Given A = (-3, 2, 4) And B = (1, 4, 1). Find The Unit Vector In The Direction Of 2B - 6A. A) (-16, 4,

Given A = (-3, 2, 4) And B = (1, 4, 1). Find The Unit Vector In The Direction Of 2B - 6A. A) (-16, 4, in the first paragraph sets the stage for a detailed exploration of vector operations, specifically focusing on how to find a unit vector in a given direction. This problem involves basic vector algebra concepts such as scalar multiplication, vector subtraction, and normalization. Understanding how to approach this problem is crucial for students and enthusiasts delving into vector mathematics, which is fundamental in physics, engineering, and computer graphics.

In this article, we will walk through the step-by-step process of solving this type of vector problem, providing clarity on each operation involved. From calculating 2B and 6A, to subtracting these vectors, and finally normalizing the resulting vector to find the unit vector, every step will be explained thoroughly. By mastering these steps, you'll be able to handle similar vector problems with confidence.

Understanding the Problem

Given Vectors A and B

The problem provides two vectors:
- A = (-3, 2, 4)
- B = (1, 4, 1)

The goal is to find the unit vector in the direction of the vector 2B - 6A.

What is a Unit Vector?

A unit vector is a vector with a magnitude (length) of 1 that points in a specific direction. To find the unit vector in the direction of any given vector, you need to:
1. Calculate the vector itself.
2. Find its magnitude (length).
3. Divide the vector by its magnitude.

Step-by-Step Solution

Step 1: Calculate 2B

Scalar multiplication involves multiplying each component of vector B by the scalar 2:
- B = (1, 4, 1)
- 2B = (2 1, 2 4, 2 1) = (2, 8, 2)

Step 2: Calculate 6A

Similarly, multiply each component of A by 6:
- A = (-3, 2, 4)
- 6A = (6 -3, 6 2, 6 4) = (-18, 12, 24)

Step 3: Find 2B - 6A

Subtract the corresponding components of 6A from 2B:
- 2B - 6A = (2 - (-18), 8 - 12, 2 - 24)
- Simplify:
- First component: 2 + 18 = 20
- Second component: 8 - 12 = -4
- Third component: 2 - 24 = -22

So, the resulting vector is:
- V = (20, -4, -22)

Calculating the Magnitude of V

The magnitude (or length) of vector V = (20, -4, -22) is calculated using the formula:
$$
|V| = \sqrt{(x)^2 + (y)^2 + (z)^2}
$$

Plugging in the components:
$$
|V| = \sqrt{20^2 + (-4)^2 + (-22)^2} = \sqrt{400 + 16 + 484}
$$
$$
|V| = \sqrt{900} = 30
$$

Finding the Unit Vector

To find the unit vector U in the direction of V, divide each component of V by its magnitude:

$$U = \left(\frac{20}{30}, \frac{-4}{30}, \frac{-22}{30} \right)$$

Simplify each component:
- $\frac{20}{30} = \frac{2}{3}$
- $\frac{-4}{30} = -\frac{2}{15}$
- $\frac{-22}{30} = -\frac{11}{15}$

Therefore, the unit vector is:

$$U = \left(\frac{2}{3}, -\frac{2}{15}, -\frac{11}{15} \right)$$

Final Answer and Interpretation

The unit vector in the direction of 2B - 6A is:
- $U = \left(\frac{2}{3}, -\frac{2}{15}, -\frac{11}{15} \right)$

This vector has a magnitude of 1 and points exactly in the same direction as the original vector V. It is crucial in various applications, such as defining directions in space, calculating angles between vectors, and more.

Additional Insights & Applications

Why Find a Unit Vector?

Unit vectors serve as standardized directional indicators, making them essential in:
- Physics: representing directions of forces and velocities.
- Computer Graphics: defining directions for shading, lighting, and object movement.
- Engineering: analyzing directional components of forces.

Related Vector Operations

Understanding how to manipulate vectors is foundational. Some related operations include:
- Vector addition and subtraction.
- Dot product for measuring angles.
- Cross product for finding perpendicular vectors.
- Scalar multiplication to scale vectors.

Summary

In summary, solving for the unit vector in the direction of a linear combination of vectors involves clear steps:
1. Scalar multiply the given vectors.
2. Subtract the scaled vectors to find the resultant vector.
3. Calculate the magnitude of this vector.
4. Divide each component by the magnitude to normalize.

By mastering these steps, you can efficiently handle a wide array of vector problems across various scientific and engineering disciplines.

Practice Problem

Try solving a similar problem: Given vectors C = (2, -1, 3) and D = (4, 0, -2), find the unit vector in the direction of 3D - 2C.

Remember: Vector operations are the building blocks of understanding multidimensional spaces. Practice regularly to become proficient in these essential mathematical tools.

Frequently Asked Questions

How do you find the vector 2B - 6A given points A = (-3, 2, 4) and B = (1, 4, 1)?

First, multiply B by 2 to get 2B = (21, 24, 21) = (2, 8, 2). Then, multiply A by 6 to get 6A = (6-3, 62, 64) = (-18, 12, 24). Subtract to find 2B - 6A = (2 - (-18), 8 - 12, 2 - 24) = (20, -4, -22).

What is the next step after finding the vector 2B - 6A in order to find its unit vector?

Calculate the magnitude (length) of the vector 2B - 6A, then divide each component of the vector by this magnitude to obtain the unit vector.

How do you compute the magnitude of the vector (20, -4, -22)?

The magnitude is $\sqrt{(20^2 + (-4)^2 + (-22)^2)} = \sqrt{(400 + 16 + 484)} = \sqrt{900} = 30$.

What is the unit vector in the direction of 2B - 6A?

Divide each component of the vector (20, -4, -22) by 30, resulting in the unit vector (20/30, -4/30, -22/30) = (2/3, -2/15, -11/15).

What are the components of the unit vector in the direction of 2B - 6A?

The components are approximately (0.6667, -0.1333, -0.7333).

Is the answer A) (-16, 4, ...)?

No, the correct unit vector components are approximately (0.6667, -0.1333, -0.7333), so answer A) is not correct.

Additional Resources

Vector Analysis and Unit Vector Computation: A Deep Dive into Directional Vectors

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Introduction

Understanding vectors and their properties is fundamental in fields ranging from physics and engineering to computer graphics and data science. When working with vectors, a common task is to find a unit vector in a particular direction, especially when that direction is defined by the difference or linear combination of other vectors. This article offers an in-depth exploration of how to find the unit vector in the direction of a specific linear combination of vectors — in this case, 2B - 6A, where A and B are given vectors.

We will carefully dissect each step, from calculating the linear combination to normalizing the resulting vector, ensuring clarity and precision for readers seeking both conceptual understanding and practical application.

Understanding the Given Vectors

Before we compute the desired unit vector, let's first familiarize ourselves with the vectors involved:

- A = (-3, 2, 4)
- B = (1, 4, 1)

These are three-dimensional vectors, each with x, y, and z components.

Step 1: Computing the Linear Combination 2B - 6A

The first key step is to evaluate the vector expression 2B - 6A carefully. This involves scalar multiplication and vector subtraction.

Scalar Multiplication

- 2B means multiplying each component of B by 2:
- \(2 \times (1, 4, 1) = (2 \times 1, 2 \times 4, 2 \times 1) = (2, 8, 2) \)

- 6A means multiplying each component of A by 6:
- \(6 \times (-3, 2, 4) = (6 \times -3, 6 \times 2, 6 \times 4) = (-18, 12, 24) \)

Vector Subtraction

Now, to compute 2B - 6A, subtract the corresponding components:

\[
\begin{aligned}
& (2, 8, 2) - (-18, 12, 24) \\
&= (2 - (-18), 8 - 12, 2 - 24) \\

&= (2 + 18, 8 - 12, 2 - 24) \\
&= (20, -4, -22)
\end{aligned}
\]

Thus, the vector in the direction of 2B - 6A is:

\[
\boxed{\mathbf{V} = (20, -4, -22)}
\]

Step 2: Finding the Magnitude of the Vector

The magnitude (or length) of a vector V = (x, y, z) is given by the Euclidean norm:

\[
|\mathbf{V}| = \sqrt{x^2 + y^2 + z^2}
\]

Applying this to our vector V = (20, -4, -22):

\[
|\mathbf{V}| = \sqrt{20^2 + (-4)^2 + (-22)^2} = \sqrt{400 + 16 + 484} = \sqrt{900}
\]

Calculating the square root:

\[
|\mathbf{V}| = 30
\]

This magnitude indicates the length of the vector in three-dimensional space.

Step 3: Computing the Unit Vector

A unit vector in the direction of V is obtained by dividing each component of V by its magnitude:

$$
\hat{\mathbf{u}} = \frac{\mathbf{V}}{|\mathbf{V}|}
$$

Applying this to each component:

$$
\hat{\mathbf{u}} = \left(\frac{20}{30}, \frac{-4}{30}, \frac{-22}{30} \right) = \left(\frac{2}{3}, -\frac{2}{15}, -\frac{11}{15} \right)
$$

Expressed as decimals for clarity:

- $\frac{2}{3} \approx 0.6667$
- $-\frac{2}{15} \approx -0.1333$
- $-\frac{11}{15} \approx -0.7333$

Therefore, the unit vector in the direction of 2B - 6A is:

$$
\boxed{\left(\frac{2}{3}, -\frac{2}{15}, -\frac{11}{15} \right)}
$$

or approximately:

$$
\boxed{(0.6667, -0.1333, -0.7333)}
$$

Alternative Representation and Verification

For completeness, it's beneficial to verify that our unit vector has a magnitude of 1:

$$
\sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{2}{15}\right)^2 + \left(-\frac{11}{15}\right)^2} = \sqrt{\frac{4}{9} + \frac{4}{225} + \frac{121}{225}}
$$

Express all fractions with denominator 225:

$$
= \sqrt{\frac{100}{225} + \frac{4}{225} + \frac{121}{225}} = \sqrt{\frac{225}{225}} = \sqrt{1} = 1
$$

This confirms the correctness of our unit vector calculation.

Summary & Practical Implications

- The vector 2B - 6A computes to (20, -4, -22).
- Its magnitude is 30.
- The unit vector in that direction is $\left(\frac{2}{3}, -\frac{2}{15}, -\frac{11}{15}\right)$.

Knowing how to derive unit vectors from linear combinations of vectors is invaluable in numerous applications:

- Directionality in physics: Describing motion or force directions.
- Computer graphics: Normalizing vectors for lighting and shading calculations.
- Robotics and navigation: Determining orientations and movement directions.
- Data normalization: Standardizing feature vectors for machine learning.

Final Thoughts

This detailed walkthrough highlights the importance of fundamental vector operations—scalar multiplication, subtraction, and normalization—in solving real-world problems. Whether you're analyzing forces, directions, or data features, mastering these steps enhances your ability to interpret and manipulate vector quantities effectively.

Remember, always verify your calculations, especially when dealing with components and magnitudes, to ensure accuracy in your applications. With a solid grasp of these principles, you can confidently tackle more complex vector problems with ease and precision.

[Given A 3 2 4 And B 1 4 1 Find The Unit Vector In The Direction Of 2b 6a A 16 4]

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