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Introduction

In the realm of vector mathematics, understanding how vectors combine and influence each other's directions is fundamental. This understanding is crucial not only for academic pursuits but also for practical applications in physics, engineering, computer graphics, and navigation systems. A common problem involves determining a specific vector that, when added to a given vector, results in a new vector aligned along a particular axis—in this case, the x-axis.

The problem statement is as follows: Given vectors $A = 4i - 10j$ and $B = 7i + 5j$, find the scalar b such that the vector $A + bB$ points solely along the x-axis, meaning it has no component in the y-direction. This problem involves understanding vector addition, scalar multiplication, and the conditions for a vector to be aligned along a particular axis.

This article aims to explore this problem in depth, providing comprehensive explanations, step-by-step solutions, and insights into the underlying vector principles. We will delve into the mathematical concepts necessary for solving such problems, including vector components, scalar multiplication, and the conditions for directional alignment. Additionally, we will discuss the significance of these concepts in various practical scenarios and extend the discussion to related vector problems to enhance understanding.

Understanding the Components of Vectors A and B

The Vector Components

Vectors in a two-dimensional plane are represented by their components along the x-axis (i-direction) and y-axis (j-direction). For the given vectors:

- $A = 4i - 10j$
- $A_x = 4$
- $A_y = -10$

- $B = 7i + 5j$
- $B_x = 7$
- $B_y = 5$

These components define the vectors' directions and magnitudes in the coordinate plane. Visualizing these vectors helps in understanding their spatial orientation.

Visual Representation

Imagine the coordinate plane with the vectors originating from the origin:

- Vector A points to the right (positive x) and downward (negative y).
- Vector B points to the right (positive x) and upward (positive y).

Understanding their directions is essential before combining them to form a new vector.

Mathematical Formulation of the Problem

The Objective

Find the scalar b such that the vector:

$$\mathbf{A} + b \mathbf{B}$$

has no y-component, i.e., it points purely along the x-axis.

Expressing the Combined Vector

The combined vector can be written in component form as:

$$\mathbf{A} + b \mathbf{B} = (A_x + b B_x) \mathbf{i} + (A_y + b B_y) \mathbf{j}$$

Substituting the components:

$$\mathbf{A} + b \mathbf{B} = (4 + 7b) \mathbf{i} + (-10 + 5b) \mathbf{j}$$

Condition for a Vector Along the X-axis

For the vector to point along the x-axis, its y-component must be zero:

$$A_y + b B_y = 0$$

Substituting known values:

$$-10 + 5b = 0$$

Solving this equation for b will give the required scalar.

Solving for the Scalar b

Step-by-Step Calculation

1. Set up the equation:

$$\begin{aligned} & -10 + 5b = 0 \end{aligned}$$

2. Solve for b:

$$\begin{aligned} & 5b = 10 \end{aligned}$$

$$\begin{aligned} & b = \frac{10}{5} \end{aligned}$$

$$\begin{aligned} & b = 2 \end{aligned}$$

Interpretation of the Result

The scalar $b = 2$ means that multiplying vector B by 2 and then adding to vector A results in a new vector that has no y-component, thus pointing purely along the x-axis.

Verifying the Solution

Calculating the Resultant Vector

Substitute $b = 2$ into the combined vector:

$$\begin{aligned} \mathbf{A} + 2\mathbf{B} &= (4 + 7 \times 2)\mathbf{i} + (-10 + 5 \times 2)\mathbf{j} \end{aligned}$$

Calculate each component:

- x-component:

$$\begin{aligned} & 4 + 14 = 18 \end{aligned}$$

\]

- y-component:

\[

$$-10 + 10 = 0$$

\]

Resultant vector:

\[

\boxed{

$$\mathbf{R} = 18 \mathbf{i} + 0 \mathbf{j}$$

}

\]

This confirms that the vector points along the x-axis, as the y-component is zero.

Magnitude of the Resultant Vector

The magnitude of R is:

\[

$$|\mathbf{R}| = \sqrt{(18)^2 + 0^2} = 18$$

\]

This is purely a horizontal vector, consistent with our goal.

Geometric Interpretation

Visualizing the Solution

- Starting with vector A, which points to (4, -10).
- Adding scalar multiple $b = 2$ of vector B shifts the vector's tip horizontally until it aligns along the x-axis.
- The process can be viewed as adjusting the length and direction of B (scaled by 2) to "cancel out" the y-component of A.

Significance of the Scalar b

- The scalar $b = 2$ indicates how many times B must be scaled before its addition to A results in a vector aligned along the x-axis.
- If b were negative, the scaled vector B would point in the opposite direction, which may alter the interpretation but still satisfy the condition for the y-component being zero.

Practical Applications of the Concept

Understanding how to manipulate vectors to achieve desired directional properties has numerous real-world applications:

1. Physics and Force Resolution

- When analyzing forces acting on an object, resolving forces into components allows engineers to understand how different forces contribute to motion along specific axes.
- For example, adjusting the magnitude of a force vector B to counteract a component of another force A to achieve equilibrium.

2. Navigation and Course Correction

- Pilots or ships can adjust their heading vectors by adding or subtracting component vectors to stay on a desired course.
- Calculating the necessary correction involves similar vector addition and scalar multiplication.

3. Robotics and Motion Planning

- Robots often need to execute movements along specific axes.
- Planning movement vectors involves combining multiple vectors and ensuring the resultant aligns with the desired direction.

4. Computer Graphics and Animation

- Moving objects along certain paths requires precise vector calculations.
- Adjusting vectors ensures objects move along specific axes or trajectories.

Extending the Problem: Additional Considerations

Varying the Vectors

- What if the vectors A and B are different? How does that affect the scalar b?
- The method remains the same: set the y-component of $A + bB$ to zero and solve for b.

Multiple Vectors and Constraints

- In scenarios with multiple vectors influencing the resultant direction, systems of equations must be solved.
- Techniques such as matrix methods or vector projection can be employed.

Directional Magnitude and Angle

- Beyond just zero y-component, one might want the resultant vector to point along a specific angle.
- This involves setting the ratio of y-component to x-component to match the tangent of that angle.

Additional Examples

Example 1: Finding b for a Different Pair of Vectors

Suppose $A = 3i + 2j$ and $B = 4i - j$, find b such that $A + bB$ points along the x-axis.

Solution:

Set the y-component to zero:

$$A_y + b B_y = 0$$

$$2 + b(-1) = 0$$

$$-1b = -2$$

$$b = 2$$

Resultant vector:

$$(3 + 4 \times 2) i + (2 - 1 \times 2) j = (3 + 8)i + (2 - 2)j = 11i + 0j$$

Example 2: Vector Along a Specific Direction

Suppose you want $A + bB$ to point at a 45-degree angle to the x-axis. How would you determine b?

Solution:

Since the vector makes a 45-degree angle:

$$\tan 45^\circ = 1 = \frac{\text{y-component}}{\text{x-component}}$$

Expressing the components:

$$\frac{A_y + b B_y}{A_x + b B_x} = 1$$

Plug in known values:

$$\frac{-10 + 5b}{4 + 7b} = 1$$

Solve for b:

$$-10 + 5b = 4 + 7b$$

$$-10 - 4 = 7b - 5b$$

Frequently Asked Questions

Given vectors $A=4i-10j$ and $B=7i+5j$, how can we find the scalar B such that the vector $A + B \cdot B$ points along the X-axis?

To ensure $A + B \cdot B$ points along the X-axis, its y-component must be zero. Set the y-component of $A + B \cdot B$ to zero and solve for B: $-10 + 5B = 0$, so $B = 2$.

What is the significance of setting the y-component of $A + B \cdot B$ to zero in this problem?

Setting the y-component to zero ensures that the resultant vector points purely along the X-axis, meaning it has no vertical (y) component.

How do you verify that the vector $A + B \cdot B$ points along the X-axis after finding B?

After calculating B, substitute it back into $A + B \cdot B$ and check that the y-component is zero, confirming the vector points along the X-axis.

What is the value of B that makes $A + B \cdot B$ align along the X-axis?

The value of B is 2.

Can the same method be used to find B if A and B are arbitrary vectors?

Yes, setting the y-component of $A + B \cdot B$ to zero and solving for B is a general approach for such problems involving vector direction alignment.

What steps are involved in solving for B in this problem?

Step 1: Write the combined vector $A + B$. Step 2: Set its y-component to zero. Step 3: Solve the resulting equation for B. Step 4: Verify the solution by checking the y-component is zero.

Is there a specific reason for choosing vector A as $4i - 10j$ in this problem?

The specific components provide a concrete example for applying the method of adjusting B to align the resultant vector along the X-axis.

What is the general formula for B if the initial vectors are different?

Generally, $B = -(\text{initial y-component of A}) / (\text{coefficient of B in the y-component of the combined vector})$.

How do the components of the vectors influence the solution for B?

The y-components determine the value of B needed to cancel out the vertical component, ensuring the resultant vector points along the X-axis.

Additional Resources

Understanding the Problem Statement

In the realm of vector algebra, operations involving vectors such as addition, scalar multiplication, and their geometric implications are foundational. The problem at hand presents two vectors:

- $A = 4i - 10j$
- $B = 7i + 5j$

and asks us to find a scalar B such that the vector $A + bB$ points along the X-axis.

At first glance, the problem appears straightforward: identify the scalar multiple of B that, when added to A, results in a vector with no Y-component, i.e., a vector pointing purely along the X-axis.

This task involves understanding the vector components, scalar multiplication, and the conditions for a vector to align with a specific axis.

Clarifying the Notation and Terminology

Vector Components and Notation

- i and j are the standard unit vectors in the Cartesian coordinate system.
- i : unit vector along the X-axis.
- j : unit vector along the Y-axis.
- A and B are expressed in component form:
- $A = 4i - 10j$ translates to $A = (4, -10)$
- $B = 7i + 5j$ translates to $B = (7, 5)$

Scalar Multiplication

- When we multiply a vector by a scalar b , each component is scaled:
- $bB = (b \cdot 7, b \cdot 5)$
- The sum of two vectors is component-wise:
- $A + bB = (4 + 7b, -10 + 5b)$

Objective

The goal is to determine b such that $A + bB$ points along the X-axis only, meaning:

- The Y-component of $A + bB$ must be zero:

$$\begin{aligned} & \backslash \\ & -10 + 5b = 0 \\ & \backslash \end{aligned}$$

- The X-component can be any value (positive, negative, or zero), as long as the vector has no Y-component.

Geometric Interpretation of the Problem

Vector Addition and Direction

- When vectors are added, the resultant vector's direction depends on the components.
- For the resultant $A + bB$ to point along the X-axis:
- Its Y-component must be zero.
- Its X-component can take any value, as that determines the direction along the X-axis.

Visualizing the Solution

- Starting at the tip of vector A , we "move" along scalar multiples of B .
- The scalar b acts as a "tuning" parameter, shifting the resultant vector's Y-component to zero.
- Once the Y-component is nullified, the resulting vector lies purely along the X-axis.

Deriving the Value of b

Step-by-Step Calculation

Given:

$$\begin{aligned} & \backslash \\ A + bB &= (4 + 7b, -10 + 5b) \\ & \backslash \end{aligned}$$

To ensure the vector points along the X-axis:

$$\begin{aligned} & \backslash \\ -10 + 5b &= 0 \\ & \backslash \end{aligned}$$

Solve for b:

$$\begin{aligned} & \backslash \\ 5b &= 10 \\ & \backslash \\ & \backslash \\ b &= \frac{10}{5} = 2 \\ & \backslash \end{aligned}$$

Result

The scalar b that satisfies the condition is 2.

Verifying the Result

Computing $A + 2B$

Substitute $b = 2$ into the expression:

$$\begin{aligned} & \backslash \\ A + 2B &= (4 + 7 \times 2, -10 + 5 \times 2) = (4 + 14, -10 + 10) = (18, 0) \\ & \backslash \end{aligned}$$

- The resultant vector is (18, 0).
- It points along the X-axis, confirming the correctness.

Direction and Magnitude

- The magnitude of the resultant vector:

$$\begin{aligned} & \backslash \\ |A + 2B| &= \sqrt{(18)^2 + 0^2} = 18 \\ & \backslash \end{aligned}$$

- The vector points along the positive X-axis, indicating the solution is valid.

Broader Implications and Related Concepts

Scalar Multiplication and Vector Direction

- The scalar b scales the vector B , changing its magnitude but not its direction unless b is negative.
- When adding bB to A , the resultant vector's direction is influenced by both vectors' components.

Conditions for a Vector to Point Along an Axis

- To point along the X-axis, the Y-component must be zero.
- The X-component can be positive or negative, depending on the context or the specific problem constraints.

General Approach for Similar Problems

1. Identify the components of the vectors involved.
2. Set the undesired component (here, the Y-component) of the resultant vector to zero.
3. Solve for the scalar multiplier that satisfies this condition.
4. Verify by substituting back into the vector sum.

Extending the Problem: What if the Vector B Had Different Components?

Suppose B was different, for example:

$$\begin{aligned} \backslash \\ B &= b_i + c_j \\ \backslash \end{aligned}$$

and the goal was to find b such that:

$$\begin{aligned} \backslash \\ A + bB &= (X, 0) \\ \backslash \end{aligned}$$

then:

$$\begin{aligned} \backslash \\ (4 + b b, -10 + b c) &= (X, 0) \\ \backslash \end{aligned}$$

- To nullify the Y-component:

$$\begin{aligned} \backslash \\ -10 + b c &= 0 \Rightarrow b = \frac{10}{c} \\ \backslash \end{aligned}$$

- The X-component then becomes:

$$\begin{aligned} \backslash \\ 4 + b b &= 4 + \frac{10}{c} \times b \\ \backslash \end{aligned}$$

which depends on b and the components of B .

This generalizes the problem, illustrating how scalar multiples can be used to align vectors along specific axes or directions.

Practical Applications of This Concept

Robotics and Navigation

- Adjusting a movement vector to eliminate lateral deviation, ensuring a robot moves directly towards a target along a specific axis.

Physics and Force Resolution

- Resolving forces into components to achieve a net force in a desired direction, such as eliminating vertical components to ensure horizontal motion.

Computer Graphics

- Aligning vectors to specific axes for rendering or animation purposes.

Summary and Key Takeaways

- Problem Restatement: Find scalar b such that $A + bB$ points along the X-axis.
- Methodology:
 - Express vector components.
 - Set the Y-component of the sum to zero.
 - Solve for b .
- Result:
 - $b = 2$ ensures $A + 2B$ points along the X-axis.
 - The resultant vector in this case is $(18, 0)$, purely along the X-axis.
- Broader Significance:
 - Highlights the utility of scalar multiplication and vector addition in controlling vector directions.
 - Demonstrates how algebraic solutions translate into geometric interpretations.

Final Remarks

This problem exemplifies essential principles in vector algebra, emphasizing how scalar manipulations can control directionality. Understanding these techniques is fundamental for students and professionals working in fields like physics, engineering, computer graphics, and robotics.

The key takeaway is that by setting the undesired component (Y-component) to zero and solving for the scalar, one can precisely control the orientation of vectors in a two-dimensional plane. This approach forms the basis for more complex vector manipulations and problem-solving strategies across various scientific and engineering disciplines.

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