

Force $F = 13\hat{j}$ N Is Exerted On A Particle At $R = (3\hat{i} + 5\hat{j})$ m. What Is The Torque On The Particle About The

Force $F = 13\hat{j}$ N Is Exerted On A Particle At $R = (3\hat{i} + 5\hat{j})$ m. What Is The Torque On The Particle About The in the first paragraph sets the stage for understanding the concept of torque in physics, particularly when a force is applied to a particle at a specific position. This article explores how to calculate the torque exerted on a particle given a force vector and position vector, emphasizing the importance of vector operations such as the cross product. Whether you're a student preparing for exams or a professional seeking a refresher, this comprehensive guide will clarify the process of computing torque in vector form.

Understanding the Basics of Force and Position Vectors

What Is a Force Vector?

A force vector describes both the magnitude and direction of a force acting on an object. In our case, the force vector is given as:

- $F = 13\hat{j}$ N

This indicates a force of 13 Newtons acting in the positive y-direction.

What Is a Position Vector?

The position vector specifies the particle's location relative to a reference point, usually the origin. Here:

- $R = (3\hat{i} + 5\hat{j})$ m

This indicates the particle is located 3 meters along the x-axis and 5 meters along the y-axis from the origin.

Calculating Torque Using Vector Cross Product

Definition of Torque

Torque (τ) about a point (commonly the origin) due to a force is a measure of the tendency of the force to rotate the object about that point. It is calculated as:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

where:

- $\mathbf{r}$ is the position vector,
- $\mathbf{F}$ is the force vector,
- $\times$ denotes the cross product.

Importance of the Cross Product

The cross product of two vectors results in a third vector that is perpendicular to the plane containing the initial vectors. Its magnitude is given by:

$$|\boldsymbol{\tau}| = |\mathbf{r}| |\mathbf{F}| \sin \theta$$

where θ is the angle between $\mathbf{r}$ and $\mathbf{F}$.

Step-by-Step Calculation of Torque

Expressing Vectors in Component Form

Given:

$$\mathbf{F} = 0\mathbf{i} + 13\mathbf{j} + 0\mathbf{k}$$

$$\mathbf{R} = 3\mathbf{i} + 5\mathbf{j} + 0\mathbf{k}$$

Performing the Cross Product

Using the determinant method:

$$\boldsymbol{\tau} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ R_x & R_y & R_z \\ F_x & F_y & F_z \end{vmatrix}$$

Substituting the component values:

```
\[
\boldsymbol{\tau} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
3 & 5 & 0 \\
0 & 13 & 0
\end{vmatrix}
\]
```

Expanding the determinant:

```
\[
\boldsymbol{\tau} = \mathbf{i} \begin{vmatrix} 5 & 0 \\ 13 & 0 \end{vmatrix}
- \mathbf{j} \begin{vmatrix} 3 & 0 \\ 0 & 0 \end{vmatrix} + \mathbf{k}
\begin{vmatrix} 3 & 5 \\ 0 & 13 \end{vmatrix}
\]
```

Calculating each minor:

- $(\mathbf{i})$ component:

```
\[
(5)(0) - (0)(13) = 0 - 0 = 0
\]
```

- $(\mathbf{j})$ component:

```
\[
(3)(0) - (0)(0) = 0 - 0 = 0
\]
```

- $(\mathbf{k})$ component:

```
\[
(3)(13) - (5)(0) = 39 - 0 = 39
\]
```

Therefore:

```
\[
\boldsymbol{\tau} = 0\mathbf{i} - 0\mathbf{j} + 39\mathbf{k}
\]
```

Final Torque Vector

```
\[
\boxed{
\boldsymbol{\tau} = 39\mathbf{k} \text{ N}\cdot\text{m}
}
\]
```

This indicates the torque is 39 Newton-meters in the positive z-direction, perpendicular to the plane of the force and position vectors.

Interpreting the Results

Magnitude of the Torque

Since the torque vector only has a $\mathbf{k}$ component:

$$|\boldsymbol{\tau}| = 39 \text{ N}\cdot\text{m}$$

Direction of the Torque

The positive $\mathbf{k}$ direction signifies that the torque tends to produce a rotation about the z-axis in the counterclockwise direction when viewed from above.

Practical Applications of Torque Calculation

Engineering and Mechanical Design

Understanding torque is essential when designing machinery, ensuring that forces do not cause unintended rotations or failures.

Physics and Dynamics

Calculating torque helps analyze rotational motion, stability, and the effects of forces in systems ranging from simple levers to complex machinery.

Summary and Key Takeaways

- Torque is calculated as the cross product of the position vector and force vector.
- The magnitude and direction of torque provide insights into the rotational effect of a force.
- In this example, the torque exerted on a particle at $\mathbf{R} = 3\mathbf{i} + 5\mathbf{j}$ m by a force $\mathbf{F} = 13\mathbf{j}$ N is 39 N·m in the positive z-direction.
- Vector operations like the cross product are crucial in physics for analyzing rotational effects.

Conclusion

Calculating torque involves understanding vector operations and their physical significance. By expressing force and position vectors in component form, performing the cross product, and interpreting the result, we can determine how forces influence rotational motion. Whether in academic settings or practical engineering applications, mastering torque calculations

is fundamental to analyzing and designing systems involving rotational dynamics.

For further learning, consider exploring topics such as moments of force, rotational equilibrium, and the applications of torque in various engineering fields.

Frequently Asked Questions

What is the force exerted on the particle at $R = (3\mathbf{i} + 5\mathbf{j})$ m?

The force exerted is $F = 13\mathbf{j}$ N, which is a force of 13 Newtons in the positive y-direction.

How do you calculate the torque about the origin for the particle?

Torque τ is calculated as $\tau = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ is the position vector and $\mathbf{F}$ is the force vector. In this case, $\tau = (3\mathbf{i} + 5\mathbf{j}) \times (0\mathbf{i} + 13\mathbf{j})$ N.

What is the cross product of $\mathbf{r} = (3\mathbf{i} + 5\mathbf{j})$ and $\mathbf{F} = (0\mathbf{i} + 13\mathbf{j})$ in vector form?

The cross product is $\tau = \mathbf{r} \times \mathbf{F} = (3\mathbf{i} + 5\mathbf{j}) \times (0\mathbf{i} + 13\mathbf{j}) = 3\mathbf{i} \times 13\mathbf{j} + 5\mathbf{j} \times 13\mathbf{j}$. Since $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ and $\mathbf{j} \times \mathbf{j} = 0$, the torque simplifies to $3 \times 13\mathbf{k} = 39\mathbf{k}$ N·m.

What is the magnitude of the torque exerted on the particle?

The magnitude of the torque is $|\tau| = 39$ N·m.

In which direction is the torque vector pointing?

The torque vector points in the positive k-direction, perpendicular to the xy-plane.

Can the torque be negative in this scenario?

Yes, if the direction of the force or position vector changes, the torque could be negative, indicating direction opposite to the positive k-axis. But in this case, it is positive.

What assumptions are made in calculating the torque in this problem?

The calculation assumes that the force acts at the given position vector in a plane, and that the force is perpendicular to the position vector components, simplifying the cross product calculation.

How does the position vector influence the magnitude of the torque?

The magnitude of torque depends linearly on the length of the position vector and the component of the force perpendicular to it. Larger distances or force components perpendicular to the position vector increase torque.

Additional Resources

Force $F = 13\hat{j}$ N Is Exerted On A Particle At $R = (3\hat{i} + 5\hat{j})$ m. What Is The Torque On The Particle About The?

Understanding the calculation of torque when a force is applied to a particle is fundamental in physics and engineering. In this guide, we will explore how to determine the torque exerted on a particle when a specific force acts at a certain position vector. We will break down the problem step-by-step, clarify essential concepts, and elucidate the mathematical procedures involved.

Introduction

When analyzing the mechanics of particles and rigid bodies, understanding the concept of torque (also known as moment of force) is crucial. Torque quantifies how much a force tends to cause an object to rotate about a specific point or axis. The problem at hand involves a force vector F and a position vector R . Our goal is to find the torque τ exerted about a particular point—presumably the origin, unless specified otherwise.

The specific problem states:

- Force $F = 13\hat{j}$ N
- Position vector $R = (3\hat{i} + 5\hat{j})$ meters

The task is to determine the torque on the particle about the origin (or the chosen point).

Fundamental Concepts

Before proceeding with calculations, let's review the core concepts involved:

1. Force Vector (F)

A force vector describes both the magnitude and direction of a force acting on a particle. It is typically expressed in unit vector notation:

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k}$$

In this problem, the force is:

$$\mathbf{F} = 13 \mathbf{j} \text{ N}$$

which indicates the force has a magnitude of 13 N acting purely in the positive y-direction.

2. Position Vector (R)

The position vector indicates the point of application of the force relative to a reference point (usually the origin). It is given as:

$$\mathbf{R} = 3 \mathbf{i} + 5 \mathbf{j} \text{ m}$$

which means the point is located 3 meters along the x-axis and 5 meters along the y-axis from the origin.

3. Torque (τ)

The torque τ about a point is given by the cross product of the position vector $\mathbf{R}$ and force vector $\mathbf{F}$:

$$\boldsymbol{\tau} = \mathbf{R} \times \mathbf{F}$$

This vector quantity indicates both the magnitude and the direction of the rotational tendency.

Step-by-Step Calculation of Torque

Let's now proceed to compute the torque step-by-step.

Step 1: Write Down the Vectors

- Force vector:

$$\mathbf{F} = 0 \mathbf{i} + 13 \mathbf{j} + 0 \mathbf{k}$$

- Position vector:

$$\mathbf{R} = 3 \mathbf{i} + 5 \mathbf{j} + 0 \mathbf{k}$$

Step 2: Recall the Cross Product Formula

The cross product of two vectors $\mathbf{A}$ and $\mathbf{B}$ in component form is:

```
\[
\mathbf{A} \times \mathbf{B} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
A_x & A_y & A_z \\
B_x & B_y & B_z
\end{vmatrix}
\]
```

which expands to:

```
\[
\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y) \mathbf{i} - (A_x B_z -
A_z B_x) \mathbf{j} + (A_x B_y - A_y B_x) \mathbf{k}
\]
```

Step 3: Apply the Formula

Given:

```
- \(\mathbf{R} = (3, 5, 0)\)
- \(\mathbf{F} = (0, 13, 0)\)
```

Compute each component:

- i-component:

```
\[
(5 \times 0) - (0 \times 13) = 0 - 0 = 0
\]
```

- j-component:

```
\[
- [ (3 \times 0) - (0 \times 0) ] = - (0 - 0) = 0
\]
```

- k-component:

```
\[
(3 \times 13) - (5 \times 0) = 39 - 0 = 39
\]
```

Step 4: Write the Resulting Torque Vector

```
\[
\boxed{
\boldsymbol{\tau} = 0 \mathbf{i} + 0 \mathbf{j} + 39 \mathbf{k} \text{ , } \text{Nm}
}
\]
```

This indicates the torque vector points in the positive z-direction with a magnitude of 39 Nm.

Interpretation of Results

The calculated torque:

$$\boldsymbol{\tau} = 39 \mathbf{k} \text{ Nm}$$

has the following implications:

- Magnitude: 39 Nm
- Direction: Along the positive z-axis, perpendicular to both the position and force vectors.

This result tells us that the force applied at point $(3, 5)$ meters causes a rotational effect about the origin in the z-direction. The positive sign indicates the direction of rotation follows the right-hand rule.

Additional Considerations

1. About Which Point Is Torque Calculated?

In this problem, the torque is computed about the origin as the position vector is given relative to the origin. If a different point was specified, the position vector would be measured from that point, and the calculation would need adjustment accordingly.

2. Effect of Force Direction

Since the force acts purely in the y-direction, and the position vector has x and y components, the cross product results in a torque in the z-direction. Had the force had components in other directions, the torque vector would reflect that.

3. Units and Consistency

Always ensure the units are consistent—here, all distances are in meters and force in newtons, so the torque is expressed in Newton-meters (Nm).

Practical Applications

Understanding how to compute torque in such scenarios is vital in fields like

mechanical engineering, robotics, and structural analysis. For example:

- Designing Rotational Systems: Ensuring a force applied at a certain point achieves desired rotational effects.
- Analyzing Mechanical Failures: Calculating the torque to understand stress and potential failure points.
- Robotics: Planning movements and forces for robotic arms or manipulators.

Summary of Key Steps

- Identify vectors: Force and position.
- Express in component form: Use unit vectors.
- Apply cross product formula: Calculate the torque vector.
- Interpret the result: Understand magnitude and direction.
- Check units: Confirm consistency.

Conclusion

Calculating the torque exerted on a particle involves understanding vector operations, primarily the cross product. In the specific problem of Force $F = 13\mathbf{j}$ N acting on a particle at $R = (3\mathbf{i} + 5\mathbf{j})$ meters, the torque about the origin is 39 Nm in the positive z-direction. Mastery of these vector techniques allows engineers and physicists to analyze and design systems involving rotational dynamics effectively.

Remember: The key to solving torque problems is carefully translating physical scenarios into vector components, applying the cross product accurately, and interpreting the results within the context of the physical system.

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