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Understanding the concepts of row space and column space of matrices is fundamental in linear algebra. These concepts are crucial for solving systems of linear equations, analyzing linear transformations, and understanding the structure of matrices. In this detailed guide, we will explore how to find bases for both the row space and the column space of the matrix:

\[C = \begin{bmatrix}
4 & 5 & 9 & -2 \\
6 & 5 & 1 & 2 \\
3 & 4 & 8 & -3
\end{bmatrix} \]

By systematically applying row operations, reducing the matrix to its row echelon form, and identifying pivotal elements, we can determine the basis vectors for these subspaces.

Understanding Row Space and Column Space

Before diving into calculations, it's essential to understand what the row space and column space represent.

Row Space

- The row space of a matrix is the subspace spanned by its row vectors.
- It contains all possible linear combinations of the row vectors.
- The basis for the row space can be obtained by selecting the linearly independent rows from the matrix.

Column Space

- The column space of a matrix is the subspace spanned by its column vectors.
- It includes all linear combinations of the columns.
- The basis for the column space can be derived from the columns corresponding to the pivot positions in the matrix's row echelon form.

Step-by-Step Procedure to Find Bases

The process involves several systematic steps:

1. Write down the matrix C.
2. Perform Gaussian elimination to reduce C to row echelon form.
3. Identify pivot positions (leading entries).
4. Determine basis for the row space from the non-zero rows of the reduced matrix.
5. Identify the columns in the original matrix corresponding to pivot columns.
6. Extract those columns to form the basis for the column space.

Step 1: Write Down the Matrix C

Given the matrix:

$$C = \begin{bmatrix}
4 & 5 & 9 & -2 \\
6 & 5 & 1 & 2 \\
3 & 4 & 8 & -3
\end{bmatrix}$$

Step 2: Row Reduction to Row Echelon Form

Applying Gaussian elimination:

First Pivot (Row 1, Column 1):

- The first pivot is 4 in position (1,1).

- Use row operations to eliminate entries below the pivot:

$$
R_2 \leftarrow R_2 - \frac{6}{4} R_1 = R_2 - 1.5 R_1
$$
$$
R_3 \leftarrow R_3 - \frac{3}{4} R_1 = R_3 - 0.75 R_1
$$

Calculations:

- Row 2:

$$
6 - 1.5 \times 4 = 6 - 6 = 0
$$

$$
5 - 1.5 \times 5 = 5 - 7.5 = -2.5
$$

$$
1 - 1.5 \times 9 = 1 - 13.5 = -12.5
$$

$$
2 - 1.5 \times (-2) = 2 + 3 = 5
$$

- Row 3:

$$
3 - 0.75 \times 4 = 3 - 3 = 0
$$

$$
4 - 0.75 \times 5 = 4 - 3.75 = 0.25
$$

$$
8 - 0.75 \times 9 = 8 - 6.75 = 1.25
$$

$$
-3 - 0.75 \times (-2) = -3 + 1.5 = -1.5
$$

Updated matrix:

$$
\begin{bmatrix}
4 & 5 & 9 & -2 \\
0 & -2.5 & -12.5 & 5 \\
0 & 0.25 & 1.25 & -1.5
\end{bmatrix}
$$

Second Pivot (Row 2, Column 2):

- Make the leading entry in row 2 equal to 1:

$$
R_2 \leftarrow R_2 / -2.5
$$

$$
R_2 = \left[0, 1, 5, -2 \right]
$$

Updated matrix:

$$
\begin{bmatrix}
4 & 5 & 9 & -2 \\
0 & 1 & 5 & -2 \\
0 & 0.25 & 1.25 & -1.5
\end{bmatrix}
$$

Eliminate entry below the second pivot (Row 3):

$$
R_3 \leftarrow R_3 - 0.25 R_2
$$

Calculations:

$$
0.25 - 0.25 \times 1 = 0.25 - 0.25 = 0
$$

$$
1.25 - 0.25 \times 5 = 1.25 - 1.25 = 0
$$

$$
-1.5 - 0.25 \times (-2) = -1.5 + 0.5 = -1
$$

Updated matrix:

$$
\begin{bmatrix}
4 & 5 & 9 & -2 \\
0 & 1 & 5 & -2 \\
0 & 0 & 0 & -1
\end{bmatrix}
$$

Third Pivot (Row 3, Column 4):

- Make the leading coefficient in row 3 equal to 1:

\[

R_3 \leftarrow R_3 / -1

\]

\[

R_3 = [0, 0, 0, 1]

\]

Final row echelon form:

\[

\begin{bmatrix}

4 & 5 & 9 & -2 \\

0 & 1 & 5 & -2 \\

0 & 0 & 0 & 1

\end{bmatrix}

\]

Step 3: Identify Pivot Positions

- Pivot positions are at:

- (1,1) in row 1
- (2,2) in row 2
- (3,4) in row 3

- These indicate the pivot columns are columns 1, 2, and 4.

Step 4: Find Basis for the Row Space

- The basis for the row space is formed by the non-zero rows of the row echelon form:

\[

\left\{

\begin{bmatrix}

4 & 5 & 9 & -2

\end{bmatrix},

\begin{bmatrix}

0 & 1 & 5 & -2

\end{bmatrix},

\begin{bmatrix}

0 & 0 & 0 & 1

\end{bmatrix}

\right\}

\]

- These vectors are linearly independent and span the row space.

Final basis for the row space:

\[

\boxed{

\left\{

(4, 5, 9, -2), \quad (0, 1, 5, -2), \quad (0, 0, 0, 1)

\right\}

}

\]

Step 5: Find Basis for the Column Space

- The basis vectors for the column space are the original columns of C corresponding to the pivot columns in the row echelon form, which are columns 1, 2, and 4 of the original matrix:

$$
\textbf{Column 1:} \quad \begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix}
$$
$$
\textbf{Column 2:} \quad \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix}
$$
$$
\textbf{Column 4:} \quad \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}
$$

Final basis for the column space:

$$
\boxed{
\left\{
\begin{bmatrix}
4 \\ 6 \\ 3
\end{bmatrix},
\begin{bmatrix}
5 \\ 5 \\ 4
\end{bmatrix},
\begin{bmatrix}
-2 \\ 2 \\ -3
\end{bmatrix}
\right\}
}

\]

Summary of Results

- Basis for the Row Space of \(C \):

\[

\boxed{

\left\{

(4, 5, 9, -2), \quad (0, 1, 5, -2), \quad (0, 0, 0

Frequently Asked Questions

What is the matrix C given in the problem?

The matrix C is:

[[459, -2],

[651, 12],

[3, 4, 8, -3]]

How do I find the bases for the row space of matrix C?

To find the basis for the row space, perform row operations to reduce C to row echelon form and identify the non-zero rows; these rows form the basis for the row space.

How can I determine the basis for the column space of matrix C?

The basis for the column space is given by the set of columns in the original matrix C that correspond to the pivot columns in its row echelon form.

What steps are involved in computing the row space basis for this matrix?

First, write down the matrix C; then, perform Gaussian elimination to reduce it to row echelon form; the non-zero rows of this form are the basis vectors for the row space.

Are the bases for the row space and column space always related?

Yes, the number of vectors in each basis equals the rank of the matrix, and the row space basis is derived from the row operations, while the column space basis comes from the pivot columns of the original matrix.

Can the third row of matrix C affect the bases for the row and column spaces?

Yes, since the third row contains four elements, it influences the row echelon form and the pivot columns, thus affecting the bases for both the row and column spaces.

Additional Resources

Given The Matrix C = $\begin{bmatrix} 459 & -2 \\ 651 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix}$: Find Bases for Both the Row Space and Column Space Of

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Introduction

In linear algebra, understanding the structure of matrices through their row and column spaces is fundamental. These subspaces reveal crucial information about the properties of the matrix, such as its rank, invertibility, and solutions to associated systems of equations. The problem at hand involves a matrix \(C \), which appears to be a non-conventional rectangular matrix with three rows and four columns, given the entries:

\[

C = \begin{bmatrix}

459 & -2 \\

651 & 12 \\

3 & 4 & 8 & -3

\end{bmatrix}

\]

However, the matrix as presented is ambiguous because the third row contains four entries, while the first two contain only two. To proceed systematically, this investigation will first clarify the matrix's structure, then compute bases for the row and column spaces, employing standard linear algebra techniques such as row reduction, rank determination, and basis extraction.

This in-depth analysis aims to serve as a comprehensive guide for students, educators, and researchers seeking to understand the methods for identifying bases of a matrix's row and column spaces, especially when dealing with matrices presented in an unconventional format.

Clarifying the Matrix Structure

The Matrix Representation

The given matrix is:

\[

C = \left[\begin{array}{cc}

459 & -2 \\

651 & 12 \\

3 & 4 & 8 & -3

\end{array}\right]

\]

This notation suggests a potential inconsistency because the third row has four entries, while the first two rows have only two. Standard matrices are rectangular with consistent row lengths. Possible interpretations include:

1. Typographical Error: The matrix may be intended as either:

- A \(3 \times 2\) matrix, with the third row having only two entries, or
- A \(3 \times 4\) matrix, with the first two rows missing entries.

2. Concatenation of two matrices: The matrix might be a block matrix or a combined structure.

3. Misprint or formatting issue: Possibly, the third row's entries are misaligned.

Assumption: Corrected Matrix

Given the context and typical matrix notation, the most reasonable assumption is that the third row is:

\[

3 \quad 4 \quad 8 \quad -3

\]

and that the first two rows are:

\[
\begin{bmatrix}
459 & -2 & 0 & 0 \\
651 & 12 & 0 & 0 \\
3 & 4 & 8 & -3
\end{bmatrix}
\]

Alternatively, it might be that the original matrix is intended to be of size \(3 \times 4\), with the first two rows being:

\[
\begin{bmatrix}
459 & -2 & 0 & 0 \\
651 & 12 & 0 & 0 \\
3 & 4 & 8 & -3
\end{bmatrix}
\]

This approach aligns with the common practice of padding rows with zeros to match dimensions.

Final Assumption

For the purposes of this analysis, we will assume that the matrix \(C \) is a \(3 \times 4\) matrix:

\[
C = \begin{bmatrix}
459 & -2 & 0 & 0 \\
651 & 12 & 0 & 0 \\
3 & 4 & 8 & -3
\end{bmatrix}

\]

This assumption allows us to proceed with standard linear algebra techniques.

Computing the Row Space

Definition and Significance

The row space of a matrix is the subspace of $\mathbb{R}^n$ spanned by its row vectors. Finding a basis for the row space involves identifying the set of linearly independent rows after row reduction, which simplifies the matrix to its row echelon form.

Step 1: Write the matrix explicitly

$$
C = \begin{bmatrix}
459 & -2 & 0 & 0 \\
651 & 12 & 0 & 0 \\
3 & 4 & 8 & -3
\end{bmatrix}
$$

Step 2: Perform row operations to find the row echelon form

- Swap rows if necessary to get a leading 1 or non-zero pivot in the first position.

- Use row operations to eliminate entries below the leading coefficient.

Row operations:

Normalize the first row

- The first row has a leading coefficient of 459.

- To simplify calculations, divide the first row by 459:

$$
R_1 \leftarrow \frac{1}{459} R_1
$$

$$
\Rightarrow R_1 = [1, -\frac{2}{459}, 0, 0]
$$

Note: For basis purposes, it's sufficient to keep the original rows; normalization is optional. Alternatively, we proceed directly with elimination.

Eliminate entries below the first pivot

- For the second row:

$$
R_2 \leftarrow R_2 - \frac{651}{459} R_1
$$

Calculate:

$$
\frac{651}{459} = \frac{651}{459} \approx 1.4196
$$

Compute:

$$
R_2 = [651, 12, 0, 0] - 1.4196 \times [459, -2, 0, 0]
$$

$$
= [651 - 1.4196 \times 459, \; 12 - 1.4196 \times (-2), \; 0, \; 0]
$$

Calculate:

- $1.4196 \times 459 \approx 651$ (since $651/459 \approx 1.4196$), so:

$$
651 - 651 = 0
$$

- $12 - 1.4196 \times (-2) = 12 + 2.8392 \approx 14.8392$

Thus,

$$
R_2 \approx [0, 14.8392, 0, 0]
$$

- For the third row:

$$
R_3 = [3, 4, 8, -3]
$$

since the first element is 3, which is less than 459, perhaps swapping rows or proceeding differently may be more straightforward. Alternatively, we can perform elimination directly.

Step 3: Focus on the third row

- Since the first row's leading coefficient is 459, and the third row's first element is 3, eliminate the third row's first element:

$$
R_3 \leftarrow R_3 - \frac{3}{459} R_1
$$

$$
\frac{3}{459} = \frac{1}{153}
$$

Calculate:

$$
R_3 - \frac{1}{153} R_1 = [3, 4, 8, -3] - \frac{1}{153} \times [459, -2, 0, 0]
$$

$$
= [3 - 3, \; 4 - \frac{1}{153} \times (-2), \; 8 - 0, \; -3 - 0]
$$

$$
= [0, 4 + \frac{2}{153}, 8, -3]
$$

Simplify:

$$
4 + \frac{2}{153} = \frac{4 \times 153 + 2}{153} = \frac{612 + 2}{153} = \frac{614}{153}
$$

Thus,

$$
R_3 = [0, \frac{614}{153}, 8, -3]
$$

Step 4: Express the row echelon form

Now, the matrix looks like:

$$
\begin{bmatrix}
459 & -2 & 0 & 0 \\
0 & 14.8392 & 0 & 0 \\
0 & \frac{614}{153} & 8 & -3
\end{bmatrix}
$$

The second row has a leading coefficient approximately 14.8392, which can be scaled to 1 for a basis.

Step 5: Extract the basis for the row space

The non-zero rows in the row echelon form are:

- Row 1: $[459, -2, 0, 0]$
- Row 2: $[0, 14.8392, 0, 0]$
- Row 3: $[0, \frac{614}{153}, 8, -3]$

The set of these three rows is linearly independent, so they form a basis for the row space.

Final basis for the row space:

\[

\left\{

\begin{bmatrix}

459 & -2 & 0 & 0

\end{bmatrix},

\quad

\begin{bmatrix}

0 & 14.8392 & 0 & 0

\end{bmatrix},

\quad

\begin{bmatrix}

0 & \frac{614}{153} & 8 & -3

Given The Matrixc 459 2 651 12 3 4 8 3 Find Bases For Both The Row Space And Column Space Of

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