Given The Matrices A=[[-2,1],[5,6]] And B=[[5,-5],[-1,0]], Find The Product AB As Well As The Product

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Understanding matrix multiplication is fundamental in various fields such as mathematics, engineering, computer science, and data analysis. When dealing with two matrices, A and B, the product AB is computed through a specific process involving the rows of A and columns of B. In this article, we will explore how to find the product of the matrices A and B, given by:

A = \(\begin{bmatrix} -2 & 1 \\ 5 & 6 \end{bmatrix}\)

B = \(\begin{bmatrix} 5 & -5 \\ -1 & 0 \end{bmatrix}\)

We will also clarify the meaning and calculation of the product BA, providing a comprehensive guide to matrix multiplication.

Understanding Matrix Multiplication

Basics of Matrix Multiplication

Matrix multiplication involves multiplying the rows of the first matrix by the columns of the second matrix. For two matrices A and B to be multipliable (i.e., for AB to be defined), the number of columns in A must equal the number of rows in B.

- If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then their product AB will be an $m \times p$ matrix.

In our case:
- Matrix A is 2×2
- Matrix B is 2×2

Since both are 2x2 matrices, both products AB and BA are defined and will result in 2x2 matrices.

Step-by-Step Calculation

The element in the ith row and jth column of the product matrix is calculated as:

$$
(AB)_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}
$$

where:
- A_{ik} is the element in row i, column k of A
- B_{kj} is the element in row k, column j of B

Now, let's apply this to our matrices.

Calculating the Product AB

Step 1: Write down the matrices

A = $\begin{bmatrix} -2 & 1 \\ 5 & 6 \end{bmatrix}$

B = $\begin{bmatrix} 5 & -5 \\ -1 & 0 \end{bmatrix}$

Step 2: Calculate each element of AB

- Element in position (1,1):

$$
AB_{11} = (-2) \times 5 + 1 \times (-1) = -10 - 1 = -11
$$

- Element in position (1,2):

$$
AB_{12} = (-2) \times (-5) + 1 \times 0 = 10 + 0 = 10
$$

- Element in position (2,1):

$$
AB_{21} = 5 \times 5 + 6 \times (-1) = 25 - 6 = 19
$$

- Element in position (2,2):

\[

$$
AB_{22} = 5 \times (-5) + 6 \times 0 = -25 + 0 = -25
$$

Step 3: Write the resulting matrix

$$
AB = \begin{bmatrix} -11 & 10 \\ 19 & -25 \end{bmatrix}
$$

Calculating the Product BA

Step 1: Write down the matrices in the order for BA

B = $\begin{bmatrix} 5 & -5 \\ -1 & 0 \end{bmatrix}$

A = $\begin{bmatrix} -2 & 1 \\ 5 & 6 \end{bmatrix}$

Step 2: Calculate each element of BA

- Element in position (1,1):

$$
BA_{11} = 5 \times (-2) + (-5) \times 5 = -10 - 25 = -35
$$

- Element in position (1,2):

\[
BA_{12} = 5 \times 1 + (-5) \times 6 = 5 - 30 = -25
\]

- Element in position (2,1):

\[
BA_{21} = (-1) \times (-2) + 0 \times 5 = 2 + 0 = 2
\]

- Element in position (2,2):

\[
BA_{22} = (-1) \times 1 + 0 \times 6 = -1 + 0 = -1
\]

Step 3: Write the resulting matrix

\[
BA = \begin{bmatrix} -35 & -25 \\ 2 & -1 \end{bmatrix}
\]

Summary of Results

- Product AB:

\[
\boxed{
AB = \begin{bmatrix} -11 & 10 \\ 19 & -25 \end{bmatrix}

}

\]

- Product BA:

\[

\boxed{

BA = \begin{bmatrix} -35 & -25 \\ 2 & -1 \end{bmatrix}

}

\]

Key Takeaways and Additional Insights

Order Matters in Matrix Multiplication

One of the most crucial aspects of matrix multiplication is that it is not commutative; that is, generally, \(AB \neq BA\). Our calculations clearly demonstrate this, as the products AB and BA are different matrices.

Applications of Matrix Multiplication

Matrix multiplication is used extensively in various domains:

- Computer Graphics: Transforming coordinates and applying rotations or scaling.
- Data Science & Machine Learning: Operations involving weight matrices in neural networks.
- Engineering: Systems modeling and signal processing.
- Mathematics: Solving systems of equations and linear transformations.

Further Topics to Explore

- Matrix Inversion: Finding the inverse of matrices when they are invertible.
- Determinants and Eigenvalues: Analyzing properties of matrices.
- Matrix Transpose: Flipping rows and columns.
- Matrix Decompositions: Such as LU, QR, and SVD for advanced computations.

Conclusion

Calculating the product of matrices A and B involves systematic multiplication of rows and columns, following the rules of linear algebra. In this instance, we derived the products AB and BA, revealing their unique matrices. Understanding these operations is essential for more complex mathematical tasks and practical applications across various scientific and engineering fields.

Remember: Always confirm the dimensions of the matrices before multiplying and be mindful that matrix multiplication is not commutative—meaning AB and BA can yield very different results. With practice, mastering these computations becomes intuitive and forms the foundation for more advanced topics in linear algebra.

Frequently Asked Questions

What is the product of matrices A = [[-2, 1], [5, 6]] and B = [[5, -5], [-1, 0]]?

The product AB is calculated as follows:

First row, first column: (-2)(5) + (1)(-1) = -10 - 1 = -11

First row, second column: (-2)(-5) + (1)(0) = 10 + 0 = 10

Second row, first column: (5)(5) + (6)(-1) = 25 - 6 = 19

Second row, second column: (5)(-5) + (6)(0) = -25 + 0 = -25

Therefore, AB = [[-11, 10], [19, -25]].

How do you compute the product of two 2x2 matrices?

To compute the product of two 2x2 matrices, multiply rows of the first matrix by columns of the second matrix and sum the products for each position. Specifically, for matrices A and B:

If A = [[a11, a12], [a21, a22]] and B = [[b11, b12], [b21, b22]],

then AB = [[a11b11 + a12b21, a11b12 + a12b22], [a21b11 + a22b21, a21b12 + a22b22]].

What is the significance of the order in matrix multiplication for matrices A and B?

Matrix multiplication is not commutative, meaning AB ≠ BA in general. The order matters because the product depends on the sequence, and reversing the order can lead to different results.

Can the matrices A = [[-2, 1], [5, 6]] and B = [[5, -5], [-1, 0]] be multiplied in reverse order?

Yes, you can multiply B by A (BA), but the resulting product will generally be different from AB. Calculating BA involves similar steps, but with the matrices swapped.

What is the product of matrices B = [[5, -5], [-1, 0]] and A = [[-2, 1], [5, 6]]?

Calculating BA:

First row, first column: (5)(-2) + (-5)(5) = -10 - 25 = -35

First row, second column: (5)(1) + (-5)(6) = 5 - 30 = -25

Second row, first column: (-1)(-2) + (0)(5) = 2 + 0 = 2

Second row, second column: (-1)(1) + (0)(6) = -1 + 0 = -1

Therefore, BA = [[-35, -25], [2, -1]].

How do you verify the correctness of matrix multiplication results?

You can verify by manually calculating each entry as shown, or using matrix multiplication functions in software tools like MATLAB, Python (NumPy), or calculators to cross-check results.

Are there any special properties of the matrices A and B that affect their product?

Given that neither matrix is diagonal, symmetric, or singular, the product is straightforward. The matrices are general 2x2 matrices, so standard multiplication applies without special properties affecting the calculation.

What is the importance of matrix multiplication in practical applications?

Matrix multiplication is fundamental in various fields such as computer graphics, engineering, physics, and machine learning. It is used to transform data, solve systems of equations, and perform linear mappings.

Can the product of matrices A and B be used to determine if they are invertible?

The invertibility of the product AB depends on whether both A and B are invertible. If both are invertible, then their product is also invertible. To check invertibility, compute their determinants; if both determinants are non-zero, they are invertible.

Additional Resources

Matrix multiplication stands as a fundamental operation in linear algebra, underpinning numerous applications across engineering, computer science, physics, and data analysis. When two matrices are multiplied, the operation combines their elements in a structured manner, producing a new matrix that encapsulates the combined transformations or relationships they represent. In this article, we delve deep into the specifics of multiplying two given matrices, A and B, with the goal of determining the product AB. We will explore the process step-by-step, analyze the results, and discuss the broader implications of matrix multiplication in mathematical and real-world contexts.

Understanding the Given Matrices

Before embarking on the multiplication process, it’s essential to understand the matrices involved and their properties. The matrices provided are:

Matrix A:

$$
A = \begin{bmatrix}
-2 & 1 \\
5 & 6
\end{bmatrix}
$$

Matrix B:

$$
B = \begin{bmatrix}
5 & -5 \\
-1 & 0
\end{bmatrix}
$$

Both matrices are 2x2 matrices, meaning they each have two rows and two columns. The dimensions are crucial because, in matrix multiplication, the number of columns in the first matrix must match the number of rows in the second matrix. Since both matrices are 2x2, their multiplication is well-defined, resulting in another 2x2 matrix.

Properties and Significance of the Matrices

- Matrix A:
- The elements are: top row $(-2, 1)$, bottom row $(5, 6)$.
- The matrix isn't symmetric or diagonal, indicating it may represent a transformation with shearing or scaling components.
- The determinant of A is $(-2)(6) - (1)(5) = -12 - 5 = -17$. Since the determinant is non-zero, A is invertible.

- Matrix B:
- Elements are: top row $(5, -5)$, bottom row $(-1, 0)$.
- It has a zero in the lower-right position, which could indicate a certain form of transformation, possibly a shear or a partial rotation.

Understanding the structure of these matrices helps in interpreting the resultant matrix after multiplication, especially in applications like computer graphics or systems of equations.

Step-by-Step Matrix Multiplication

The core of this article is to compute the product AB, which involves multiplying matrix A by matrix B in that order.

The Mathematical Process

Given matrices:

$$
A = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
= \begin{bmatrix}
-2 & 1 \\
5 & 6
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix}
= \begin{bmatrix}
5 & -5 \\
-1 & 0
\end{bmatrix}
$$

The product AB is computed as:

$$
AB = \begin{bmatrix}
a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\
a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22}
\end{bmatrix}
$$

Breaking this down into steps:

1. Calculating the element at position (1,1):

$$
(-2)(5) + (1)(-1) = -10 - 1 = -11
$$

2. Calculating the element at position (1,2):

$$
(-2)(-5) + (1)(0) = 10 + 0 = 10
$$

3. Calculating the element at position (2,1):

$$
(5)(5) + (6)(-1) = 25 - 6 = 19
$$

4. Calculating the element at position (2,2):

$$
(5)(-5) + (6)(0) = -25 + 0 = -25
$$

Final Result of the Product AB

$$
AB = \begin{bmatrix}
-11 & 10 \\
19 & -25
\end{bmatrix}
$$

This matrix encapsulates the combined effect of the transformations represented by matrices A and B when applied sequentially.

Interpretation and Analysis of the Result

The resulting matrix:

$$
AB = \begin{bmatrix}
-11 & 10 \\
19 & -25
\end{bmatrix}
$$

has several noteworthy properties:

Mathematical Properties

- Determinant:

$$
\det(AB) = (-11)(-25) - (10)(19) = 275 - 190 = 85
$$

The determinant being positive and non-zero indicates that the combined transformation is invertible and preserves orientation (no reflection).

- Eigenvalues and Eigenvectors:

Analyzing the eigenvalues of AB can reveal the nature of the transformation—whether it involves scaling, rotation, shear, or a combination thereof.

Geometric Interpretation

- The original matrices A and B can be viewed as linear transformations in a 2D space:
- A could represent a shear and scaling.
- B might represent rotation and scaling.
- Their product, AB, embodies the combined transformation, which could involve a more complex distortion or scaling pattern.

- The signs and magnitudes of the entries suggest that the transformation involves reflection or inversion components, along with stretching/shrinking along certain axes.

Practical Implications

Understanding the product of matrices like A and B is critical in fields such as:

- Computer Graphics: Combining transformations like rotation, scaling, and shearing to manipulate images or models.
- Control Systems: Modeling the combined effect of different system parameters.
- Physics: Describing compound transformations in space, such as successive rotations or reflections.
- Data Science: Applying multiple linear transformations to datasets for normalization or feature extraction.

Broader Context: The Significance of Matrix Operations

Matrix multiplication isn't merely a computational task; it's a gateway to understanding how complex systems evolve or how different transformations combine. In many scientific domains, the ability to accurately multiply matrices enables scientists and engineers to simulate real-world phenomena, optimize systems, and analyze data.

Matrix Multiplication in System Modeling

In systems theory, matrices are used to represent state transitions. Multiplying these matrices models the evolution of a system over time or through stages. For example, in a Markov process, the transition matrix, when multiplied by a state vector, predicts future states.

Applications in Computer Graphics and Animation

Transformations in 3D space—rotation, scaling, translation—are represented by matrices. Combining these matrices allows for complex animations and rendering. The multiplication of transformation matrices ensures that multiple effects can be applied sequentially and efficiently.

Theoretical Significance

The process of matrix multiplication also illustrates fundamental algebraic properties:
- Associativity: $(AB)C = A(BC)$
- Distributivity: $A(B + C) = AB + AC$
- Non-commutativity: Generally, $AB \neq BA$, which underscores the importance of order in transformations.

Challenges and Considerations

While matrix multiplication is straightforward conceptually, it can become computationally intensive for larger matrices, especially in high-dimensional data. Optimized algorithms and computational techniques, such as Strassen's algorithm, are used to accelerate calculations.

Conclusion: The Power of Matrix Operations

The detailed calculation of the product AB from matrices A and B exemplifies the core principles of linear algebra and their profound impact across disciplines. The resulting matrix encapsulates a compounded transformation, with properties that reveal much about the underlying systems or models it represents. Whether in theoretical mathematics, engineering, or applied sciences, understanding how to correctly and efficiently multiply matrices is fundamental.

This exploration underscores the importance of mastering matrix operations—not only as a mathematical exercise but as a vital tool for analyzing and navigating complex systems. As technology advances and datasets grow larger, the ability to manipulate and interpret matrices will remain a cornerstone of scientific progress and innovation.

In summary, the product AB, derived from the given matrices, is:

\[

\boxed{

\begin{bmatrix}

-11 & 10 \\

19 & -25

\end{bmatrix}

}

\]

and serves as a testament to the power and elegance of linear algebra in modeling and understanding the world around us.

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