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Understanding the behavior of functions and their derivatives is a fundamental aspect of calculus. A common problem involves determining whether a function is increasing or decreasing over a certain interval and how the derivatives of inverse functions relate to the original functions. In this context, we are asked to analyze specific functions and identify which among them is increasing, and then to evaluate the derivative of their inverse at a particular point. This problem combines the concepts of monotonicity, inverse functions, derivatives, and the application of the inverse function theorem.

In this article, we will explore the criteria for a function to be increasing, examine the properties of the provided functions, and determine which functions satisfy the condition that the derivative of their inverse at 20 equals 1/5. We will also delve into the calculations involved in finding inverse functions and their derivatives, providing a comprehensive understanding of the topic.

Understanding Increasing Functions and the Inverse Function Theorem

What Does It Mean for a Function to Be Increasing?

A function $f(x)$ is said to be increasing on an interval if, for any two points x_1 and x_2 within that interval, whenever $x_1 < x_2$, it follows that $f(x_1) \leq f(x_2)$. If the inequality is strict, $f(x_1) < f(x_2)$, the function is strictly increasing.

Mathematically, this can be checked using the derivative:
- If $f'(x) > 0$ for all x in an interval, then f is strictly increasing on that interval.
- Conversely, if $f'(x) < 0$, then f is decreasing.

The Inverse Function and Its Derivative

Given a function f that is invertible (i.e., has an inverse f^{-1}), the inverse function theorem states that:
\[

$$
(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}
$$
for all y in the range of f.

This relation allows us to compute the derivative of the inverse function at a point once we know the derivative of the original function at the corresponding point:
- To find $(f^{-1})'(20)$, we first need to determine x such that $f(x) = 20$.
- Then, $(f^{-1})'(20) = \frac{1}{f'(x)}$.

Our goal is to find the functions f among the options that are increasing and satisfy:
$$
(f^{-1})'(20) = \frac{1}{5A}
$$
which implies:
$$
f'(x) = 5A
$$
at the point where $f(x) = 20$.

Analyzing the Given Functions

The problem provides two candidate functions:
- $F_1(x) = x + 5B$
- $F_2(x) = x^3 + 5x$

The question is: For which of these functions does the derivative of the inverse at 20 equal $1/5A$?

To answer this, we will analyze each function separately.

Function 1: $F_1(x) = x + 5B$
- This is a linear function.
- Its derivative:
$$
F_1'(x) = 1
$$
- Since the derivative is constant and positive, F_1 is strictly increasing everywhere.
- To find $F_1^{-1}(x)$:
$$
y = x + 5B \Rightarrow x = y - 5B
$$

- The inverse function:
$$
F_1^{-1}(x) = x - 5B
$$
- The derivative of the inverse:
$$
(F_1^{-1})'(x) = 1
$$
- Therefore:
$$
(F_1^{-1})'(20) = 1
$$
- Comparing this with the desired derivative $1/5A$:
$$
1 = \frac{1}{5A} \Rightarrow 5A = 1 \Rightarrow A = \frac{1}{5}
$$
- Since A is a parameter, for any choice of B, the inverse derivative at 20 is 1, which equals $1/5A$ only if $A = 1/5$.

Conclusion for F_1:
- The function is increasing everywhere.
- The inverse derivative at 20 is always 1.
- To satisfy $(f^{-1})'(20) = 1/5A$, A must be $1/5$.

Function 2: $F_2(x) = x^3 + 5x$

- This is a cubic function.
- Its derivative:
$$
F_2'(x) = 3x^2 + 5
$$
- Since $3x^2 \geq 0$, the derivative:
$$
F_2'(x) = 3x^2 + 5 \geq 5 > 0
$$
for all real x.

- Because the derivative is always positive, F_2 is strictly increasing over $\mathbb{R}$.

- To find the inverse and evaluate its derivative at 20:
1. Find x such that:
$$
F_2(x) = 20 \Rightarrow x^3 + 5x = 20
$$
2. Solve for x:
$$
x^3 + 5x - 20 = 0
$$

$$
$$
- This is a cubic equation. Let's see if there's an obvious root:
$$
x=2 \Rightarrow 8 + 10 = 18 \neq 20
$$
$$
x=3 \Rightarrow 27 + 15 = 42 \neq 20
$$
$$
x=1 \Rightarrow 1 + 5 = 6 \neq 20
$$
$$
x= -2 \Rightarrow -8 - 10 = -18 \neq 20
$$
$$
x= 2.5 \Rightarrow (2.5)^3 + 5(2.5) = 15.625 + 12.5 = 28.125 \neq 20
$$
- Since no simple rational root appears, approximate x numerically or use the cubic formula. For simplicity, approximate:

Try $x \approx 2.2$:
$$
(2.2)^3 + 5(2.2) = 10.648 + 11 = 21.648 \approx 20
$$
Close enough for an estimate. Let's accept $x \approx 2.18$.

- Now, compute the derivative at this x:
$$
F_2'(x) = 3x^2 + 5
$$
At $x \approx 2.18$:
$$
F_2'(2.18) \approx 3 \times (2.18)^2 + 5 \approx 3 \times 4.75 + 5 = 14.25 + 5 = 19.25
$$
- Using the inverse function theorem:
$$
(F_2^{-1})'(20) = \frac{1}{F_2'(x)} \approx \frac{1}{19.25} \approx 0.052
$$
- Recall the target derivative:
$$
\frac{1}{5A}
$$
Set:
$$
0.052 \approx \frac{1}{5A}
$$
Solve for A:
$$
5A \approx \frac{1}{0.052} \approx 19.23
$$

\[
A \approx \frac{19.23}{5} \approx 3.846
\]

Conclusion for \(F_2\):
- The inverse derivative at 20 depends on the value of \(A\). If \(A \approx 3.846\), then the condition holds.

Summary and Final Conclusions

- Both functions \(F_1(x) = x + 5B\) and \(F_2(x) = x^3 + 5x\) are strictly increasing over \(\mathbb{R}\).

- For \(F_1\):
- The inverse derivative at 20 is always 1.
- To satisfy \((f^{-1})'(20) = 1/5A\), the parameter \(A\) must be \(1/5\).

- For \(F_2\):
- The derivative of the inverse at 20 is approximately \(1/19.25 \approx 1/5A\), implying \(A \approx 3.846\).

Therefore:

- If the parameter \(A\) is set to \(1/5\), then the linear function \(F_1\) satisfies the condition

Frequently Asked Questions

Given the functions F(x) = x + 5 and F(x) = x^3 + 5x, for which of these is the derivative of the inverse function at 20 equal to 1/5?

To find this, we need to compute (f^{-1})'(20) = 1/5 for each function. Recall that (f^{-1})'(a) = 1 / f'(f^{-1}(a)). So, first, find x such that F(x) = 20, then compute f'(x). For F(x) = x + 5, x = 15; for F(x) = x^3 + 5x, solve x^3 + 5x = 20.

How do you determine whether the inverse of a function is increasing at a specific point?

An inverse function is increasing at a point if the original function is increasing at the corresponding point, which depends on the sign of its derivative. If the original function's derivative is positive there, its inverse is also increasing at that point.

What is the derivative of F(x) = x + 5, and what does it imply about its inverse?

The derivative of $F(x) = x + 5$ is $F'(x) = 1$, which is positive everywhere. This means the inverse function exists, is increasing, and its derivative at a point can be found using $(f^{-1})'(a) = 1 / F'(f^{-1}(a))$.

For F(x) = x^3 + 5x, how do you find the value of x where F(x) = 20?

Solve the cubic equation $x^3 + 5x = 20$. This can be approached by trial, factoring, or numerical methods to find the specific x-value where the function equals 20.

How is the derivative of the inverse function related to the derivative of the original function?

The derivative of the inverse function at a point a is given by $(f^{-1})'(a) = 1 / f'(f^{-1}(a))$, provided f is differentiable and invertible at that point.

What are the key properties of increasing functions that ensure their inverses are also increasing?

If a function is strictly increasing and differentiable with a positive derivative everywhere, its inverse is also increasing and differentiable, with the inverse's derivative given by the reciprocal of the original function's derivative at corresponding points.

Which of the given functions, F(x) = x + 5 or F(x) = x^3 + 5x, has a derivative that leads to (f^{-1})'(20) = 1/5?

For $F(x) = x + 5$, the inverse derivative at the point where $F^{-1}(20)$ is 1/5 because $F'(x) = 1$. For $F(x) = x^3 + 5x$, solving for x where $F(x)=20$ and computing $1 / F'(x)$ at that point determines the inverse derivative. The function whose derivative at the pre-image of 20 is 5 will satisfy $(f^{-1})'(20) = 1/5$.

Additional Resources

For Which Of The Following Increasing Functions F Does (f-1)'(20) = 1/5?

Understanding the relationship between a function and its inverse is a fundamental aspect of calculus, particularly when analyzing the derivative of inverse functions at specific points. The question, "For which of the following increasing functions F does (f-1)'(20) = 1/5?" invites us to explore how the derivative of the inverse function relates to the original function, and how to determine the specific function(s) that satisfy this condition. In this comprehensive review, we will examine the properties of the given functions, delve into the concept of inverse functions and their derivatives, analyze the provided options, and discuss the implications of the derivative at particular points.

Understanding the Derivative of an Inverse Function

Fundamental Concept

The derivative of an inverse function, often denoted as $(f^{-1})'(x)$, has a well-known relationship with the derivative of the original function f(x). Specifically, if f is invertible and differentiable at a point x, then at the corresponding point y = f(x), the derivatives are related by:

$$ (f^{-1})'(y) = \frac{1}{f'(x)} $$

This relationship implies that to find the derivative of the inverse at a specific y-value, we need to identify the corresponding x-value such that f(x) = y, and then compute the reciprocal of f'(x).

Application to Our Problem

Given that $(f^{-1})'(20) = 1/5$, it follows that:

$$ \frac{1}{f'(x)} = \frac{1}{5} \Rightarrow f'(x) = 5 $$

where x is the point such that f(x) = 20. Therefore, the problem reduces to:

1. Find x such that f(x) = 20.
2. Verify whether f'(x) = 5 at that point.

This approach allows us to analyze each candidate function to determine if the condition holds.

Analyzing the Candidate Functions

The two functions provided are:

A) $F(x) = x + 5$

B) $F(x) = x^3 + 5x$

We will analyze each to determine whether the inverse's derivative at 20 is 1/5.

Function A: $F(x) = x + 5$

Step 1: Find x such that $F(x) = 20$

$$ x + 5 = 20 \Rightarrow x = 15 $$

Step 2: Compute $F'(x)$

$$ F'(x) = \frac{d}{dx}(x + 5) = 1 $$

Since $F'(x) = 1$ everywhere, at $x = 15$, $F'(15) = 1$.

Step 3: Confirm whether $(f^{-1})'(20) = 1/5$

Applying the inverse derivative formula:

$$ (F^{-1})'(20) = \frac{1}{F'(x)} = \frac{1}{1} = 1 $$

But the question asks whether this derivative equals 1/5; since it equals 1, the answer is no for this function.

Pros:
- Simple linear function, easy to invert.
- Derivative is constant, making analysis straightforward.

Cons:
- The inverse's derivative at 20 is 1, not 1/5, thus does not satisfy the condition.

Function B: $F(x) = x^3 + 5x$

Step 1: Find x such that $F(x) = 20$

$$ x^3 + 5x = 20 $$

This is a cubic equation:

$$ x^3 + 5x - 20 = 0 $$

Test possible rational roots, such as x = 2:
$$ 8 + 10 - 20 = -2 \neq 0 $$
x = 3:
$$ 27 + 15 - 20 = 22 \neq 0 $$
x = 1:
$$ 1 + 5 - 20 = -14 \neq 0 $$
x = 2.5:
$$ (2.5)^3 + 5(2.5) = 15.625 + 12.5 = 28.125 \neq 20 $$

Between x = 1.5 and 2, let's check x = 1.7:
$$ 1.7^3 + 5(1.7) = 4.913 + 8.5 = 13.413 $$
x = 2:
$$ 8 + 10 = 18 $$
x = 2.2:
$$ 10.648 + 11 = 21.648 $$

Since at x=2, F(x)=18, and at x=2.2, F(x)=21.648, the value 20 is between these. Let's interpolate:

Estimate x:
$$ x \approx 2 + \frac{20 - 18}{21.648 - 18} \times 0.2 $$
$$ \approx 2 + \frac{2}{3.648} \times 0.2 $$
$$ \approx 2 + 0.109 \approx 2.109 $$

To be precise, approximate $x \approx 2.11$. Now, compute F'(x):

$$ F'(x) = 3x^2 + 5 $$

At $x \approx 2.11$:

$$ F'(2.11) = 3 \times (2.11)^2 + 5 $$
$$ = 3 \times 4.45 + 5 $$
$$ = 13.35 + 5 = 18.35 $$

Now, compute the inverse derivative at y=20:

$$ (F^{-1})'(20) = \frac{1}{F'(x)} \approx \frac{1}{18.35} \approx 0.0545 $$

This is approximately 1/18.35, which is about 0.0545, not 1/5 (which is 0.2). Therefore, this function does not satisfy the condition either.

Summary and Conclusions

Key Findings

- The derivative of the inverse function at 20 is given by the reciprocal of the derivative of the original function at the corresponding x-value where f(x) = 20.
- For the linear function F(x) = x + 5, the derivative is constant (1), so (F^{-1})'(20) = 1, which does not satisfy the condition (must be 1/5).
- For the cubic function $F(x) = x^3 + 5x$, the derivative at the relevant x-value (~2.11) is approximately 18.35, so the derivative of the inverse at 20 is about 1/18.35, which is far from 1/5.

Implications

- Neither of the provided functions satisfies the condition (f-1)'(20) = 1/5.
- To satisfy the condition, the original function must have a derivative at the corresponding x-value equal to 5, i.e., \(f'(x) = 5\), and must pass through the point where \(f(x) = 20\).

Designing a Suitable Function

- Consider a function with a constant derivative of 5, such as \(f(x) = 5x + C\). To pass through the point where \(f(x) = 20\):

\[5x + C = 20 \]

If we select \(C = 0\):

\[5x = 20 \Rightarrow x = 4 \]

- The inverse function in this case is:

\[f^{-1}(y) = \frac{y - C}{5} \]

- At \(y = 20 \), the inverse derivative is:

\[(f^{-1})'(20) = \frac{1}{f'(x)} = \frac{1}{5} = 0.2 \]

which matches the desired value.

Pros:
- Linear functions with slope 5 can satisfy the derivative condition easily.
- The inverse function's derivative at the specific point aligns with the requirement.

Cons:
- It is a very specific case; real-world functions may not be linear.
- The simplicity may not reflect more complex applications.

Final Remarks

In conclusion, neither of the provided functions—F(x) = x + 5 nor F(x) = x^3 + 5x—satisfies the condition \((f^{-1})'(20) = 1/5\). To meet this criterion, the original function must have a derivative of 5 at the point where it attains the value 20. Linear functions with a constant slope of 5, such as \(f(x) = 5x + C\), are ideal candidates, as they guarantee the inverse derivative condition is met at the corresponding x-value. This analysis underscores the importance of understanding the relationship between a function and its inverse, especially how derivatives interplay

For Which Of The Following Increasing Functions F Does F 1 20 1 5a F X X 5b F X X 3 5x

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