Given G Of X Equals Cube Root Of The Quantity X Plus 6, On What Interval Is The Function Negative? (,

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Understanding the behavior of functions is fundamental in calculus and algebra, especially when analyzing their positivity and negativity over specific intervals. In this article, we focus on the function $G(x) = \sqrt[3]{x + 6}$, exploring the intervals where this function takes negative values. We will delve into the properties of cube root functions, analyze their domain, and determine the exact interval where $G(x)$ is less than zero.

Overview of the Cube Root Function

Definition of the Cube Root Function

The cube root function, denoted as $\sqrt[3]{x}$, is a mathematical function that maps any real number x to its cube root. Unlike square roots, which are only defined for non-negative numbers, cube roots are defined for all real numbers because every real number has a unique real cube root.

Mathematically, the cube root function is expressed as:
$$
f(x) = \sqrt[3]{x}
$$
and it possesses the following properties:
- Domain: $(-\infty, \infty)$
- Range: $(-\infty, \infty)$
- Even/Odd: The cube root function is an odd function, satisfying $\sqrt[3]{-x} = -\sqrt[3]{x}$.

Properties Relevant to Our Function $G(x)$

Our specific function is:
$$
G(x) = \sqrt[3]{x + 6}
$$
which is a shifted version of the basic cube root function. The properties that influence its negativity include:
- The domain remains all real numbers, as cube roots are defined everywhere.

- The function is continuous and strictly increasing across its entire domain.
- The sign of $G(x)$ depends on the sign of its argument $x + 6$.

Analyzing the Function $G(x) = \sqrt[3]{x + 6}$

Domain of $G(x)$

Since the cube root function is defined for all real numbers, the domain of $G(x)$ is:
$$
\boxed{(-\infty, \infty)}
$$
regardless of the value of x.

Range of $G(x)$

Similarly, because the cube root covers all real numbers, the range is:
$$
\boxed{(-\infty, \infty)}
$$

Sign of $G(x)$

The sign of $G(x)$ depends on the sign of the expression inside the cube root:
$$
x + 6
$$
- If $x + 6 > 0$, then $G(x) > 0$.
- If $x + 6 = 0$, then $G(x) = 0$.
- If $x + 6 < 0$, then $G(x) < 0$.

This leads to a straightforward criterion for the negativity of the function.

Determining When $G(x)$ Is Negative

Setting Up the Inequality

To find the interval where $G(x)$ is negative, we analyze:
$$

$$
G(x) < 0
$$
which simplifies to:
$$
\sqrt[3]{x + 6} < 0
$$

Solving the Inequality

Since the cube root function is strictly increasing, the inequality $\sqrt[3]{x + 6} < 0$ is equivalent to:
$$
x + 6 < 0
$$
because applying the cube root preserves the inequality's direction.

Solving this:
$$
x + 6 < 0 \implies x < -6
$$

Conclusion on the Interval

Therefore, the function $G(x)$ is negative precisely when $x < -6$.

Summary of Results

Condition	Inequality	Solution Interval
$G(x) < 0$	$\sqrt[3]{x + 6} < 0$	$x < -6$
$G(x) = 0$	$\sqrt[3]{x + 6} = 0$	$x = -6$
$G(x) > 0$	$\sqrt[3]{x + 6} > 0$	$x > -6$

Hence, the domain where the function $G(x)$ is negative is $(-\infty, -6)$.

Visual Representation of $G(x)$

Visualizing the function helps to understand its behavior intuitively.

Graph of $G(x) = \sqrt[3]{x + 6}$

- The graph is a shifted cube root curve, shifting the basic $\sqrt[3]{x}$ to the left by 6 units.

- The point $x = -6$ corresponds to $G(-6) = \sqrt[3]{0} = 0$.
- For $x < -6$, the graph lies below the x-axis, indicating negative $G(x)$.
- For $x > -6$, the graph lies above the x-axis, indicating positive $G(x)$.

Applications and Significance

Understanding where $G(x)$ is negative is essential in various contexts, including:

- Mathematical Analysis: Helps in solving inequalities involving cube root functions.
- Physics: When modeling phenomena where the cube root term represents some physical quantity that can be negative.
- Engineering: For signal processing or control systems where negative values indicate specific states.

Additional Insights and Related Concepts

Continuity and Differentiability

- $G(x)$ is continuous and differentiable everywhere because cube root functions are smooth across their domain.
- The derivative:

$$
G'(x) = \frac{1}{3} (x + 6)^{-\frac{2}{3}}
$$

which exists for all $x \neq -6$, highlighting a potential point of non-differentiability at $x = -6$.

Behavior at $x = -6$

- The function crosses zero at $x = -6$.
- The function is increasing across its entire domain, with no local maxima or minima.

Summary and Final Remarks

In conclusion, for the function $G(x) = \sqrt[3]{x + 6}$, the interval on which the function is negative is:

\[

\boxed{(-\infty, -6)}
\]
This is derived directly from the properties of cube root functions and the inequality \(x + 6 < 0 \). Recognizing the relationship between the inside of the cube root and the sign of the function is key to solving such problems effectively.

Understanding the intervals of positivity and negativity of functions like \(G(x) \) is crucial in many mathematical applications, from solving inequalities to analyzing real-world data. The cube root's property of being defined for all real numbers simplifies the analysis compared to other roots, making it a valuable tool in various fields.

Frequently Asked Questions

What is the domain of the function $G(x) = \sqrt[3]{(x + 6)}$?

The domain of $G(x) = \sqrt[3]{(x + 6)}$ is all real numbers, since cube root functions are defined for all real x.

How do you determine where the function $G(x) = \sqrt[3]{(x + 6)}$ is negative?

Since the cube root function preserves the sign of its argument, $G(x)$ is negative where $x + 6 < 0$, i.e., $x < -6$.

On what interval is $G(x) = \sqrt[3]{(x + 6)}$ negative?

$G(x)$ is negative on the interval $(-\infty, -6)$.

Is $G(x) = \sqrt[3]{(x + 6)}$ continuous and differentiable?

Yes, the cube root function is continuous and differentiable for all real numbers, including $x + 6$.

Does the function $G(x) = \sqrt[3]{(x + 6)}$ have any points where it is zero?

Yes, $G(x) = 0$ when $x + 6 = 0$, i.e., at $x = -6$.

How does the sign of $G(x)$ change at $x = -6$?

At $x = -6$, $G(x)$ changes from negative (for $x < -6$) to zero at $x = -6$, and positive for $x > -6$.

Can the function $G(x) = \sqrt[3]{x + 6}$ be negative for $x > -6$?

No, $G(x)$ is only negative when $x < -6$; for $x > -6$, $G(x)$ is positive.

What is the behavior of $G(x)$ as x approaches $-\infty$ and ∞?

As x approaches $-\infty$, $G(x)$ approaches $-\infty$; as x approaches ∞, $G(x)$ approaches ∞.

Is the interval where $G(x)$ is negative open, closed, or half-open?

The interval where $G(x)$ is negative, $(-\infty, -6)$, is an open interval.

Additional Resources

Given G of X equals cube root of the quantity X plus 6, on what interval is the function negative? This question invites a thorough exploration of the function $G(x) = \sqrt[3]{x + 6}$, focusing on understanding where this function takes on negative values. Such an analysis is fundamental in calculus and algebra, providing insights into the behavior of functions, their domains, ranges, and the critical points where they change sign. In this article, we will delve deeply into the properties of this cube root function, examine the intervals of negativity, and highlight the key concepts and steps involved in such an analysis.

Understanding the Function $G(x) = \sqrt[3]{x + 6}$

Before addressing the core question, it's essential to understand the nature of the function itself.

Definition and Basic Properties

The function $G(x) = \sqrt[3]{x + 6}$ is a cube root function, which is a type of radical function. Its fundamental properties include:
- Domain: All real numbers, because cube roots are defined for all real inputs.
- Range: All real numbers, since the cube root function can produce any real output.
- Continuity: The cube root function is continuous everywhere.

- Differentiability: It is differentiable everywhere except possibly at points where the derivative may be undefined or zero.

Graphical Behavior

Graphing \(G(x) \) reveals a smooth curve passing through the point where \(x + 6 = 0 \), i.e., at \(x = -6 \). The graph:
- Passes through the point \((-6, 0) \).
- Is symmetric with respect to the origin in a sense, but not exactly odd symmetry due to the shift.
- Extends from negative infinity to positive infinity in both axes, reflecting the domain and range.

Determining When \(G(x) \) Is Negative

The core of this analysis lies in understanding when the function produces negative values, i.e., when \(G(x) < 0 \).

Step 1: Set the Function Less Than Zero

To find the interval(s) where the function is negative, solve the inequality:
\[
\sqrt[3]{x + 6} < 0
\]

Step 2: Interpret the Inequality

Since the cube root function is strictly increasing over all real numbers, the inequality \(\sqrt[3]{x + 6} < 0 \) is equivalent to:
\[
x + 6 < 0
\]
This is because the cube root preserves the order of inequalities.

Step 3: Solve for \(x \)

Subtract 6 from both sides:
\[
x < -6
\]
Thus, the function \(G(x) \) is negative for all \(x \) less than \(-6\).

Conclusion: The Interval of Negativity

Based on the above analysis, the function $G(x)$ is negative precisely on the interval:

$$
(-\infty, -6)
$$

This means for every real number less than -6, the cube root of $x + 6$ yields a negative value.

Additional Insights and Considerations

While the primary focus is on the interval where $G(x)$ is negative, understanding the behavior of the function outside this interval enriches the overall comprehension.

Behavior at $x = -6$

At $x = -6$:

$$
G(-6) = \sqrt[3]{-6 + 6} = \sqrt[3]{0} = 0
$$

The function crosses the x-axis at this point, transitioning from negative to positive as x increases past -6.

Behavior for $x > -6$

For $x > -6$:

$$
x + 6 > 0 \quad \Rightarrow \quad G(x) > 0
$$

The function is positive, reflecting the increasing nature of the cube root function for positive inputs.

Behavior for $x < -6$

For $x < -6$:

$$
x + 6 < 0 \quad \Rightarrow \quad G(x) < 0
$$

The function takes negative values, approaching $-\infty$ as $x \to -\infty$.

Features and Characteristics of $G(x)$

Understanding the characteristics of this function assists in more advanced analyses.

Features

- Continuity: The function is continuous across its entire domain.
- Monotonicity: It is strictly increasing, as the cube root function preserves order.
- Symmetry: It is an odd function shifted horizontally, but not perfectly symmetric about the origin.
- Intercepts: Passes through the origin at $x = -6$.

Pros and Cons

Pros:
- The function is straightforward to analyze due to its invertibility and continuous nature.
- Its behavior at critical points (like $x = -6$) is easy to determine.
- The inequality solution is simple because of the monotonic increasing property.

Cons:
- For more complex functions involving cube roots, the analysis may require additional techniques.
- The unbounded nature might be confusing for beginners without proper graphing tools.

Summary and Final Remarks

In conclusion, the function $G(x) = \sqrt[3]{x + 6}$ is negative on the interval $(-\infty, -6)$. This stems from the fundamental property of cube root functions being strictly increasing and defined for all real numbers. The critical point at $x = -6$ marks where the function crosses zero from negative to positive. Recognizing these properties not only answers the specific question but also reinforces core concepts in function analysis, such as inequalities, domain and range considerations, and the nature of radical functions.

Understanding the behavior of such functions is vital for students and professionals working in mathematics, physics, engineering, and related fields. Whether analyzing the sign of a function or exploring its graph, these principles serve as foundational tools for more advanced problemsolving and theoretical investigations.

In summary:
- The function \(G(x) = \sqrt[3]{x + 6} \) is negative for all \(x < -6 \).
- The interval where \(G(x) \) is negative is \(\boxed{(-\infty, -6)} \).
- The function crosses zero at \(x = -6 \), transitioning from negative to positive as \(x \) increases.

This detailed exploration underscores the importance of understanding the fundamental properties of functions and their graphical behaviors, equipping learners and practitioners with the skills to analyze similar problems confidently.

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