For The Follow Second Order System With A Unit Step Input, Find The Damping Ratio, Natural Frequency,

For The Follow Second Order System With A Unit Step Input, Find The Damping Ratio, Natural Frequency, understanding the dynamic behavior of second-order systems is fundamental in control engineering and system analysis. These systems are ubiquitous in mechanical, electrical, and aerospace engineering, where predicting how a system responds to inputs is crucial for design and stability. When subjected to a unit step input, a second-order system’s response—characterized by parameters like damping ratio and natural frequency—determines whether the system is underdamped, overdamped, or critically damped, and how quickly it reaches steady state.

In this comprehensive guide, we will explore the concepts of damping ratio and natural frequency, explain how to derive these parameters from the system's transfer function, and analyze their influence on system response. Whether you are a student, a control engineer, or someone interested in system dynamics, this detailed discussion aims to clarify the process of analyzing second-order systems with step inputs.

Understanding Second Order Systems

Definition and Standard Form

A second-order system can generally be described by a differential equation of the form:

$$ \frac{d^2 y(t)}{dt^2} + 2 \zeta \omega_n \frac{dy(t)}{dt} + \omega_n^2 y(t) = \omega_n^2 u(t) $$

where:
- $y(t)$ is the system output,
- $u(t)$ is the input (here, a unit step),
- ω_n is the natural frequency,
- ζ is the damping ratio.

The corresponding transfer function from the input $U(s)$ to the output $Y(s)$ is:

$$ G(s) = \frac{\omega_n^2}{s^2 + 2 \zeta \omega_n s + \omega_n^2} $$

This standard form is fundamental for analyzing system behavior.

Significance of Parameters

- Natural Frequency (ω_n): Indicates how fast the system oscillates in the absence of damping.
- Damping Ratio (ζ): Measures the extent of damping; determines whether the system is oscillatory or not.

Step Response of a Second Order System

Response to a Unit Step Input

When a unit step input $u(t)$ (i.e., $u(t) = 1$ for $t \geq 0$) is applied to a second-order system, the output $y(t)$ exhibits characteristic behaviors based on the damping ratio:

- Underdamped ($0 < \zeta < 1$): Oscillatory response with overshoot.
- Critically damped ($\zeta = 1$): Fastest response without overshoot.
- Overdamped ($\zeta > 1$): Slow response without oscillations.

The standard step response for an underdamped system is:

$$y(t) = 1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t + \phi)$$

where:
- $\omega_d = \omega_n \sqrt{1 - \zeta^2}$ is the damped natural frequency,
- $\phi = \arccos(\zeta)$ is the phase angle.

Finding the Damping Ratio and Natural Frequency

Given the Transfer Function

Suppose you are provided with the transfer function of a second-order system:

$$G(s) = \frac{\omega_n^2}{s^2 + 2 \zeta \omega_n s + \omega_n^2}$$

or in a more general form, from experimental data or a system model, you have:

$$ G(s) = \frac{K}{s^2 + as + b} $$

To find ζ and ω_n, align the denominator with the standard form:

$$ s^2 + 2 \zeta \omega_n s + \omega_n^2 $$

by comparing coefficients.

Step-by-Step Procedure

1. Identify the denominator coefficients:

From the transfer function denominator, note the coefficients of s^2, s, and the constant term.

2. Calculate the natural frequency (ω_n):

$$ \omega_n = \sqrt{b} $$

assuming the denominator is normalized such that the coefficient of s^2 is 1.

3. Calculate the damping ratio (ζ):

$$ \zeta = \frac{a}{2 \omega_n} $$

where a is the coefficient of s.

Example Calculation

Suppose the transfer function is:

$$ G(s) = \frac{25}{s^2 + 4s + 25} $$

Step 1: Compare with standard form:

$$ s^2 + 2 \zeta \omega_n s + \omega_n^2 $$

Step 2: Identify coefficients:

- $a = 4$
- $b = 25$

Step 3: Find ω_n:

$$\omega_n = \sqrt{25} = 5 \, \text{rad/sec}$$

Step 4: Find ζ:

$$\zeta = \frac{a}{2 \omega_n} = \frac{4}{2 \times 5} = \frac{4}{10} = 0.4$$

This damping ratio indicates an underdamped system with oscillatory response.

Impact of Damping Ratio and Natural Frequency on System Response

Underdamped System ($0 < \zeta < 1$)

- Exhibits oscillations with a decaying amplitude.
- The overshoot and settling time depend on ζ and ω_n.
- Lower ζ leads to larger overshoot and longer oscillations.

Critically Damped System ($\zeta = 1$)

- No oscillations.
- Fastest response to reach steady state without overshoot.
- Often desired in control applications requiring quick stabilization.

Overdamped System ($\zeta > 1$)

- No oscillations.
- Response is slow with a longer settling time.
- Suitable in systems where oscillations are undesirable.

Practical Applications and Considerations

- Designing Controllers: Engineers often tune controllers (like PD, PID, or lead-lag compensators) to achieve desired damping ratios and natural frequencies.
- Stability Analysis: Ensuring that the damping ratio is positive guarantees system stability.

- Performance Metrics: Overshoot, rise time, settling time, and steady-state error are influenced by these parameters.

Conclusion

Understanding how to determine the damping ratio and natural frequency from a second-order system's transfer function is essential in analyzing and designing control systems. By comparing the system’s denominator to the standard form, engineers can predict how the system will respond to step inputs, optimize performance, and ensure stability. Whether dealing with mechanical vibrations, electrical circuits, or aerospace control systems, these parameters serve as foundational tools in system dynamics analysis.

Summary of Key Steps

1. Obtain the transfer function of the system.

2. Express the denominator in the standard second-order form.

3. Identify the coefficients of s and the constant term.

4. Calculate the natural frequency ω_n as the square root of the constant term.

5. Compute the damping ratio ζ using the coefficient of s and ω_n.

6. Use these parameters to analyze or design the system response to step inputs.

By following these steps, engineers and students can effectively analyze second-order systems and predict their behavior under step inputs, leading to better control system design and stability assurance.

Frequently Asked Questions

How do you determine the damping ratio and natural

frequency for a second order system with a unit step input?

The damping ratio (ζ) and natural frequency (ω_n) can be found by analyzing the system's characteristic equation or transfer function. For a standard second order system, the transfer function is $G(s) = \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2)$. Using the system's response to a unit step input, you can identify the overshoot, settling time, and oscillation period to calculate ζ and ω_n.

What is the significance of the damping ratio in the response of a second order system to a unit step input?

The damping ratio determines the nature of the system's response: whether it is underdamped ($\zeta < 1$), critically damped ($\zeta = 1$), or overdamped ($\zeta > 1$). It influences overshoot, oscillations, and settling time. For a unit step input, the damping ratio helps predict how quickly and smoothly the system reaches its steady-state value.

How can the overshoot and settling time in the step response be used to find the damping ratio and natural frequency?

The percentage overshoot (PO) relates to ζ via $PO = \exp(-\zeta\pi / \sqrt{1 - \zeta^2}) \times 100\%$. The settling time (usually for 2% criterion) relates to ζ and ω_n through $T_s \approx 4 / (\zeta\omega_n)$. By measuring these parameters from the response, you can solve for ζ and ω_n.

What is the typical approach to extract damping ratio and natural frequency from a step response graph?

Identify the peak overshoot to estimate ζ using the overshoot formula. Measure the oscillation period between successive peaks to find the damped natural frequency (ω_d), then use the relation $\omega_d = \omega_n\sqrt{1 - \zeta^2}$ to calculate ω_n. These steps allow estimation of both parameters from the response curve.

Why is understanding the damping ratio and natural frequency important in control system design?

Knowing ζ and ω_n helps in designing controllers that achieve desired transient and steady-state performance. Proper tuning of these parameters ensures minimal overshoot, quick settling time, and system stability when responding to inputs like a unit step.

Additional Resources

Second Order System Response to Unit Step Input: Damping Ratio and Natural Frequency Analysis

Understanding the dynamic behavior of second order systems is fundamental in control engineering and systems analysis. When such a system is subjected to a unit step input, analyzing parameters like the damping ratio and natural frequency becomes crucial to predict system performance, stability, and transient response characteristics. This comprehensive review explores these concepts in detail, providing clarity on how they influence the system's behavior and how they can be determined from the system's transfer function.

Introduction to Second Order Systems

A second order system is characterized by a differential equation of the form:

$$
\frac{d^2 y(t)}{dt^2} + 2 \zeta \omega_n \frac{dy(t)}{dt} + \omega_n^2 y(t) = \omega_n^2 u(t)
$$

where:

- $y(t)$ is the system output,
- $u(t)$ is the input (in this case, a unit step),
- ω_n is the natural frequency,
- ζ is the damping ratio.

The transfer function for such a system, assuming zero initial conditions, is:

$$
G(s) = \frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2 \zeta \omega_n s + \omega_n^2}
$$

This canonical form allows us to analyze the system's response characteristics based on the parameters ζ and ω_n.

Understanding Key Parameters: Natural Frequency and Damping Ratio

Natural Frequency (ω_n)

- Definition: The natural frequency is the frequency at which the system would oscillate if there were no damping ($\zeta = 0$). It reflects the intrinsic oscillatory tendency of the system.
- Units: Radians per second (rad/sec).
- Physical Significance: A higher ω_n indicates a faster response, meaning the system reaches its steady-state more quickly after a disturbance.

Damping Ratio (ζ)

- Definition: The damping ratio measures how oscillations in a system decay after a disturbance. It quantifies the degree of damping relative to critical damping.
- Range:
- $\zeta = 0$: Undamped (pure oscillation).
- $0 < \zeta < 1$: Underdamped (oscillatory response with decay).
- $\zeta = 1$: Critically damped (fastest non-oscillatory response).
- $\zeta > 1$: Overdamped (non-oscillatory, slower response).
- Physical Significance: The damping ratio influences overshoot, settling time, and stability of the system.

Response of Second Order System to a Unit Step Input

Applying a unit step input $u(t) = 1 \cdot \text{(for } t \geq 0)$, the system's output $y(t)$ exhibits characteristic transient behaviors determined by the values of ζ and ω_n.

Standard Time-Domain Responses Based on Damping Ratio

- Underdamped ($0 < \zeta < 1$):

- Oscillatory response with exponential decay.
- Overshoot occurs, with the maximum overshoot M_p given by:

$$
M_p = e^{\left(-\frac{\pi \zeta}{\sqrt{1 - \zeta^2}} \right)} \times 100\%
$$

- Settling time (T_s) approximately:

$$
T_s \approx \frac{4}{\zeta \omega_n}
$$

- Critically Damped ($\zeta = 1$):
- Fastest response without overshoot.
- No oscillations; output reaches steady state as quickly as possible without crossing the final value.

- Overdamped ($\zeta > 1$):
- No oscillation; response is sluggish.
- Response involves two real poles, leading to a longer settling time.

- Undamped ($\zeta = 0$):
- Persistent oscillations; the response continues indefinitely.

Mathematical Derivation of Response and Parameters

Step Response of Second Order System

For a standard second order system, the step response can be expressed as:

$$
y(t) = 1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin \left(\omega_d t + \phi \right)
$$

where:

- $\omega_d = \omega_n \sqrt{1 - \zeta^2}$ is the damped natural frequency,
- $\phi = \arccos(\zeta)$ is the phase angle.

In cases where $\zeta < 1$, the response exhibits oscillations at \(

ω_d with an exponential decay envelope determined by $e^{-\zeta \omega_n t}$.

Determining ζ and ω_n from the Transfer Function

Given the transfer function:

$$
G(s) = \frac{\omega_n^2}{s^2 + 2 \zeta \omega_n s + \omega_n^2}
$$

the denominator polynomial's coefficients can be used to extract the parameters:

- Identify the coefficients: For the denominator polynomial $s^2 + 2 \zeta \omega_n s + \omega_n^2$,

- $a_1 = 2 \zeta \omega_n$,

- $a_0 = \omega_n^2$.

- Calculate ω_n:

$$
\omega_n = \sqrt{a_0}
$$

- Calculate ζ:

$$
\zeta = \frac{a_1}{2 \omega_n}
$$

Example:

Suppose the denominator of the transfer function is $s^2 + 4 s + 25$.

- $a_1 = 4$,
- $a_0 = 25$.

Then:

$$
\omega_n = \sqrt{25} = 5 \text{ rad/sec}
$$

$$
\zeta = \frac{4}{2 \times 5} = \frac{4}{10} = 0.4
$$

This indicates an underdamped system with moderate damping.

Practical Methods for Determining Damping Ratio and Natural Frequency

1. Using Step Response Data

One of the most straightforward approaches involves analyzing the system's step response:

- Overshoot (M_p):

$$
M_p = \frac{y_{max} - y_{ss}}{y_{ss}} \times 100\%
$$

where y_{max} is the peak value, and y_{ss} is the steady-state value.

- Calculating ζ from overshoot:

$$
\zeta = - \frac{\ln \left(\frac{M_p}{100} \right)}{\sqrt{\pi^2 + \left(\ln \left(\frac{M_p}{100} \right)\right)^2}}
$$

- Settling Time (T_s):

$$
T_s \approx \frac{4}{\zeta \omega_n}
$$

Rearranged to find ω_n:

$$
\omega_n = \frac{4}{\zeta T_s}
$$

Note: This method assumes a near second order dominant response and negligible higher-order effects.

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2. Pole Location Method

- Procedure:

- Obtain the system's pole locations from the transfer function denominator.
- The poles are complex conjugates in underdamped cases: $s = -\zeta \omega_n \pm j \omega_d$.
- From the pole locations:

$$
\text{Real part} = -\zeta \omega_n
$$

$$
\text{Imaginary part} = \omega_d = \omega_n \sqrt{1 - \zeta^2}
$$

- Calculations:

$$
\zeta = - \frac{\text{Real part}}{\sqrt{\text{Real part}^2 + \text{Imaginary part}^2}}
$$

$$
\omega_n = \sqrt{\text{Real part}^2 + \text{Imaginary part}^2}
$$

Impacts of Damping Ratio and Natural Frequency on System Performance

Transient Response Characteristics

- Rise Time (\(

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