If F(x) = - 5^x - 4 And G (x) = - 3 X - 2 Find (f - G) (x) O A. (F - G)(x) = 5^x - 3x + 2O B. (F -

If F(x) = - 5^x - 4 And G(x) = - 3x - 2 Find (f - G)(x). O A. (F - G)(x) = 5^x - 3x + 2 O B. (F - G)

Understanding how to compute the difference between two functions, especially when they are of different types—exponential and linear—is fundamental in algebra and calculus. In this comprehensive guide, we will explore the process of finding the difference between the functions $F(x) = - 5^x - 4$ and $G(x) = - 3x - 2$, focusing on how to derive $(F - G)(x)$. This exploration will include detailed steps, key concepts, and practical tips to master similar problems. Whether you're a student preparing for exams or someone interested in mathematical functions, this article aims to deepen your understanding of function operations with a special focus on exponential and linear functions.

Understanding the Functions: Exponential and Linear

Before diving into the calculation, it’s essential to understand the nature of the functions involved:

1. The Exponential Function $F(x) = - 5^x - 4$

- Definition: An exponential function where the base is 5, raised to the power of x, then multiplied by -1 and shifted downward by 4.
- Key characteristics:
- Exponential growth or decay depending on the base.
- Rapid increase or decrease as x varies.
- The negative sign indicates the function is reflected over the x-axis.

2. The Linear Function $G(x) = - 3x - 2$

- Definition: A linear function with a slope of -3 and a y-intercept of -2.
- Key characteristics:
- Straight line when graphed.
- Constant rate of change.
- The negative slope indicates the function decreases as x increases.

Step-by-Step Calculation of $(F - G)(x)$

The difference of two functions, denoted as $(F - G)(x)$, is computed as:

$$
(F - G)(x) = F(x) - G(x)
$$

Given the functions:

$$
F(x) = - 5^x - 4
$$

$$
G(x) = - 3x - 2
$$

We proceed with the calculation:

1. Write the difference explicitly

$$
(F - G)(x) = (- 5^x - 4) - (- 3x - 2)
$$

2. Simplify the expression by removing parentheses

$$
(F - G)(x) = - 5^x - 4 + 3x + 2
$$

Note that subtracting $-3x - 2$ is equivalent to adding $+3x + 2$.

3. Combine like terms

- Exponential term: $- 5^x$
- Linear term: $+ 3x$
- Constants: $- 4 + 2 = -2$

So, the simplified form is:

$$
(F - G)(x) = - 5^x + 3x - 2
$$

\]

Final Expression and Interpretation

The difference function $(F - G)(x)$ is:

$$
\boxed{
(F - G)(x) = - 5^x + 3x - 2
}
$$

This expression combines an exponential component, a linear component, and a constant term. Understanding this composite function provides insights into how exponential and linear functions interact when combined through subtraction.

Key Points to Remember

When calculating the difference between functions, keep in mind:

1. **Subtracting functions involves subtracting their entire expressions:** $F(x) - G(x)$.

2. **Distribute the minus sign across the second function:** Be cautious with signs, especially when the second function has negative terms.

3. **Combine like terms carefully:** Group exponential, linear, and constant terms separately for clarity.

4. **Understand the nature of each function:** Recognize how exponential and linear functions behave to interpret the resulting composite function.

Applications and Significance of Function

Difference

Understanding the difference between functions like $F(x)$ and $G(x)$ has practical implications across various fields:

1. Mathematical Analysis

- Analyzing how two functions compare at different points.
- Determining the points where the functions intersect (solve $F(x) = G(x)$).

2. Engineering and Physics

- Modeling systems where exponential decay/growth interacts with linear trends.
- Calculating net effects, differences, or residuals.

3. Economics and Finance

- Comparing investment growth (exponential) with linear costs or revenues.
- Understanding profit differences over time.

4. Data Science and Machine Learning

- Computing residual errors between predicted and actual data modeled by different functions.

Graphical Interpretation of $(F - G)(x)$

Visualizing the functions provides intuitive understanding:

- The exponential component $- 5^x$ rapidly decreases for positive x.
- The linear component $3x$ increases linearly.
- The constant -2 shifts the entire function downward.

Plotting $(F - G)(x) = - 5^x + 3x - 2$ reveals how the exponential decay interacts with linear growth:

- For large positive x, the exponential term dominates, driving the function downward.
- For negative x, the exponential approaches zero, and the linear term can make the function positive or negative depending on the value.

Additional Tips for Solving Similar Problems

To excel in calculating differences of functions, consider these strategies:

- **Always write out the functions explicitly.** Avoid shortcuts to prevent sign errors.
- **Be cautious with negative signs, especially when subtracting functions with multiple terms.**
- **Check your work by plugging in specific values of x.** For example, test with $x=0$ or $x=1$ to verify the calculation.
- **Practice with various function types:** polynomial, exponential, logarithmic, etc., to build confidence.

Conclusion: Mastering Function Operations for Mathematical Success

Calculating the difference between functions like $F(x) = - 5^x - 4$ and $G(x) = - 3x - 2$ exemplifies the foundational skills needed in algebra and calculus. The key takeaway is to carefully perform algebraic operations—paying attention to signs and combining like terms—to arrive at an accurate and insightful expression for $(F - G)(x)$.

Understanding how exponential and linear functions interact when subtracted deepens your comprehension of their behaviors and prepares you for more complex mathematical modeling. Whether analyzing data, solving equations, or exploring mathematical theories, mastering such operations is essential to advancing your mathematical proficiency.

Remember: Practice makes perfect. Work through similar problems regularly to become confident in handling diverse function combinations and operations.

Frequently Asked Questions

Given F(x) = -5^x - 4 and G(x) = -3x - 2, what is (F - G)(x)?

(F - G)(x) = (-5^x - 4) - (-3x - 2) = -5^x - 4 + 3x + 2 = -5^x + 3x - 2

How do you compute (F - G)(x) when F(x) and G(x) are given functions?

Subtract G(x) from F(x): $(F - G)(x) = F(x) - G(x)$, then simplify the expression.

What is the simplified form of (F - G)(x) for the given functions?

The simplified form is $(F - G)(x) = -5^x + 3x - 2$.

Are there any common factors or terms in (F - G)(x)?

No, the terms -5^x, $3x$, and -2 are distinct and cannot be combined further.

If F(x) = -5^x - 4 and G(x) = -3x - 2, what is the value of (F - G)(x) at x=0?

At $x=0$: $F(0) = -5^0 - 4 = -1 - 4 = -5$; $G(0) = -3(0) - 2 = -2$; thus, $(F - G)(0) = -5 - (-2) = -5 + 2 = -3$.

Can the expression (F - G)(x) be written in a different form?

Yes, $(F - G)(x) = -5^x + 3x - 2$, which clearly shows the exponential, linear, and constant parts.

What type of function is (F - G)(x)?

It is a combination of an exponential function (-5^x) and a linear function ($3x$) with a constant shift (-2).

Is the answer to the given options correctly provided in the question?

Based on the calculation, the correct $(F - G)(x)$ is $-5^x + 3x - 2$, which matches option B if it states that.

What is the importance of understanding function subtraction in algebra?

It helps in analyzing combined behaviors of functions, solving equations, and understanding function transformations.

What are common mistakes to avoid when subtracting

functions?

Ensure to subtract each term properly, especially paying attention to signs and not combining unlike terms incorrectly.

Additional Resources

Understanding the Composition of Functions: A Deep Dive into F(x) and G(x)

In the realm of mathematics, functions serve as foundational constructs that describe relationships between variables. When these functions are combined—through operations such as addition, subtraction, or composition—the resulting expressions reveal intricate behaviors and properties. This article explores a particular scenario involving two functions, F(x) and G(x), and aims to analyze the combined function (F - G)(x). By systematically dissecting the given functions, performing algebraic manipulations, and interpreting the results, we provide a comprehensive understanding suitable for both students and enthusiasts seeking to deepen their grasp of function operations.

Introduction to the Functions F(x) and G(x)

Before delving into the combination of functions, it is crucial to understand their individual definitions and fundamental characteristics.

Function F(x): Exponential Decay with a Shift

Given:

$$F(x) = -5^x - 4$$

- Exponential Component: The term 5^x signifies exponential growth or decay, depending on the base and sign. In this case, the base is 5, a positive real number greater than 1, indicating exponential growth as x increases.
- Negative Sign: The leading negative sign before 5^x inverses the exponential growth, producing exponential decay when considering the overall behavior.
- Vertical Shift: Subtracting 4 shifts the entire graph downward by 4 units. This affects the function's range and intercepts but maintains the exponential decay pattern.

Graphical insight: The graph of $F(x)$ is a reflection of the standard exponential growth graph 5^x across the x-axis, shifted downward by 4 units. As x approaches positive infinity, $F(x)$ decreases toward $-\infty$ due to the dominant exponential term. Conversely, as x approaches negative infinity, 5^x approaches 0, so $F(x)$ approaches -4.

Function G(x): Linear Function with a Negative Slope

Given:
$$
G(x) = -3x - 2
$$

- Linear Nature: The function is linear with a slope of -3 and a y-intercept at -2.
- Behavior: As x increases, $G(x)$ decreases linearly; as x decreases, $G(x)$ increases linearly.
- Graph: The line cuts the y-axis at -2 and descends with a slope of -3.

Graphical insight: The linear function is straightforward, with predictable behavior—decreasing steadily as x increases, and crossing the y-axis at -2.

Objective: Find (F - G)(x)

The core goal is to compute the difference:
$$
(F - G)(x) = F(x) - G(x)
$$
Given the definitions:
$$
F(x) = -5^x - 4
$$
$$
G(x) = -3x - 2
$$

Substituting:
$$
(F - G)(x) = (-5^x - 4) - (-3x - 2)
$$

Our task involves algebraic simplification and interpretative analysis to understand the resulting function's behavior.

Step-by-Step Derivation of (F - G)(x)

1. Expand the expression

Applying the distributive property:
\[
(F - G)(x) = -5^x - 4 + 3x + 2
\]

Note that subtracting \(-3x - 2\) is equivalent to adding \(3x + 2\).

2. Combine like terms

Rearranged, the expression becomes:
\[
(F - G)(x) = -5^x + 3x - 4 + 2
\]

Simplify the constants:
\[
-4 + 2 = -2
\]

Thus:
\[
(F - G)(x) = -5^x + 3x - 2
\]

Final simplified form:
\[
\boxed{
(F - G)(x) = -5^x + 3x - 2
}
\]

Interpreting the Resulting Function

The expression:
\[
(F - G)(x) = -5^x + 3x - 2
\]
is a blend of exponential and linear components. Let's analyze this hybrid function's properties:

Behavior at Extreme Values of x

- As $x \to +\infty$:
- $5^x \to +\infty$, so $-5^x \to -\infty$.
- $3x \to +\infty$, but grows linearly.
- Overall, the exponential decay dominates, and the function tends toward $-\infty$.

- As $x \to -\infty$:
- $5^x \to 0$, so $-5^x \to 0$.
- $3x \to -\infty$, linearly decreasing.
- Therefore, the dominant term is $3x$, and the function tends toward $-\infty$.

This indicates the function has a global tendency toward $-\infty$ at both ends of the real line.

Critical Points and Turning Behavior

To understand where the function reaches local maxima or minima, we analyze its derivative.

Calculus-Based Analysis: Derivative of (F - G)(x)

1. Compute the derivative:

$$
\frac{d}{dx} (F - G)(x) = \frac{d}{dx} \left(-5^x + 3x - 2\right)
$$

Recall:
- Derivative of a^x: $\frac{d}{dx} a^x = a^x \ln a$
- Derivative of $k x$: k

Applying:

$$
\frac{d}{dx} (-5^x) = -5^x \ln 5
$$

$$
\frac{d}{dx} (3x) = 3
$$

$$
\frac{d}{dx} (-2) = 0
$$

Thus:
$$
\frac{d}{dx} (F - G)(x) = -5^x \ln 5 + 3
$$

2. Find critical points by setting the derivative to zero:

$$
-5^x \ln 5 + 3 = 0
$$
$$
-5^x \ln 5 = -3
$$
$$
5^x \ln 5 = 3
$$
$$
5^x = \frac{3}{\ln 5}
$$

Since $5^x > 0$ for all real x, and $\ln 5 > 0$, the right side is positive, and the solution exists.

3. Solve for x:

$$
x = \log_5 \left(\frac{3}{\ln 5} \right)
$$

This is the critical point where the function may attain a local maximum or minimum.

Implications of the Analysis

The critical point indicates where the function transitions from increasing to decreasing or vice versa. To classify it:

- Evaluate the second derivative or analyze the sign change of the first derivative around the critical point.
- The presence of an exponential term combined with a linear term results in a complex but predictable behavior: the function will have a single extremum, likely a maximum, due to the dominant exponential decay at large x.

Comparison with the Multiple-Choice Options

The question provides options for the expression of \((F - G)(x)\):

A. \((F - G)(x) = 5^x - 3x + 2\)

B. \((F - G)(x) = \) (not fully specified)

From our derivation, the correct expression is:
\[
(F - G)(x) = -5^x + 3x - 2
\]

This matches none of the options exactly as written, but a close inspection suggests that option A is not correct because:

- Our derived expression has \(-5^x\), whereas option A has \(5^x\).
- The signs before the linear and constant terms differ.

However, examining the options further, if the options are incorrectly formatted or incomplete, the key takeaway is that the correct combined function, based on algebraic operations, is:

\[
\boxed{
(F - G)(x) = -5^x + 3x - 2
}
\]

which is consistent with the derivation.

Conclusion and Final Remarks

The process of combining functions is fundamental in algebra and calculus, providing insights into the behavior of complex systems modeled by multiple relationships.

If F X 5 X 4 And G X 3 X 2 Find F G X O A F G X 5 X 3x 2o B F

If F X 5 X 4 And G X 3 X 2 Find F G X O A F G X 5 X 3x 2o B F

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