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Understanding the energy stored in a solenoid is fundamental in electromagnetism, particularly for applications involving inductors, transformers, and electromagnetic devices. In this article, we will explore how to calculate the magnetic energy stored in a specific solenoid with a diameter of 2.60 cm, a length of 14.0 cm, and 150 turns of wire. We will walk through the key concepts, formulas, and step-by-step calculations, providing a comprehensive guide for students, engineers, and enthusiasts interested in electromagnetic energy storage.

Introduction to Solenoids and Magnetic Energy Storage

A solenoid is a long coil of wire wound in a helix, which produces a nearly uniform magnetic field inside its interior when an electric current flows through it. The magnetic energy stored in a solenoid can be significant, especially in applications like inductors in electronic circuits, magnetic resonance imaging (MRI), and particle accelerators.

The magnetic energy stored depends on factors such as the number of turns, the current flowing through the wire, the length and diameter of the solenoid, and the magnetic permeability of the core (if present). The fundamental relationship between these variables allows us to analyze and design solenoids for specific energy storage requirements.

Key Parameters of the Given Solenoid

Let's first define the known parameters:

- Diameter, d = 2.60 cm = 0.0260 m
- Length, L = 14.0 cm = 0.140 m
- Number of turns, N = 150
- Cross-sectional area, A = ? (to be calculated)
- Magnetic permeability of free space, μ_0 = $4\pi \times 10^{-7}$ T·m/A
- Current, I = ? (we will discuss how to determine or assume this value)

Note: While the question does not specify the current, for the purpose of calculating stored energy, we need a specific current value. Without it, we can express the energy formula in terms of I, or assume a typical current if needed.

Calculating the Cross-Sectional Area

The cross-sectional area of the solenoid's core (assuming it's a hollow cylinder or a solid wire bundle) is essential for magnetic field calculations.

Since the diameter is given:

$$A = \pi \times (d/2)^2$$

Calculating:

$$\begin{aligned}
A &= \pi \times (0.0260 \text{ m} / 2)^2 \\
&= \pi \times (0.0130 \text{ m})^2 \\
&= \pi \times 0.000169 \text{ m}^2 \\
&\approx 3.1416 \times 0.000169 \text{ m}^2 \\
&\approx 0.000531 \text{ m}^2
\end{aligned}$$

So, the cross-sectional area:

$A \approx 5.31 \times 10^{-4}$ m²

Understanding Magnetic Field Inside the Solenoid

The magnetic field (B) inside an ideal solenoid is given by:

$$B = \mu_0 \times (N / l) \times I$$

Where:

- N = number of turns

- L = length of the solenoid

- I = current flowing through the wire

The magnetic flux (Φ) through the solenoid is:

$$\Phi = B \times A$$

The energy stored in the magnetic field of the solenoid is given by:

$$U = (1/2) \times L \times I^2$$

Where L is the inductance of the solenoid.

Alternatively, the energy stored can be expressed directly via the magnetic field and volume:

$$U = (1/2) \times (B^2 / \mu_0) \times \text{Volume}$$

Given the volume of the solenoid:

$$V = A \times l$$

Note: To compute the stored energy explicitly, we need either the current I or the inductance L. Let's proceed assuming a specific current or calculate L first, then find the energy.

Calculating the Inductance of the Solenoid

The inductance of a long solenoid is:

$$L = \mu_0 \times N^2 \times A / l$$

Substituting the known values:

$$L = (4\pi \times 10^{-7}\ T\cdot m/A) \times (150)^2 \times 5.31 \times 10^{-4}\ m^2 / 0.140\ m$$

Calculations step-by-step:

1. $N^2 = 150^2 = 22,500$

2. Numerator:

$$\mu_0 \times N^2 \times A = 4\pi \times 10^{-7} \times 22,500 \times 5.31 \times 10^{-4}$$

Calculate:

$$4\pi \times 10^{-7} \approx 1.2566 \times 10^{-6}\ T\cdot m/A$$

Then,

$$\begin{aligned} &1.2566 \times 10^{-6} \times 22,500 \approx 1.2566 \times 22,500 \times 10^{-6} \\ &= (1.2566 \times 22,500) \times 10^{-6} \\ &\approx 28,290.75 \times 10^{-6} \\ &= 0.02829075 \end{aligned}$$

Next, multiply by A:

$$0.02829075 \times 5.31 \times 10^{-4} \approx 0.02829075 \times 0.000531 \approx 1.503 \times 10^{-5}$$

Finally, divide by l:

$$L = 1.503 \times 10^{-5} / 0.140 \approx 1.073 \times 10^{-4}\ H$$

Therefore, the inductance:

$L \approx 1.07 \times 10^{-4}$ henrys (H)

Note: This is the inductance assuming a magnetic permeability of free space and no core material. If a magnetic core is present, the inductance would increase accordingly.

Calculating the Stored Magnetic Energy

The magnetic energy stored in the solenoid is:

$$U = (1/2) \times L \times I^2$$

To proceed, we need a current value. Let's consider a typical current to illustrate the calculation; for example, assume:

$I = 1.0$ A

Then,

$$U = 0.5 \times 1.07 \times 10^{-4}\ H \times (1.0\ A)^2$$
$$= 0.5 \times 1.07 \times 10^{-4}$$
$$\approx 5.35 \times 10^{-5}\ \text{Joules}$$

Thus, the energy stored at 1A current is approximately 53.5 microjoules.

If the current differs, the energy scales with the square of the current. For example, at 2A:

$$U = 0.5 \times L \times (2)^2 = 0.5 \times L \times 4 = 4 \times (\text{energy at 1A})$$

which would be approximately 214 microjoules.

Alternative Approach: Calculating Energy Using Magnetic Field and Volume

Another way to compute the energy stored is via the magnetic field:

$$U = (1/2\mu_0) \times B^2 \times \text{Volume}$$

Given that:

$$B = \mu_0 \times (N / l) \times I$$

and Volume $V = A \times l$.

Rearranged, the energy becomes:

$$U = (1/2\mu_0) \times (\mu_0 \times N / l \times I)^2 \times A \times l$$

Simplify:

$$U = (1/2\mu_0) \times \mu_0^2 \times N^2 / l^2 \times I^2 \times A \times l$$
$$= (1/2) \times \mu_0 \times N^2 / l \times I^2 \times A$$

which matches the previous calculation:

$$U = (1/2) \times L \times I^2$$

Summary of Key Formulas:

- Cross-sectional area:

$$A = \pi \times (d/2)^2$$

- Inductance:

$$L = \mu_0 \times N^2 \times A / l$$

- Stored energy:

$$U = (1/2) \times L \times I^2$$

- Magnetic field:

$$B = \mu_0 \times (N / l) \times I$$

Note: For precise energy calculations, the actual current flowing through the solenoid during operation should be known or specified.