If F(x) = Cos X And G(x)= Tan X, Which Of These Lines Is An Asymptote Of The Graph Of (f-g)(x)? A. Cx

If F(x) = Cos X And G(x)= Tan X, Which Of These Lines Is An Asymptote Of The Graph Of (f-g)(x)? A. Cx In the exploration of mathematical functions, understanding asymptotes is crucial for analyzing the behavior of graphs, especially when dealing with combinations of functions like differences, sums, or products. Given the functions F(x) = cos x and G(x) = tan x, we are tasked with determining which lines serve as asymptotes of the graph of (f - g)(x). Specifically, the question asks whether the line A. Cx is an asymptote for this combined function. This article delves into the concepts of asymptotes, analyzes the behavior of F(x) and G(x), and provides a detailed step-by-step approach to identify the asymptote of (f - g)(x).

Understanding the Functions: Cosine and Tangent

Properties of Cosine Function

- The cosine function, F(x) = cos x, is a periodic function with a period of 2□.
- Its range is [-1, 1], meaning it oscillates between these values.
- Cos x is continuous and smooth everywhere, with no asymptotes.

Properties of Tangent Function

- The tangent function, G(x) = tan x, is also periodic with a period of □.
- It has vertical asymptotes where the cosine function equals zero, i.e., at x = □/2 + n□, where n is an integer.
- G(x) is unbounded near its asymptotes, approaching infinity or negative infinity.

Analyzing the Function (f - g)(x) = cos x - tan x

Expression of (f - g)(x)

The combined function is:

(f - g)(x) = cos x - tan x

Rewriting Tan x in terms of sine and cosine

- Recall that tan x = sin x / cos x.
- Therefore, (f - g)(x) = cos x - (sin x / cos x).

Expressing (f - g)(x) as a single fraction

To analyze the behavior, combine into a common denominator:

$(f - g)(x) = (\cos^2 x - \sin x) / \cos x$

This form clearly shows the function's behavior near points where the denominator approaches zero, which are potential asymptotes.

Identifying Potential Asymptotes of (f - g)(x)

Vertical Asymptotes

Vertical asymptotes occur where the denominator is zero and the numerator is not zero at those points. Here, the denominator is cos x, so potential vertical asymptotes are at $x = \pi/2 + n\pi$, where n is any integer, since $\cos x = 0$ at these points.

Behavior Near $x = \pi/2 + n\pi$

- As x approaches these points, tan x tends to infinity or negative infinity.
- The numerator, $\cos^2 x - \sin x$, can be evaluated at these points:
  - At $x = \pi/2 + n\pi$, $\cos x = 0$.
  - $\sin x = \pm 1$ depending on n.

- Thus, numerator at these points is:
  - $\cos^2 x = 0$
  - $\sin x = \pm 1$

- Numerator becomes $0 - (\pm 1) = \mp 1$, which is non-zero.

Therefore, at these points, the numerator remains finite and non-zero while the denominator approaches zero, leading to the entire function tending toward infinity or negative infinity. This confirms the presence of vertical asymptotes at $x = \pi/2 + n\pi$.

Are There Any Oblique or Horizontal Asymptotes?

Horizontal Asymptotes

As x approaches infinity or negative infinity:

- cos x oscillates between -1 and 1, so it remains bounded.

- tan x becomes unbounded near its asymptotes, but away from them, it oscillates wildly.

Since tan x does not tend toward a finite value, and cos x remains bounded, their difference does not tend to a finite limit; it oscillates without approaching a horizontal line.

Oblique Asymptotes

Oblique (slant) asymptotes typically occur when the degree of the numerator exceeds that of the denominator in rational functions. Since our function involves trigonometric functions, and the dominant behavior near asymptotes is unbounded, there is no oblique asymptote present.

Conclusion: Which Line Is an Asymptote of (f - g)(x)?

Summary of Findings

- Vertical asymptotes occur at $x = \pi/2 + n\pi$ due to the zeroes of cos x in the denominator.
- These asymptotes are lines $x = \pi/2 + n\pi$, not linear functions like Cx.
- The expression A. Cx suggests a potential asymptote along a line of the form y = Cx, which is a slant line rather than a vertical line.
- Since the asymptotes are vertical lines, the line y = Cx does not serve as an asymptote in this context.

Final Insight: Is the Line A. Cx an Asymptote?

Given the analysis, the asymptotes of (f - g)(x) are vertical lines at $x = \pi/2 + n\pi$, rather than any slant line like y = Cx. Therefore, the correct conclusion is that the graph of (f - g)(x) does not have the line y = Cx as an asymptote. Instead, it has vertical asymptotes at specific x-values where cos x = 0.

Additional Tips for Analyzing Asymptotes of Trigonometric Functions

1. Always identify points where denominators become zero in rational expressions involving trigonometric functions.

2. Understand the periodic nature of trig functions to anticipate the location of asymptotes.

3. Evaluate the behavior of the function near these critical points to confirm the presence of vertical asymptotes.

4. Recall that horizontal and oblique asymptotes are less common in functions involving unbounded oscillations like tan x.

Summary

In the case of the functions F(x) = cos x and G(x) = tan x, the combined function (f - g)(x) exhibits vertical asymptotes at $x = \pi/2 + n\pi$ due to the zeros of cos x in the denominator of its simplified form. These asymptotes are lines $x = \pi/2 + n\pi$, not lines of the form y = Cx. Therefore, the line A. Cx is not an asymptote of (f - g)(x). Understanding these properties helps in grasping the behavior of complex trigonometric functions and their graphs.

Frequently Asked Questions

Given F(x) = Cos x and G(x) = Tan x, what is the expression for (f – g)(x)?

(f - g)(x) = Cos x - Tan x

Which lines can serve as asymptotes for the graph of (f – g)(x) = Cos x – Tan x?

Vertical asymptotes occur where Tan x is undefined, i.e., at x = (□/2) + n□, where n is an integer.

Are lines like Cx (a straight line) asymptotes for (f – g)(x) = Cos x – Tan x?

No, lines of the form Cx are not asymptotes for this function; asymptotes are typically vertical or horizontal lines where the function diverges.

What is the significance of the line x = (□/2) + n□ for the graph of (f – g)(x)?

These are the vertical asymptotes because Tan x tends to infinity at these points, causing the function to diverge.

Does the difference (f – g)(x) = Cos x – Tan x have any horizontal asymptotes?

No, because as x approaches infinity or negative infinity, Tan x oscillates and Cos x remains bounded, so the difference does not approach a finite limit.

What is the role of the line Cx in relation to the asymptotes of (f –

g)(x)?

The line Cx does not serve as an asymptote for this function; it is unrelated to the vertical or horizontal asymptotes of (f - g)(x).

Based on the options, which line is an asymptote of (f - g)(x)?

Vertical asymptotes occur at x = (□/2) + n□; the line Cx is not an asymptote unless specified as x = (□/2) + n□, but as given, Cx is not a valid asymptote for this graph.

Additional Resources

Understanding Asymptotes in the Context of (f - g)(x): A Detailed Analysis

In the realm of mathematical analysis, particularly in the study of functions and their behaviors, asymptotes serve as critical indicators of a graph's tendencies and limits. When examining the function \((f - g)(x) \), where \(f(x) = \cos x \) and \(g(x) = \tan x \), determining the presence and nature of asymptotes becomes an essential step in understanding the graph's overall structure. This article delves deeply into the characteristics of these functions, explores how their difference behaves, and critically analyzes which lines—if any—act as asymptotes of \((f - g)(x) \). We will also interpret the given option, "A. Cx," in the context of asymptotic behavior, providing a comprehensive overview suitable for both students and enthusiasts of mathematical analysis.

Fundamental Concepts: Functions, Asymptotes, and Their Significance

Before diving into the specifics of \((f - g)(x) \), it's important to clarify some foundational concepts:

What is an Asymptote?

An asymptote is a line that a graph approaches arbitrarily closely as the independent variable x tends toward a certain value—either finite or infinite. Asymptotes can be:

- Vertical Asymptotes: Lines $x = a$ where the function diverges to infinity or negative infinity as $x \to a$.
- Horizontal Asymptotes: Lines $y = L$ which the function approaches as $x \to \pm \infty$.
- Oblique (Slant) Asymptotes: Lines $y = mx + c$ that the graph approaches but are neither horizontal nor vertical.

Understanding the presence and type of asymptotes helps in sketching the graph accurately and predicting the behavior of functions at critical points or limits.

Functions Involved: Cosine and Tangent

- Cosine Function ($\cos x$): A periodic, bounded function oscillating between -1 and 1 with a period of 2π. It is continuous everywhere and does not have asymptotes.
- Tangent Function ($\tan x$): A periodic function with period π, which has vertical asymptotes at points where $\cos x = 0$, i.e., at $x = \frac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$. $\tan x$ is unbounded near these points.

Analyzing $(f - g)(x) = \cos x - \tan x$

The core of this analysis revolves around understanding the behavior of the composite function:

$$
h(x) = (f - g)(x) = \cos x - \tan x
$$

\]

which combines the properties of both the cosine and tangent functions.

Step 1: Basic Domain Considerations

Since $\tan x$ is undefined where $\cos x = 0$, the domain of $h(x)$ excludes points where $\cos x = 0$:

$$
x \neq \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}
$$

At these points, $\tan x \to \pm \infty$, and thus $h(x)$ cannot be defined at these points.

Step 2: Behavior Near Vertical Asymptotes

As $x \to \frac{\pi}{2} + n\pi$, $\tan x$ approaches $\pm \infty$, and the dominant term in $h(x)$ is $- \tan x$, which diverges to $\mp \infty$. Consequently, $h(x)$ exhibits vertical asymptotes at these points.

- For $x \to \frac{\pi}{2}^-$, $\tan x \to +\infty$, so $h(x) \to -\infty$.
- For $x \to \frac{\pi}{2}^+$, $\tan x \to -\infty$, so $h(x) \to +\infty$.

Similarly, this pattern repeats at other vertical asymptotes.

Step 3: Behavior at Infinity

To understand the potential for horizontal or oblique asymptotes, examine $h(x)$ as $x \to \pm \infty$.

- Since $\cos x$ oscillates between -1 and 1, it remains bounded.
- $\tan x$ does not have a limit as $x \to \pm \infty$ because it is periodic and unbounded near asymptotes.

Thus, $h(x)$ does not tend to a finite value at infinity; instead, it oscillates wildly due to the unbounded nature of $\tan x$. Therefore, there are no horizontal asymptotes.

Is There an Asymptote of the Form $y = Cx$?

The question posed is: "Which of these lines is an asymptote of the graph of $(f - g)(x)$? A. Cx."

This suggests considering the possibility of a slant (oblique) asymptote of the form:

$$
y = Cx
$$

where C is a constant.

Step 1: Testing for Slant Asymptotes

A line $y = Cx$ is a candidate asymptote if:

$$
\lim_{x \to \pm \infty} \left[h(x) - Cx \right] = 0
$$

which indicates that $h(x)$ approaches the line $y = Cx$ as $x \to \pm \infty$.

Given the behavior of $h(x)$, especially the unbounded oscillations of $\tan x$, it is unlikely that such a limit exists, because:

- The oscillations of $\tan x$ prevent $h(x)$ from settling close to any linear function Cx.
- The boundedness of $\cos x$ is insignificant compared to the unbounded nature of $\tan x$.

Hence, no linear asymptote of the form $y = Cx$ exists for $h(x)$.

Step 2: Conclusion on Asymptotic Lines

Given the above, $(f - g)(x) = \cos x - \tan x$ does not have any oblique asymptote of the form $y = Cx$. It exhibits vertical asymptotes at points where $\tan x$ diverges but lacks horizontal or oblique asymptotes.

Implications for the Given Multiple Choice: "A. Cx"

The multiple-choice answer "A. Cx" suggests a line of the form $y = Cx$. Based on our detailed analysis, this is not an asymptote of $(f - g)(x)$.

- Vertical asymptotes occur at specific finite points $x = \frac{\pi}{2} + n\pi$, but these are lines $x = \text{constant}$, not of the form $y = Cx$.
- Horizontal asymptotes are absent because the function oscillates unboundedly as $x \to \pm \infty$.
- Oblique or slant asymptotes of the form $y = Cx$ are ruled out because the oscillatory behavior of $\tan x$—and consequently of $h(x)$—prevents the function from approaching such a line at infinity.

Therefore, the statement "Which of these lines is an asymptote of $(f - g)(x)$? A. Cx" is incorrect in the context of asymptotic behavior.

Summary and Final Remarks

This comprehensive analysis reveals that:

- The function \((f - g)(x) = \cos x - \tan x \) exhibits vertical asymptotes at points where \(\cos x = 0 \), i.e., \(x = \frac{\pi}{2} + n\pi \), due to the divergence of \(\tan x \).
- There are no horizontal asymptotes because the oscillatory nature of \(\tan x \) prevents the function from approaching a finite constant as \(x \to \pm \infty \).
- There are no oblique (slant) asymptotes of the line \(y = Cx \) because the unbounded, oscillatory behavior of \(\tan x \) precludes the function from approaching such linear lines at infinity.
- The proposed line \(y = Cx \) as an asymptote is therefore not supported by the behavior of \(h(x) \).

In conclusion, understanding the asymptotic behavior of combined functions like \(\cos x - \tan

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