially in algebra, calculus, and number theory. The sequence given—{-1, 2, 5, 8, 11, 14...}—serves as a classic example for exploring the mechanics of arithmetic progressions, their properties, and how to compute their sums efficiently. This detailed review delves into the structure of this sequence, the principles underlying its sum, and practical methods to determine the total of its terms.

Introduction to Arithmetic Sequences

An arithmetic sequence is a list of numbers where each term after the first is obtained by adding a fixed number, called the common difference, to the previous term. These sequences are foundational in mathematics because they model numerous real-world phenomena—including financial calculations, population growth, and more.

Key Characteristics of Arithmetic Sequences:

- First Term (a_1): The starting point of the sequence.
- Common Difference (d): The constant amount added to each term to get the next.
- General Term (a_n): The formula to find any term in the sequence.
- Sum of n Terms (S_n): The total when summing the first n terms.

Dissecting the Sequence {-1, 2, 5, 8, 11, 14...}

Let's analyze the specific sequence provided:

- Sequence terms: -1, 2, 5, 8, 11, 14, ...
- First term (a_1): -1
- Common difference (d): To find d, subtract the first term from the second:
- 2 - (-1) = 3
- Verification of the common difference: Check subsequent pairs:
- 5 - 2 = 3
- 8 - 5 = 3
- 11 - 8 = 3
- 14 - 11 = 3

Since the difference remains consistent at 3, this confirms the sequence is an arithmetic progression with:

- $a_1 = -1$
- d = 3

General Formula for the nth Term

The nth term of an arithmetic sequence is given by:

$$a_n = a_1 + (n - 1)d$$

Applying this to our sequence:

$$a_n = -1 + (n - 1) \times 3$$
$$a_n = -1 + 3n - 3$$
$$a_n = 3n - 4$$

Implication:

- To find the nth term, plug in the value of n:
- For example, when n=1:
$$a_1 = 3(1) - 4 = -1$$
- When n=2:
$$a_2 = 3(2) - 4 = 6 - 4 = 2$$
- When n=3:
$$a_3 = 3(3) - 4 = 9 - 4 = 5$$
- And so on.

Understanding the Sum of the Sequence

The sum of the first n terms of an arithmetic sequence, denoted as S_n, is given by the formula:

$$S_n = \frac{n}{2} (a_1 + a_n)$$

Alternatively, since a_n can be calculated directly, the formula is often rewritten as:

$$S_n = \frac{n}{2} [2a_1 + (n - 1)d]$$

Why does this formula work?

- Because pairing the first and last terms, second and second-last, and so on, each pair sums to the same total, and the total sum is the average of the first and last terms multiplied by the number of pairs.

Calculating the Sum for a Specific Number of Terms

Suppose you want to find the sum of the first n terms, for example, the first 10 terms:

1. Identify the first term a_1: -1
2. Calculate the nth term a_n:

$$a_n = 3n - 4$$

For n=10:

$$a_{10} = 3(10) - 4 = 30 - 4 = 26$$

3. Apply the sum formula:

$$S_{10} = \frac{10}{2} (-1 + 26) = 5 \times 25 = 125$$

Therefore, the sum of the first 10 terms is 125.

Generalizing the Sum for Any Number of Terms

Using the sum formula:

$$S_n = \frac{n}{2} (a_1 + a_n)$$

Substituting the general expression for a_n:

$$S_n = \frac{n}{2} \left(a_1 + 3n - 4 \right)$$

Since $a_1 = -1$:

$$S_n = \frac{n}{2} \left(-1 + 3n - 4 \right) = \frac{n}{2} (3n - 5)$$

Final formula:

$$\boxed{S_n = \frac{n}{2} (3n - 5)}$$

This formula allows us to find the sum of any number of terms in the sequence directly, without needing to compute each term individually.

Practical Applications and Insights

Understanding this sequence and its sum formula has several practical implications:

- Efficient calculations: Instead of summing each term manually, the formula provides a quick way to compute the total.
- Predicting growth: The sequence grows linearly with n, which models scenarios such as evenly increasing savings, linear acceleration, or resource allocation.
- Mathematical modeling: Recognizing the pattern allows for modeling complex systems with simple formulas.

Exploring the Sequence’s Behavior

Let's analyze some aspects of the sequence:

- Growth rate: The sequence increases by 3 with each subsequent term.
- Sum growth: The sum grows quadratically with n, as seen in the formula $S_n = \frac{n}{2}(3n - 5)$.

Example:

- For $n=5$:

$$a_5 = 3(5) - 4 = 15 - 4 = 11$$

$$S_5 = \frac{5}{2} (3 \times 5 - 5) = \frac{5}{2} (15 - 5) = \frac{5}{2} \times 10 = 25$$

- The sum after five terms is 25, matching the direct addition:

$$-1 + 2 + 5 + 8 + 11 = 25$$

Extensions and Variations

1. Finite vs. Infinite Sums:

- The sequence is finite if we specify a particular n.
- An infinite arithmetic sequence would involve considering an infinite sum, which diverges unless the common difference is zero.

2. Summation with Different Starting Points:

- If the sequence started with a different initial term but maintained the same difference, the sum formula would adjust accordingly.

3. Summing a Subsequence:

- Sometimes, only a subset of the sequence is summed, for example, summing every other term or only terms between certain indices.

Summary of Key Takeaways

- The sequence {-1, 2, 5, 8, 11, 14...} is an arithmetic progression with:

- First term $a_1 = -1$,
- Common difference $d = 3$,
- General term $a_n = 3n - 4$.

- The sum of the first n terms is:

$$S_n = \frac{n}{2} (3n - 5)$$

- This formula simplifies calculations and provides insights into the sequence’s growth behavior.

Conclusion

The arithmetic sequence starting with -1 and increasing by 3 each time exemplifies the elegance and utility of mathematical formulas in simplifying seemingly complex calculations. Whether you're summing a finite number of terms or analyzing the sequence's growth, understanding the underlying structure and formulas provides clarity and efficiency. Recognizing the pattern, deriving the general formulas, and applying them correctly are essential skills for students and professionals working in fields that rely on sequence analysis and summation techniques.

This deep dive into the sequence {-1, 2, 5, 8, 11, 14...} not only clarifies the specific question about its sum but also reinforces broader concepts applicable across mathematics and related disciplines.

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