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Introduction to Steady-State Error in Control Systems

In control system engineering, understanding the steady-state error is crucial for designing systems that meet specific accuracy requirements. The steady-state error measures how accurately a system follows a desired input signal after transient effects have subsided. It indicates the long-term deviation between the input and output signals and is essential for ensuring system performance, especially in applications requiring high precision.

This article focuses on calculating the steady-state error for a specific closed-loop control system characterized by given transfer functions. The system's transfer functions are provided as Gc(s) and Gp(s). We will analyze the system thoroughly, derive the relevant formulas, and compute the steady-state error for different types of input signals.

Understanding the Given Transfer Functions

Before delving into the calculation process, it is vital to understand the components of the control system:

- Controller Transfer Function (Gc(s)):

Given as Gc(s) = 2, which indicates a proportional controller with a gain of 2.

- Plant Transfer Function (Gp(s)):

Given as Gp(s) = 2 / (s (s + 7)^2), which models the dynamics of the system being controlled.

The overall system is a unity feedback system, where the output is fed back and compared with the input to generate an error signal.

Formulating the Closed-Loop Transfer Function

The standard form of a unity feedback control system's transfer function is:

$$
T(s) = \frac{G_{total}(s)}{1 + G_{total}(s)}
$$

where $G_{total}(s)$ is the product of the controller and plant transfer functions:

$$
G_{total}(s) = G_c(s) \times G_p(s)
$$

Given:

$$
G_c(s) = 2
$$

$$
G_p(s) = \frac{2}{s(s+7)^2}
$$

The open-loop transfer function:

$$
G_{OL}(s) = G_c(s) \times G_p(s) = 2 \times \frac{2}{s(s+7)^2} = \frac{4}{s(s+7)^2}
$$

The closed-loop transfer function:

$$
T(s) = \frac{G_{OL}(s)}{1 + G_{OL}(s)} = \frac{\frac{4}{s(s+7)^2}}{1 + \frac{4}{s(s+7)^2}} = \frac{4}{s(s+7)^2 + 4}
$$

This transfer function describes how the output responds to the input.

Steady-State Error and Its Calculation

The steady-state error e_{ss} depends on the type of input signal applied to the system:

- Step input ($R(s) = \frac{1}{s}$)
- Ramp input ($R(s) = \frac{1}{s^2}$)
- Parabolic input ($R(s) = \frac{1}{s^3}$)

The general formula for steady-state error:

$$
e_{ss} = \lim_{t \to \infty} e(t)
$$

which, in the Laplace domain, becomes:

$$
e_{ss} = \lim_{s \to 0} s E(s)
$$

where $E(s)$ is the Laplace transform of the error signal:

$$
E(s) = R(s) - Y(s)
$$

and the output $Y(s)$ is:

$$
Y(s) = T(s) R(s)
$$

Thus,

\[

$$
E(s) = R(s) - T(s) R(s) = R(s) [1 - T(s)]
$$

Therefore, the steady-state error:

$$
e_{ss} = \lim_{s \to 0} s R(s) [1 - T(s)]
$$

Calculating Steady-State Error for Different Inputs

1. Step Input ($R(s) = \frac{1}{s}$)

Substituting into the formula:

$$
e_{ss} = \lim_{s \to 0} s \times \frac{1}{s} \times [1 - T(s)] = \lim_{s \to 0} [1 - T(s)]
$$

The key is to evaluate $T(s)$ as $s \to 0$.

Calculate $T(0)$:

$$
T(0) = \frac{4}{0 \times (0 + 7)^2 + 4} = \frac{4}{0 + 4} = 1
$$

Thus,

$$
e_{ss} = 1 - 1 = 0
$$

Result: The steady-state error for a step input is zero, indicating Type 0 system with zero error for step inputs.

---

2. Ramp Input ($R(s) = \frac{1}{s^2}$)

Using the same approach:

$$
e_{ss} = \lim_{s \to 0} s \times \frac{1}{s^2} \times [1 - T(s)] = \lim_{s \to 0} \frac{1}{s} [1 - T(s)]
$$

Since $\frac{1}{s} \to \infty$ as $s \to 0$, the behavior depends on $1 - T(s)$.

Evaluate $T(s)$ near zero:

$$
T(0) = 1 \quad \text{(as previously calculated)}
$$

To find e_{ss}, expand $T(s)$ for small s:

$$

T(s) = \frac{4}{s(s+7)^2 + 4}

\]

As \(s \to 0 \):

\[

s(s+7)^2 \to 0

\]

\[

T(0) = 1

\]

The limit becomes:

\[

e_{ss} = \lim_{s \to 0} \frac{1 - T(s)}{s}

\]

Since \(T(s) \to 1 \), and the numerator approaches zero, we can apply L'Hôpital's Rule or analyze the small \(s \) behavior.

Expanding \(T(s) \) near \(s=0 \):

\[

T(s) \approx 1 - \text{(small term)}

\]

Therefore,

\[

1 - T(s) \sim \text{small term}

\]

Calculating the exact small s behavior:

$$
T(s) = \frac{4}{s(s+7)^2 + 4}
$$

At small s:

$$
s(s+7)^2 \approx s \times 49 = 49s
$$

$$
T(s) \approx \frac{4}{4 + 49s} \approx 1 - \frac{49s}{4}
$$

Thus,

$$
1 - T(s) \approx \frac{49s}{4}
$$

Now,

$$
e_{ss} = \lim_{s \to 0} \frac{1 - T(s)}{s} \approx \lim_{s \to 0} \frac{\frac{49s}{4}}{s} = \frac{49}{4} = 12.25
$$

Result: The steady-state error for a ramp input is 12.25, indicating the system cannot perfectly track ramp inputs and exhibits a finite steady-state error — typical for a Type 0 system.

---

3. Parabolic Input ($R(s) = \frac{1}{s^3}$)

For a parabolic input:

$$
e_{ss} = \lim_{s \to 0} s \times \frac{1}{s^3} \times [1 - T(s)] = \lim_{s \to 0} \frac{1}{s^2} [1 - T(s)]
$$

As $s \to 0$, $\frac{1}{s^2} \to \infty$, and since $1 - T(s) \to 0$, the overall limit depends on how fast $1 - T(s)$ approaches zero.

From the previous expansion:

$$
1 - T(s) \approx \frac{49s}{4}
$$

Thus,

$$
e_{ss} \approx \lim_{s \to 0} \frac{\frac{49s}{4}}{s^2} = \lim_{s \to 0} \frac{49}{4s} \to \infty
$$

Result: The steady-state error for parabolic inputs tends to infinity, indicating the system cannot track parabolic inputs accurately.

Summary of Steady-State Errors

Frequently Asked Questions

What is the steady-state error in a unity feedback system with Gc(s) = 2 and Gp(s) = 2 / (s (s + 7)^2)?

The steady-state error can be determined using the Final Value Theorem and the system's Type. Since Gp(s) has two integrators (due to s in the denominator), the system is Type 2, which results in zero steady-state error for a step input.

How do you calculate the steady-state error for a given transfer function in a feedback system?

You use the Final Value Theorem: e_ss = lim_{t□□} e(t) = lim_{s□0} s E(s), where E(s) = R(s) - T(s) G(s) R(s). For standard inputs like step, ramp, or parabolic, specific formulas involve the system's Type and position error constant.

What is the significance of the system being Type 2 in this control system?

A Type 2 system, with two integrators in the open-loop transfer function, has zero steady-state error for step and ramp inputs, but a finite error for parabolic inputs. Given Gp(s), the system is Type 2, ensuring zero error for step inputs.

How does the transfer function $Gp(s) = 2 / (s (s + 7)^2)$ affect the system's steady-state error?

The transfer function indicates two poles at $s = -7$ and one at $s = 0$, making the system Type 2. This results in a zero steady-state error for step inputs and finite error for ramp inputs, assuming unity feedback.

What role does the controller Gc(s) = 2 play in the steady-state error calculation?

The controller Gc(s) affects the open-loop transfer function and system dynamics. In this case, Gc(s)=2 adds a gain but doesn't introduce additional poles or zeros, so it influences the magnitude of the steady-state error but not its qualitative nature for a Type 2 system.

Can you determine the exact numerical value of the steady-state error for a step input in this system?

Yes. For a step input R(s)=1/s, the steady-state error e_ss = 0 since the system is Type 2, which provides zero error for step inputs regardless of gain, assuming no additional disturbances.

What is the general approach to find the steady-state error in systems with multiple poles and zeros?

Identify the system type based on the number of integrators (poles at s=0), compute the relevant error constant (Kp, Kv, Ka), and then use the standard formulas for steady-state error for different input types. The Final Value Theorem simplifies this process.

Additional Resources

Understanding the steady-state error for a control system is crucial in the design and analysis of feedback systems. In this article, we will analyze the steady-state error for the system characterized by the transfer functions Gc(s) = 2 and Gp(s) = 2 / (s (s + 7)^2). With this in-depth examination, you will learn how to determine the steady-state error of a closed-loop system given specific transfer functions and input types.

Introduction to Steady-State Error in Control Systems

Before diving into the specifics of the transfer functions, it’s essential to understand what steady-state error (SSE) is and why it matters. The steady-state error refers to the difference between the desired input and the actual output of a control system after all transient effects have decayed and the system has reached equilibrium. It provides insight into the accuracy and performance of the system under steady conditions.

Key points about steady-state error:

- It indicates how well the system can track a given input.
- It depends on the system's transfer functions and the type of input signal.
- Reducing steady-state error is often a primary goal, especially in precision control applications.

System Overview and Transfer Functions

The given system consists of two transfer functions:

- Controller transfer function: Gc(s) = 2
- Plant transfer function: Gp(s) = 2 / (s (s + 7)^2)

The combined open-loop transfer function is:

$$ G_{OL}(s) = Gc(s) \times Gp(s) = 2 \times \frac{2}{s (s+7)^2} = \frac{4}{s (s+7)^2} $$

The closed-loop transfer function, assuming unity feedback, is:

$$ G_{cl}(s) = \frac{G_{OL}(s)}{1 + G_{OL}(s)} = \frac{\frac{4}{s (s+7)^2}}{1 + \frac{4}{s (s+7)^2}} $$

Types of Inputs and Their Effects on Steady-State Error

The steady-state error depends heavily on the input type. Common input signals include:

- Step input: $R(s) = \frac{A}{s}$

- Ramp input: $R(s) = \frac{A}{s^2}$
- Parabolic input: $R(s) = \frac{A}{s^3}$

The system's ability to track these inputs without error is characterized by the system type, which relates to the number of integrators (poles at the origin) in the open-loop transfer function.

Determining the System Type

The system type is determined by the number of poles at the origin (s=0) in $G_{OL}(s)$.

Given:

$$ G_{OL}(s) = \frac{4}{s (s+7)^2} $$

Since there is a single pole at $s=0$, this is a Type 1 system.

Implication:

- For a step input, the steady-state error will be finite.
- For a ramp input, the steady-state error will be non-zero but finite.
- For a parabolic input, the steady-state error will tend to infinity.

Calculating Steady-State Error for Different Inputs

The general approach involves using the Final Value Theorem:

$$ e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s E(s) $$

where:

$$ E(s) = R(s) - Y(s) $$

$$ Y(s) = G_{cl}(s) R(s) $$

$$ E(s) = R(s) - G_{cl}(s) R(s) = R(s) (1 - G_{cl}(s)) $$

Thus,

$$ e_{ss} = \lim_{s \to 0} s R(s) \left(1 - G_{cl}(s) \right) $$

Step 1: Compute $G_{cl}(s)$ at $s=0$

Recall:

$$ G_{cl}(s) = \frac{G_{OL}(s)}{1 + G_{OL}(s)} = \frac{\frac{4}{s (s+7)^2}}{1 + \frac{4}{s (s+7)^2}} $$

At $s=0$:

$$ G_{OL}(0) = \frac{4}{0 \times (7)^2} \rightarrow \text{infinite} $$

Therefore:

$$ G_{cl}(0) = \frac{\infty}{1 + \infty} = 1 $$

This indicates that the system is capable of perfect tracking of a step input, as expected for a Type 1 system.

Step 2: Calculate Steady-State Error for Various Inputs

---

1. Step Input: $R(s) = \frac{A}{s}$

Using the formula:

$$e_{ss} = \lim_{s \to 0} s \times \frac{A}{s} \times (1 - G_{cl}(s)) = A \times (1 - G_{cl}(0))$$

Since $G_{cl}(0) = 1$:

$$e_{ss} = A \times (1 - 1) = 0$$

Result: The steady-state error for a step input is zero. The system perfectly tracks step inputs, confirming its Type 1 classification.

---

2. Ramp Input: $R(s) = \frac{A}{s^2}$

$$e_{ss} = \lim_{s \to 0} s \times \frac{A}{s^2} \times (1 - G_{cl}(s)) = \lim_{s \to 0} \frac{A}{s} (1 - G_{cl}(s))$$

Since $G_{cl}(0) = 1$, we need the first derivative of $G_{cl}(s)$ as $s \to 0$ to evaluate the limit. The steady-state error for a ramp input in a Type 1 system is given by:

$$e_{ss} = \frac{A}{K_v}$$

where K_v is the velocity error constant:

$$K_v = \lim_{s \to 0} s G_{OL}(s)$$

Calculate:

$$s G_{OL}(s) = s \times \frac{4}{s (s+7)^2} = \frac{4}{(s+7)^2}$$

At $s = 0$:

$$K_v = \frac{4}{(7)^2} = \frac{4}{49} \approx 0.0816$$

Therefore,

$$e_{ss} = \frac{A}{K_v} = \frac{A}{4/49} = A \times \frac{49}{4} = 12.25 A$$

Result: The steady-state error for a ramp input is approximately 12.25 times the input amplitude.

---

3. Parabolic Input: $R(s) = \frac{A}{s^3}$

In a Type 1 system, the steady-state error for a parabolic input tends to infinity, indicating the system cannot accurately track such a signal.

Result: The steady-state error diverges, making tracking of parabolic inputs impossible in this configuration.

Summary of Steady-State Errors

Input Type	Steady-State Error e_{ss}	Notes
Step ($\frac{A}{s}$)	0	Perfect tracking
Ramp ($\frac{A}{s^2}$)	12.25 A	Finite, system follows ramp with a proportional error
Parabolic ($\frac{A}{s^3}$)	Infinity	Not trackable

Conclusion and Practical Implications

The analysis reveals that our control system with $G_c(s) = 2$ and $G_p(s) = 2 / (s (s+7)^2)$ is a Type 1 system, capable of perfectly tracking step inputs with zero steady-state error. However, it exhibits a finite, but relatively large, steady-state error for ramp inputs, which scales with the input amplitude. For higher-order inputs like parabolas, the error diverges, indicating the system's inability to track such signals accurately.

Practical Takeaways:

- To improve ramp tracking, increase the system's velocity constant K_v by modifying the controller or plant.
- For applications requiring minimal steady-state error for ramps, consider adding integral action (e.g., increasing system type).
- The system's current configuration is suitable for systems where step accuracy is critical, but less so for continuous acceleration or velocity tracking.

Final Remarks:

Understanding how to determine the steady-state error based on transfer functions is fundamental in control system design

Gc S 2 Gp S 2 S S 7 S 7 Determine The Steady State Error For The Closed Loop System With A

Gc S 2 Gp S 2 S S 7 S 7 Determine The Steady State Error For The Closed Loop System With A

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