If 0.6 J Of Work Is Needed To Stretch A Spring From 7 Cm To 9 Cm And Another 1 J Is Needed To Stretch

If 0.6 J Of Work Is Needed To Stretch A Spring From 7 Cm To 9 Cm And Another 1 J Is Needed To Stretch the spring further, it provides an interesting scenario for understanding the principles of elastic potential energy, Hooke's Law, and work-energy relationships in physics. This article explores these concepts in detail, analyzing what the given data reveals about the spring’s properties, calculating its spring constant, and applying these principles to real-world problems. Whether you're a student studying physics or an enthusiast seeking to deepen your understanding of springs and energy, this comprehensive guide will clarify the essential ideas behind spring mechanics and work done during deformation.

Understanding the Basics: Work, Elastic Potential Energy, and Hooke's Law

Work Done in Stretching a Spring

When you stretch or compress a spring, work is done against the spring's restoring force. The amount of work done is stored as elastic potential energy within the spring, which can be recovered when the spring returns to its original length. The work done on a spring from an initial displacement to a final displacement is directly related to the elastic potential energy stored in the spring.

Mathematically, the work W done in stretching a spring from an initial length x_i to a final length x_f can be expressed as:

$$
W = \frac{1}{2} k (x_f^2 - x_i^2)
$$

where:
- k is the spring constant,
- x_i and x_f are the initial and final displacements from the spring's natural length.

This equation assumes the spring obeys Hooke's Law, which states that the restoring force F is proportional to the displacement:

$$
F = -k x

\]

The negative sign indicates the force opposes the displacement.

Calculating the Spring Constant from Given Data

Given the data:

- Work to stretch from 7 cm to 9 cm: \(W_1 = 0.6 \) Joules
- Further work needed to stretch beyond 9 cm (say to some unknown length): \(W_2 = 1 \) Joule

Let's analyze the first part: stretching from 7 cm to 9 cm.

Converting Lengths to Displacements

Assuming the natural length of the spring is \(L_0 \), and the displacements are measured from this natural length, the initial and final displacements are:

\[
x_i = 7\, \text{cm} - L_0
\]
\[
x_f = 9\, \text{cm} - L_0
\]

However, since the problem provides only the lengths, and the work depends on the change in energy between these lengths, the actual natural length \(L_0 \) cancels out when calculating differences, provided the displacements are measured from the natural length.

Alternatively, if the initial length is taken as the reference (say 7 cm), then the displacements are:

\[
x_i = 0.0\, \text{cm} \quad \text{(initial at 7 cm)}
\]
\[
x_f = 2\, \text{cm}
\]

But to be precise, we need to define the displacements relative to the natural length. Since the problem involves changes in length, and energy stored depends on the square of the displacement, the difference in potential energy can be expressed as:

\[

W = \frac{1}{2} k (x_f^2 - x_i^2)
\]

Given that, and assuming the natural length corresponds to 7 cm (i.e., initial displacement zero), then:

\[
x_i = 0\, \text{cm}
\]
\[
x_f = 2\, \text{cm} = 0.02\, \text{m}
\]

Now, applying the formula:

\[
0.6\, \text{J} = \frac{1}{2} k (0.02)^2
\]
\[
0.6 = \frac{1}{2} k \times 0.0004
\]
\[
k = \frac{0.6 \times 2}{0.0004} = \frac{1.2}{0.0004} = 3000\, \text{N/m}
\]

Thus, the spring constant is approximately 3000 N/m.

Understanding Energy Changes for Further Stretching

The second part of the problem states that an additional 1 Joule of work is needed to stretch the spring further. Using this, we can analyze how much further displacement is associated with this additional work, assuming the spring constant remains constant within this range.

Calculating the Additional Displacement

Suppose the initial displacement at 9 cm is:

\[
x_1 = 2\, \text{cm} = 0.02\, \text{m}
\]

The work required to stretch from 9 cm to some new length \(x_2 \) is:

\[

$$
W_{additional} = \frac{1}{2} k (x_2^2 - x_1^2)
$$

Given:

$$
W_{additional} = 1\, \text{J}
$$
$$
k = 3000\, \text{N/m}
$$
$$
x_1 = 0.02\, \text{m}
$$

Plugging in the values:

$$
1 = \frac{1}{2} \times 3000 \times (x_2^2 - 0.0004)
$$
$$
1 = 1500 \times (x_2^2 - 0.0004)
$$
$$
x_2^2 - 0.0004 = \frac{1}{1500} \approx 0.0006667
$$
$$
x_2^2 = 0.0006667 + 0.0004 = 0.0010667
$$
$$
x_2 = \sqrt{0.0010667} \approx 0.0327\, \text{m} = 3.27\, \text{cm}
$$

This indicates the spring is stretched to approximately 3.27 cm beyond the natural length after the additional work.

Note: Since the initial stretch was 2 cm, the total displacement from the natural length after the additional work is 3.27 cm, meaning the total stretch from the natural length is about 3.27 cm.

Implication: The work done to stretch the spring from 7 cm to 9 cm is 0.6 J, and an additional 1 J extends the spring further, demonstrating the proportional relationship between work and the square of displacement.

Applications and Real-World Implications of Spring Mechanics

Designing Mechanical Systems

Understanding how much work is needed to stretch a spring to a certain length is vital in designing mechanical systems such as vehicle suspensions, watch mechanisms, and industrial machinery. Engineers use the spring constant k to select appropriate springs that can withstand specific loads without permanent deformation.

Energy Storage and Harvesting

Springs are often employed as energy storage components. Knowing the work required to stretch a spring helps in calculating the energy stored, which can be harnessed in various applications like clock mechanisms, energy harvesting devices, and shock absorbers.

Safety Considerations

Overstretching a spring beyond its elastic limit can cause permanent deformation or failure. Calculating the work involved in stretching can help determine safe operational limits, ensuring the longevity and safety of mechanical components.

Summary of Key Concepts

- The work done to stretch a spring from one length to another is given by $W = \frac{1}{2} k (x_f^2 - x_i^2)$.
- Calculating the spring constant k from known work and displacement allows engineers and physicists to predict the behavior of springs under various loads.
- The elastic potential energy stored in a spring is directly proportional to the square of its displacement from the natural length.
- Understanding the energy-work relationship in springs is essential for designing safe and efficient mechanical systems.

Conclusion

Analyzing the work needed to stretch a spring from a specific length and further stretching scenarios provides valuable insights into the fundamental physics of elastic materials. By calculating the spring constant and understanding how work relates to displacement, engineers and students can better grasp the principles governing elastic potential energy and Hooke's Law. Whether designing complex machinery or learning foundational physics concepts, these calculations form the backbone of understanding how springs function within various systems. Remember, always consider the limits of elasticity to ensure safety and optimal performance in practical applications.

Frequently Asked Questions

What is the spring constant given that 0.6 J of work stretches a spring from 7 cm to 9 cm?

First, convert lengths to meters: 7 cm = 0.07 m, 9 cm = 0.09 m. The work done to stretch a spring is $W = (1/2)\ k\ (x^2 - x_0^2)$. Plugging in the values: $0.6 = (1/2)\ k\ (0.09^2 - 0.07^2)$. Calculate the difference: $0.09^2 = 0.0081$, $0.07^2 = 0.0049$, difference = 0.0032. Thus, $0.6 = (1/2)\ k\ 0.0032 \rightarrow 0.6 = 0.0016\ k \rightarrow k = 0.6 / 0.0016 = 375$ N/m.

How much work is needed to stretch the same spring from 9 cm to 11 cm?

Calculate the work using $W = (1/2)\ k\ (x_2^2 - x_1^2)$. With k = 375 N/m, $x_1 = 0.09$ m, $x_2 = 0.11$ m: $W = (1/2)\ 375\ (0.11^2 - 0.09^2) = 187.5\ (0.0121 - 0.0081) = 187.5\ 0.004 = 0.75$ J. Therefore, 0.75 Joules of work are needed.

What is the total work done when stretching the spring from 7 cm to 11 cm?

Calculate the work from 7 cm to 9 cm (0.6 J) and from 9 cm to 11 cm (0.75 J), then sum: 0.6 J + 0.75 J = 1.35 J. So, the total work needed is 1.35 Joules.

What does the work done tell us about the energy stored in the spring during stretching?

The work done on the spring is stored as elastic potential energy, calculated as $(1/2)\ k\ x^2$ at the final stretch. For example, when stretched to 9 cm, the stored energy is $(1/2)\ 375\ 0.09^2$ = approximately 1.52 J, but since the work done is 0.6 J, the actual energy stored corresponds to that amount, indicating some energy is lost or not stored perfectly due to other factors.

If the work needed to stretch from 7 cm to 9 cm is 0.6 J, what is the work required to return the spring from 9 cm back to 7 cm?

The work required to return the spring from 9 cm to 7 cm is equal in magnitude but opposite in sign to the work done to stretch it. Therefore, 0.6 J of work is released during the return, meaning the energy stored is released back to the environment or the doing force during the compression.

Additional Resources

Spring Mechanics and Energy: Unveiling the Intricacies of Work and Stretching

---

Introduction

When exploring the world of springs, one quickly realizes that their behavior isn't just about elasticity; it's a fascinating interplay of physics, energy transfer, and material properties. The question at hand—"If 0.6 J of work is needed to stretch a spring from 7 cm to 9 cm, and another 1 J is needed to stretch it further"—serves as an intriguing gateway into understanding spring mechanics beyond the basic Hooke's Law. This article delves deep into the principles underlying these energy requirements, exploring what they reveal about the spring's properties, the work-energy relationship, and the practical implications in engineering and design.

Understanding the Basics: Hooke's Law and Elastic Potential Energy

Before dissecting the specific energy values, it's essential to revisit fundamental concepts:

- Hooke's Law states that the restoring force exerted by an ideal spring is proportional to its extension or compression from its equilibrium position:

$$
F = -k x
$$

where:
- F is the force,
- k is the spring constant (a measure of stiffness),
- x is the displacement from the equilibrium position.

- Elastic Potential Energy stored in a spring is given by:

$$
U = \frac{1}{2} k x^2
$$

This energy is what is required to stretch or compress the spring from its unstressed state to a displacement x.

The Significance of Work in Stretching a Spring

In physics, the work done on a spring in stretching it from an initial displacement x_i to a final displacement x_f is equal to the change in elastic potential energy:

$$
W = \Delta U = U_{f} - U_{i} = \frac{1}{2}k x_f^2 - \frac{1}{2}k x_i^2
$$

This relation presumes the spring is ideal, with no energy losses due to damping or internal friction.

Analyzing the Given Data

The problem provides two key pieces of information:

1. Work to stretch from 7 cm to 9 cm: 0.6 Joules
2. Additional work to stretch further: 1 Joule (implying from some larger initial stretch)

Let’s interpret and analyze these in detail.

Calculating the Spring Constant from the Initial Data

Step 1: Convert the displacements into meters for SI consistency:

- $x_1 = 7\,cm = 0.07\,m$
- $x_2 = 9\,cm = 0.09\,m$

Step 2: Use the work formula:

$$
W_{1\to 2} = \frac{1}{2}k x_2^2 - \frac{1}{2}k x_1^2
$$

Given:

$$
0.6\,J = \frac{1}{2}k (0.09)^2 - \frac{1}{2}k (0.07)^2
$$

Calculating the squares:

\[

0.6 = \frac{1}{2}k (0.0081 - 0.0049) = \frac{1}{2}k \times 0.0032
\]

Simplify:

\[
0.6 = 0.0016 k
\]

Solve for \(k\):

\[
k = \frac{0.6}{0.0016} = 375\, \text{N/m}
\]

Result: The spring constant \(k \approx 375\, \text{N/m}\).

Determining the Initial and Final Displacements for the Additional Work

The problem states that another 1 J is needed to stretch the spring further from some position. To analyze this, we need to understand from what initial displacement this additional work applies and to what final displacement.

Assumption: The "another 1 J" refers to the work needed to stretch the spring from the position at 9 cm to some further extension, say, \(x_3\).

Step 1: Establish the initial points:

- Starting point: \(x_2 = 0.09\,m\)

- Work to further stretch from \(x_2\) to \(x_3\):

\[
W_{2 \to 3} = \frac{1}{2}k x_3^2 - \frac{1}{2}k x_2^2 = 1\,J
\]

Plugging in \(k = 375\, \text{N/m}\):

\[
1 = \frac{1}{2} \times 375 \times (x_3^2 - 0.09^2)
\]

\[
1 = 187.5 \times (x_3^2 - 0.0081)
\]

Solve for \(x_3^2\):

\[

$$
x_3^2 = \frac{1}{187.5} + 0.0081 \approx 0.00533 + 0.0081 = 0.01343
$$

$$
x_3 = \sqrt{0.01343} \approx 0.116\,m
$$

Final extension: approximately 11.6 cm.

Interpretation: To stretch the spring from 9 cm to about 11.6 cm requires an additional 1 Joule of work.

The Energy Profile and Nonlinearity

The calculations reveal an intriguing aspect: the energy stored in the spring increases quadratically with extension, consistent with Hooke’s Law. However, the difference in work required for consecutive stretches reveals the nature of the energy landscape.

- From 7 cm to 9 cm: 0.6 J
- From 9 cm to approximately 11.6 cm: 1 J

The work-to-displacement ratio is not linear in the sense of simple proportionality; it depends on the squares of displacements:

$$
W = \frac{1}{2}k (x_f^2 - x_i^2)
$$

This quadratic relation explains why larger stretches require disproportionately more work, even if the incremental extensions are similar.

Practical Implications and Real-World Applications

Understanding the energy requirements for stretching a spring has practical consequences across multiple fields:

- Designing Mechanical Systems: Engineers must account for the energy stored in springs when designing machinery, ensuring that actuators and energy absorbers can handle the expected loads.

- Material Selection: The value of k influences the choice of spring materials—stiffer springs with higher k values store more energy but may also require more work to compress or stretch.

- Energy Storage Devices: Springs serve as simple energy storage units; knowing the work-energy relationship helps optimize their use in devices like shock absorbers, watches, and sensors.

- Safety Considerations: Overstretching springs beyond their elastic limit risks permanent deformation or failure, emphasizing the importance of understanding energy thresholds.

Limitations and Real-World Deviations

While the idealized calculations provide a solid foundation, real-world springs often exhibit:

- Nonlinear Behavior: At large displacements, springs may deviate from Hooke’s Law due to material properties or geometric nonlinearities.

- Internal Friction: Energy losses due to internal damping mean that actual work input exceeds the stored elastic potential energy.

- Material Fatigue: Repeated stretching can weaken the spring, altering \(k\) over time.

Understanding these factors is crucial for accurate system design and longevity.

Summary and Key Takeaways

- The spring constant \(k\) for the system is approximately 375 N/m based on the given work data.

- The quadratic relationship between extension and stored energy emphasizes that larger displacements require exponentially more work.

- Stretching from 7 cm to 9 cm requires 0.6 J, while an additional 1 J suffices to extend the spring further from 9 cm to approximately 11.6 cm.

- These insights highlight the importance of energy considerations in spring applications, from simple mechanical devices to complex engineering systems.

Final Thoughts

The analysis of work needed to stretch a spring reveals much about its intrinsic properties and behavior under load. Recognizing the quadratic energy-displacement relationship allows engineers and designers to predict how springs will perform in real-world scenarios, ensuring safety, efficiency, and optimal functionality. Whether in precision instruments or large-scale machinery, understanding these fundamental principles remains vital for harnessing the full potential of elastic systems.

If 0 6 J Of Work Is Needed To Stretch A Spring From 7 Cm To 9 Cm And Another 1 J Is Needed To Stretch

If 0 6 J Of Work Is Needed To Stretch A Spring From 7 Cm To 9 Cm And Another 1 J Is Needed To Stretch

Related Articles

- In The Mac OS, The Resource ____ Stores Information About The Data.a)file B)fork C)segment D)metadata
- Implement The Following Operation Using Shift And Arithmetic Instructions. Assume That All Parameters
- Identify And Briefly Describe The Five Key Organizational Complements That Must Be In Place To Ensure

Back to Home