For What Values Of K Does The Function Y = Cos Kt Satisfy The Differential Equation 16y'' = ?49y? For

For What Values Of K Does The Function Y = Cos Kt Satisfy The Differential Equation 16y'' = ?49y? For understanding this question, we need to analyze the relationship between the function $y = \cos(kt)$ and the differential equation $16 y'' = ?49 y$. Determining the values of k that satisfy this equation involves differentiating the function, substituting into the differential equation, and solving for k. This process is fundamental in differential equations, especially when examining solutions involving trigonometric functions.

Understanding the Differential Equation and the Function

The Given Differential Equation

The differential equation in question is:

$$16 y'' = ?49 y$$

While the exact form appears to have a typographical placeholder "?49", the most common interpretation in differential equations involving such notation suggests it might be:

$$16 y'' = 49 y$$

or

$$16 y'' = -49 y$$

since differential equations involving cosine functions often relate to negative constants, corresponding to oscillatory solutions. For the purpose of this article, we will explore both possibilities:

1. $16 y'' = 49 y$
2. $16 y'' = -49 y$

and determine for which values of k the function $y = \cos(k t)$ satisfies these equations.

The Function $y = \cos(k t)$

The function $y = \cos(k t)$ is a standard solution to second-order linear differential equations with constant coefficients involving sine and cosine functions. Its derivatives are:

$$y' = -k \sin(k t)$$
$$y'' = -k^2 \cos(k t)$$

Understanding these derivatives is crucial because substituting y'' into the differential equation allows us to find the conditions on k.

Deriving Conditions for k

Substituting y and y'' into the Differential Equation

Starting with the second derivative:

$$y'' = -k^2 \cos(k t)$$

Substitute into the differential equation:

1. For $16 y'' = 49 y$:

$$16(-k^2 \cos(k t)) = 49 \cos(k t)$$

2. For $16 y'' = -49 y$:

$$16(-k^2 \cos(k t)) = -49 \cos(k t)$$

Since $\cos(k t)$ appears on both sides, and assuming $\cos(k t) \neq 0$, we can divide both sides by $\cos(k t)$ to simplify.

Solving for k

Let's analyze both cases separately:

Case 1: $16 y'' = 49 y$

$$16(-k^2) = 49$$
$$-16 k^2 = 49$$
$$k^2 = -\frac{49}{16}$$

Since k^2 is negative, k must be imaginary:

$$k = \pm i \frac{7}{4}$$

This implies that the solution involving $y = \cos(k t)$ with real k does not satisfy the differential equation $16 y'' = 49 y$. Instead, solutions are exponential or hyperbolic functions due to complex k.

Case 2: $16 y'' = -49 y$

$$16(-k^2) = -49$$
$$-16 k^2 = -49$$
$$16 k^2 = 49$$
$$k^2 = \frac{49}{16}$$
$$k = \pm \frac{7}{4}$$

In this case, k is real and positive or negative, and the function $y = \cos(k t)$ with $k = \pm \frac{7}{4}$ satisfies the differential equation.

Summary of Results

- For the differential equation $16 y'' = 49 y$, the function $y = \cos(k t)$ does not satisfy the equation when k is real because it leads to imaginary k. The solutions involve exponential functions instead.

- For the differential equation $16 y'' = -49 y$, the function $y = \cos(k t)$ satisfies the equation when $k = \pm \frac{7}{4}$.

Implications for the Solutions and Applications

Oscillatory Solutions with Real k

The case where $16 y'' = -49 y$ corresponds to simple harmonic motion, which is typical in physics for systems like springs and pendulums. Here, the solutions are sinusoidal functions:

$$y(t) = A \cos\left(\frac{7}{4} t\right) + B \sin\left(\frac{7}{4} t\right)$$

where A and B are constants determined by initial conditions.

Exponential Solutions for $16 y'' = 49 y$

When $16 y'' = 49 y$, solutions involve exponential functions due to imaginary k:

$$y(t) = C e^{\frac{7}{4} t} + D e^{-\frac{7}{4} t}$$

which describe growth or decay processes depending on initial conditions.

Conclusion: Key Takeaways

- The function $y = \cos(k t)$ satisfies the differential equation $16 y'' = -49 y$ when $k = \pm \frac{7}{4}$.
- For the differential equation $16 y'' = 49 y$, the solutions involve exponential functions, not trigonometric ones, when considering real k.
- Understanding the relationship between the form of the differential equation and the type of functions that satisfy it is fundamental in differential equations and mathematical modeling.

Final Notes for Students and Enthusiasts

When solving second-order linear differential equations, always analyze the characteristic equation to determine the form of the solutions. Recognize that the sign of the constant on the right side influences whether solutions are oscillatory or exponential. The process demonstrated here can be applied generally to similar problems involving trigonometric and exponential solutions, enabling you to analyze a wide array of physical and mathematical systems.

By mastering these concepts, you'll be better equipped to tackle differential equations in physics, engineering, and applied mathematics, and to interpret the behavior of systems modeled by such equations.

Frequently Asked Questions

What is the general form of the differential equation involving y'' and y in the context of y = cos kt?

The differential equation is $16y'' = -49y$, which relates the second derivative of y to y itself, with coefficients indicating a harmonic oscillator type equation.

For which values of k does y = cos kt satisfy the differential equation 16y'' = -49y?

Y = cos kt satisfies the differential equation when k^2 = 49/16, so k = ±7/4.

How do you derive the values of k for the function y = cos kt to satisfy the differential equation?

By substituting y = cos kt into the differential equation, computing y'', and setting 16(-k^2 cos kt) = -49 cos kt, leading to k^2 = 49/16.

Does the differential equation 16y'' = -49y imply simple harmonic motion, and how is k related?

Yes, it implies simple harmonic motion, with k = ±7/4 representing the angular frequency of oscillation.

Can the solution y = cos kt satisfy the differential equation for k = 0?

No, because if k = 0, y = cos 0 = 1, which results in y'' = 0, and substituting into the equation would not satisfy it unless the constant term matches, which it does not in this case.

Is the differential equation 16y'' = -49y homogeneous, and what are its characteristic roots?

Yes, it is homogeneous, and the characteristic roots are r = ±7/4 i, indicating oscillatory solutions involving sine and cosine functions.

Additional Resources

For What Values Of K Does The Function Y = Cos Kt Satisfy The Differential Equation 16y'' = ?49y?

Understanding the relationship between functions and differential equations is a fundamental aspect of mathematical analysis, particularly in the study of oscillatory systems, wave phenomena, and engineering applications. The question posed—"For what values of k does the function \(Y = \cos kt\) satisfy the differential equation \(16 y'' = ?49 y\)?"—serves as an insightful exploration into the interplay between parameters within differential equations and their corresponding solutions. This analysis not only reveals the specific values of k that satisfy the given equation but also underscores the underlying principles that govern such relationships in higher mathematics.

In this comprehensive review, we will systematically analyze the problem, interpret the differential equation, examine the properties of the cosine function as a potential solution, and derive the conditions on k. Our approach will include detailed explanations, mathematical derivations, and contextual insights to facilitate a deep understanding of the topic.

Understanding the Differential Equation: Context and Significance

Nature of the Differential Equation

The given differential equation is expressed as:

$$
16 y'' = ?49 y
$$

At first glance, the notation appears somewhat ambiguous, possibly due to a typographical or formatting issue. Typically, in differential equations involving second derivatives, the standard form is:

$$
a y'' + b y' + c y = 0
$$

or

$$
y'' + \text{(some constant)} \times y = 0
$$

Given the context and the structure presented, it is reasonable to interpret the equation as a second-order linear differential equation with constant coefficients, likely of the form:

$$
16 y'' = -49 y
$$

This assumption is supported by the presence of positive constants and the typical form of equations describing harmonic oscillations. The notation "?49" probably indicates a negative sign, making the equation:

$$
16 y'' = -49 y
$$

which is a standard second-order differential equation describing simple harmonic motion.

Significance of the Differential Equation:

This form—$y'' + \frac{49}{16} y = 0$—models systems where the acceleration (second derivative of position with respect to time) is proportional and opposite in direction to displacement, such as a mass-spring system or oscillating pendulum under ideal conditions.

The solutions to such equations are well-known and involve sinusoidal functions, primarily sine and cosine functions. Understanding the specific values of k for which $y = \cos kt$ satisfies this differential equation provides insight into the natural frequencies of oscillation and the mathematical structure underlying these physical phenomena.

Standard Form and Solution Approach

To analyze the solution, rewriting the differential equation in a standard form is necessary:

$$
16 y'' + 49 y = 0
$$

which simplifies to:

$$
y'' + \frac{49}{16} y = 0
$$

This is a homogeneous linear differential equation with constant coefficients, whose characteristic equation is:

$$
r^2 + \frac{49}{16} = 0
$$

Solving this characteristic equation yields the roots:

$$
r = \pm i \sqrt{\frac{49}{16}} = \pm i \frac{7}{4}
$$

The general solution to the differential equation is then:

$$
y(t) = C_1 \cos \left(\frac{7}{4} t\right) + C_2 \sin \left(\frac{7}{4} t\right)
$$

where C_1 and C_2 are arbitrary constants determined by initial conditions.

Analyzing the Candidate Solution: $Y = \cos Kt$

Properties of the Cosine Function as a Solution

The question centers on whether the function:

$$
Y(t) = \cos Kt
$$

can satisfy the differential equation, and if so, for which values of K. To determine this, it is essential to verify if $Y(t)$ fits into the general solution of the differential equation and whether it satisfies the differential equation when substituted.

First derivative:

$$
Y'(t) = -K \sin Kt
$$

Second derivative:

$$
Y''(t) = -K^2 \cos Kt
$$

Substituting into the differential equation:

$$
16 y'' = -49 y
$$

becomes:

$$
16 (-k^2 \cos kt) = -49 \cos kt
$$

which simplifies to:

$$
-16 k^2 \cos kt = -49 \cos kt
$$

Assuming $\cos kt \neq 0$, we can divide both sides by $\cos kt$:

$$
-16 k^2 = -49
$$

which simplifies to:

$$
16 k^2 = 49
$$

and hence:

$$
k^2 = \frac{49}{16}
$$

Taking square roots:

$$
k = \pm \frac{7}{4}
$$

Implication:

The cosine function $y = \cos kt$ satisfies the differential equation $16 y'' = -49 y$ if and only if:

$$
k = \pm \frac{7}{4}
$$

This critical result indicates that the particular values of k are directly tied to the natural frequencies of the system described by the differential equation.

Physical and Mathematical Interpretation

The derived values of k correspond to the system's angular frequencies, which in this case are $\pm 7/4$. These frequencies determine the oscillatory behavior of solutions that are purely cosine (or sine) functions, reflecting undamped harmonic motion.

Mathematically, this matches the eigenvalues of the differential operator, and physically, it indicates the system's resonance conditions. For instance, if the system is driven at these frequencies, it will resonate, leading to large amplitude oscillations.

Broader Implications and Contextual Insights

Connection to Physical Systems

The analysis reveals how specific parameter choices influence solution forms. In classical mechanics, the parameters $k = \pm 7/4$ correspond to the natural angular frequency of oscillation for a mass-spring system with effective parameters derived from the coefficients in the differential equation.

This is especially relevant in engineering disciplines such as mechanical vibrations, electrical circuits (LC circuits), and wave mechanics, where understanding the natural frequencies informs design and analysis.

Mathematical Significance and Eigenvalues

From a mathematical standpoint, the values of k represent the eigenvalues of the differential operator associated with the problem. The eigenfunctions, which in this case are sinusoidal functions, form a basis for representing solutions to more complex boundary value problems.

Furthermore, this analysis exemplifies how the structure of the differential equation dictates the form of solutions and how parameter values determine whether particular functions are solutions.

Extensions and Related Problems

The problem can be extended in numerous directions:

- Damped oscillations: Introducing damping terms, such as y', modifies the solution structure.

- Forced oscillations: Applying external forcing functions leads to resonance phenomena at specific frequencies.

- Higher-order equations: Exploring differential equations of higher order or with variable coefficients expands the analytical complexity.

- Boundary conditions: Imposing boundary conditions transforms the problem into an eigenvalue problem, with solutions dependent on boundary constraints.

Summary and Final Remarks

The question of determining the values of k for which $y = \cos kt$ satisfies the differential equation $16 y'' = -49 y$ hinges on fundamental principles of differential equations and harmonic analysis. The key steps involve transforming the differential equation into its characteristic form, deriving the eigenvalues, and interpreting these in the context of sinusoidal solutions.

Final conclusion:

$$
\boxed{
k = \pm \frac{7}{4}
}
$$

These are the precise values of k for which the cosine function is a solution to the differential equation. This outcome underscores the deep connection between the parameters within differential equations and the functional forms of their solutions—a cornerstone concept in both theoretical and applied mathematics.

Understanding these relationships enhances our ability to analyze oscillatory systems, design stable structures, and interpret wave phenomena, reinforcing the vital role of mathematical analysis in scientific inquiry and engineering innovation.

In essence, the exploration of the values of k in this context exemplifies the harmony between mathematical theory and physical intuition, demonstrating how seemingly abstract parameters encode fundamental physical properties like frequency and stability.

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