

Graph The Image Of This Figure After A Dilation With A Scale Factor Of $\frac{1}{2}$ Centered At The Origin.Use

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Understanding how geometric transformations affect figures on the coordinate plane is fundamental in both pure and applied mathematics. Among these transformations, dilation (or scaling) plays a critical role in altering the size of a figure while maintaining its shape and orientation. When a figure undergoes a dilation centered at the origin with a specific scale factor—in this case, $\frac{1}{2}$ —the entire figure is proportionally reduced, leading to a new, smaller image that preserves the original's proportions. This article explores the process of graphing such a dilation, the mathematical principles involved, and practical tips to accurately perform this transformation.

Understanding Dilation in the Coordinate Plane

What Is Dilation?

Dilation is a transformation that enlarges or reduces a figure by a certain scale factor relative to a fixed point called the center of dilation. When the center is at the origin $(0,0)$, the process simplifies because each point's coordinates are scaled directly from the origin.

Mathematically, if we have a point $P(x, y)$, then its image $P'(x', y')$ after dilation with a scale factor k centered at the origin is given by:

$$\begin{aligned} x' &= kx \quad \text{and} \quad y' = ky \end{aligned}$$

This means every coordinate of the original figure is multiplied by the scale factor.

Impact of a Scale Factor of $\frac{1}{2}$

A scale factor of $\frac{1}{2}$ indicates the figure will be reduced to half its original size. Every point's distance from the origin will be halved, and the shape will remain similar to the original figure. For example:

- A point at $(4, 6)$ maps to $(2, 3)$
- A point at $(-8, 10)$ maps to $(-4, 5)$

This proportional reduction is crucial for understanding how the figure's dimensions change and how to graph it accurately.

Step-by-Step Guide to Graphing the Dilation

Step 1: Identify the Coordinates of the Original Figure

Begin by noting all the vertex points of the original figure. Whether the figure is a triangle, rectangle, or any polygon, list all vertices with their (x, y) coordinates.

Example:

Suppose the original figure is a triangle with vertices:

- $(A(2, 4))$
- $(B(6, 8))$
- $(C(4, 10))$

Step 2: Apply the Dilation Formula

Using the formula $(x', y') = (kx, ky)$, multiply each coordinate by $1/2$:

Original Point	Calculation	Dilation Result
$(A(2, 4))$	$(1/2)(2), (1/2)(4)$	$(1, 2)$
$(B(6, 8))$	$(1/2)(6), (1/2)(8)$	$(3, 4)$
$(C(4, 10))$	$(1/2)(4), (1/2)(10)$	$(2, 5)$

Repeat this step for all vertices to find the new set of points.

Step 3: Plot the Original and Dilated Figures

Using graph paper or digital graphing tools:

- Plot the original points (A, B, C) .
- Draw the original figure connecting these points.
- Plot the dilated points (A', B', C') .
- Connect the dilated points to form the new figure.

This visual comparison helps in understanding how the dilation affects the figure's size and position.

Step 4: Analyze the Resulting Figure

Observe that:

- The dilated figure is similar to the original, with all sides proportionally reduced.
- The shape is preserved, but the size is halved.
- All vertices are closer to the origin, reflecting the reduction.

Key Properties of Dilation with Scale Factor $\frac{1}{2}$

Preservation of Shape and Orientation

Dilation preserves the shape of the figure; only its size changes. The angles remain the same, and the figure's orientation is unchanged because the transformation is uniform in all directions.

Reduction in Dimensions

Each length in the figure is scaled by $\frac{1}{2}$:

- Side lengths are halved.
- Coordinates are multiplied by $\frac{1}{2}$.
- The figure shrinks toward the origin.

Implications for Centered at the Origin

Since the dilation is centered at the origin, every point's position is scaled directly from the origin. This means:

- Points closer to the origin move less than those farther away.
- The entire figure shrinks uniformly towards the origin.

Practical Applications of Dilation

Design and Engineering

Dilation helps in creating scaled models of structures, machinery parts, or landscapes. By understanding how to graph and perform dilations, engineers can visualize and plan in different scales.

Art and Graphics

Artists and graphic designers utilize dilation to resize images and figures while maintaining proportions, ensuring visual consistency across different media.

Mathematics Education

Teaching concepts of similarity, scale, and transformations is foundational in geometry. Dilation exercises reinforce understanding of proportional reasoning.

Tips for Accurate Graphing of Dilated Figures

- Always double-check coordinate calculations to avoid errors.
- Use graph paper or digital tools for precision.
- Label original and dilated points clearly for comparison.
- Remember that the center of dilation is at the origin; if different, additional translation steps are needed.
- Practice with various figures to reinforce the concept of proportional scaling.

Conclusion

Graphing the image of a figure after a dilation with a scale factor of $\frac{1}{2}$ centered at the origin involves understanding the fundamental principles of dilation, applying the scaling formula to each vertex, and accurately plotting the points to visualize the transformation. This process highlights the importance of proportional reasoning in geometry and provides a practical method for resizing figures while preserving their shape. Whether for academic purposes, design, or engineering, mastering the concept of dilation is essential for students and professionals working with geometric figures on the coordinate plane. With careful calculation and plotting, you can confidently perform and interpret dilations, deepening your understanding of geometric transformations.

Frequently Asked Questions

What is the effect of a dilation with a scale factor of $\frac{1}{2}$ centered at the origin on a figure?

The figure is reduced in size by half, with all points moving closer to the origin, maintaining the same shape and proportions.

How do you graph a figure after applying a dilation with a scale factor of $\frac{1}{2}$ centered at the origin?

Multiply the coordinates of each point in the original figure by $\frac{1}{2}$, then plot the new points to form the dilated image.

What is the mathematical formula for dilating a point

(x, y) with a scale factor of 1/2 centered at the origin?

The dilated point is $(x/2, y/2)$, obtained by multiplying each coordinate by the scale factor.

Does the shape of the figure change after a dilation centered at the origin?

No, the shape remains similar; only the size changes, proportional to the scale factor.

What are common mistakes to avoid when graphing a dilation with a scale factor of 1/2?

Common mistakes include forgetting to multiply both x and y coordinates by 1/2, or misplacing the points after scaling.

Can a dilation with a scale factor of 1/2 be applied to any type of figure?

Yes, the dilation can be applied to any geometric figure, including lines, polygons, and circles, to reduce their size proportionally.

How does the dilation with a scale factor of 1/2 affect the area of the figure?

The area of the figure is reduced to one-quarter (since $(1/2)^2 = 1/4$) of the original area.

Is the center of dilation always at the origin in this type of problem?

In this specific problem, yes, the dilation is centered at the origin; otherwise, the center would need to be specified.

What is the purpose of performing a dilation with a scale factor of 1/2 in geometric problems?

It helps in understanding similarity, proportionality, and how figures change size while maintaining shape.

How do you verify that the dilation has been performed correctly on a figure?

Check that each point has been scaled appropriately by 1/2 and that the new figure maintains the same shape, just smaller, with all points correctly plotted.

Additional Resources

Graph the image of this figure after a dilation with a scale factor of $\frac{1}{2}$ centered at the origin. Use

An In-Depth Exploration of Geometric Dilation and its Impact on Graphs

Understanding the transformation of figures via dilation is fundamental in the study of geometry, particularly in the context of coordinate geometry. When a figure undergoes dilation centered at the origin with a specific scale factor, its size and position are affected in predictable ways, providing insight into concepts such as similarity, symmetry, and proportionality. This article aims to provide a comprehensive, detailed analysis of how a figure's graph changes after a dilation with a scale factor of $\frac{1}{2}$ centered at the origin.

Introduction to Dilation in Coordinate Geometry

What is Dilation?

Dilation, also known as a homothety, is a transformation that enlarges or reduces a figure by a certain scale factor with respect to a fixed point called the center of dilation. The essence of dilation is that it preserves the shape of the figure (it's a similarity transformation) but alters its size.

The Center of Dilation: The Origin

Centering the dilation at the origin $(0,0)$ simplifies calculations because the transformation applies uniformly to all points in the plane relative to the origin. When the center is at the origin, each point's image under dilation can be computed straightforwardly by multiplying its coordinates by the scale factor.

The Scale Factor

The scale factor determines the degree of enlargement or reduction:

- A scale factor greater than 1 (>1) results in an enlarged figure.
- A scale factor between 0 and 1 (<1) results in a reduced figure.
- A scale factor of exactly 1 results in the original figure, with no change.

In this specific case, the scale factor is $\frac{1}{2}$, meaning the figure will be reduced to half its original size.

Mathematical Foundation of Dilation at the Origin with Scale Factor $\frac{1}{2}$

Coordinates Transformation

Suppose a point $P(x, y)$ lies on the original figure. The image P' after dilation

centered at the origin with scale factor $(k = 1/2)$ is given by:

$$P'(x', y') = (k \times x, k \times y)$$

This simple relation results because the origin is the center of dilation, and the transformation scales all points relative to it.

Implications for the Entire Figure

- Every point on the figure is scaled uniformly.
- The shape remains similar to the original, preserving angles and proportionality.
- The size is halved, leading to a figure that is similar but smaller.

Example

If the original point is $(4, 6)$, then the image point after dilation becomes:

$$(1/2 \times 4, 1/2 \times 6) = (2, 3)$$

This scaling applies to all points, thereby shrinking the entire figure by half.

Visualizing the Dilation: Graphical Impact

Effect on Basic Shapes

The impact of dilation can be visualized most straightforwardly with common geometric figures:

- Line segments: Both endpoints are scaled, resulting in a line segment half as long, maintaining the same slope.
- Triangles: All vertices are scaled, creating a similar, smaller triangle.
- Circles: The radius is scaled by the factor $(1/2)$, producing a circle with half the radius.

Preservation of Shape and Orientation

Since dilation is a similarity transformation centered at the origin, it preserves:

- The shape of the figure.
- The angles between sides.
- The relative proportions of the figure.

However, the figure's size is decreased, and its position relative to the origin remains proportional, with all points pulled closer to the center.

Step-by-Step Procedure to Graph the Dilated Figure

1. Identify All Coordinates of the Original Figure

The initial step involves cataloging the coordinates of all key points defining the figure.

2. Apply the Dilation Formula

For each point (x, y) , compute:

$$(x', y') = \left(\frac{1}{2} x, \frac{1}{2} y\right)$$

3. Plot the Image Points

Using graph paper or graphing software, plot each scaled point.

4. Connect the Image Points

Reconstruct the figure by connecting the scaled points in the same order as the original, ensuring the shape's integrity.

5. Verify the Transformation

Check that the scaled figure maintains the same shape and proportionality, just at a reduced scale.

Analytical Example: Transforming a Specific Figure

Suppose the original figure is a triangle with vertices at $A(2, 4)$, $B(4, 8)$, and $C(6, 2)$.

Original Coordinates:

Vertex	Coordinates
A	(2, 4)
B	(4, 8)
C	(6, 2)

Dilated Coordinates:

Applying the scale factor $\frac{1}{2}$:

$$A' = \left(\frac{1}{2} \times 2, \frac{1}{2} \times 4\right) = (1, 2)$$

$$B' = \left(\frac{1}{2} \times 4, \frac{1}{2} \times 8\right) = (2, 4)$$

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$$C' = \left(\frac{1}{2} \times 6, \frac{1}{2} \times 2\right) = (3, 1)$$

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Graphical Representation:

- The original triangle spans from points $(2, 4)$, $(4, 8)$, and $(6, 2)$.
- The scaled triangle spans from points $(1, 2)$, $(2, 4)$, and $(3, 1)$.

The image triangle is similar to the original but precisely half the size, maintaining the same angles and side ratios.

Theoretical Significance and Applications

Similarity and Proportionality

Dilation is a core concept in similarity transformations, which are fundamental in geometric proofs and constructions. The fact that all lengths are scaled uniformly by the same factor ensures the figure's shape remains unchanged.

Real-World Applications

- Engineering and Design: Scaling architectural plans or mechanical parts.
- Computer Graphics: Resizing images or models while preserving proportions.
- Mathematics Education: Visualizing proportional relationships and transformations.

Limitations and Considerations

- Dilation centered at points other than the origin results in different transformations, often involving translation components.
- When dealing with irregular figures, attention must be paid to all vertices to ensure proper scaling.

Summary and Key Takeaways

- Dilation centered at the origin with a scale factor of $\frac{1}{2}$ shrinks the figure by half, maintaining shape similarity.
- All points (x, y) are mapped to $(x/2, y/2)$.
- The transformation preserves angles and proportional relationships, emphasizing the concept of geometric similarity.
- Graphically, the figure appears "pulled" closer to the origin, with its size reduced uniformly.

Final Thoughts

Understanding how to graph a figure after a dilation with a specific scale factor is crucial in mastering coordinate geometry. It deepens comprehension of similarity, proportionality, and transformation principles. The case of dilation centered at the origin with a scale factor of $\frac{1}{2}$ offers a clear, straightforward example of these concepts in action. Whether in academic contexts or practical applications, being able to visualize and compute these transformations enhances spatial reasoning and mathematical literacy.

In conclusion, the process of graphing the image of a figure after dilation involves recognizing the fundamental principles of coordinate transformations, applying the correct scale factor, and accurately plotting the resulting points. This not only reinforces geometric concepts but also provides essential skills applicable in various scientific, engineering, and artistic fields.

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