If Is An Acute Angle In Standard Position And B (-3, 4) Is A Point On The Terminal Side Of The Angle,

If Is An Acute Angle In Standard Position And B (-3, 4) Is A Point On The Terminal Side Of The Angle, understanding how to analyze and interpret the given information is essential in trigonometry. Whether you're a student tackling geometry problems or a mathematics enthusiast exploring the properties of angles and coordinate points, this scenario offers a valuable opportunity to deepen your understanding of angles in standard position, reference angles, and the relationships between coordinate points and angles.

In this article, we'll explore the fundamental concepts involved in identifying an acute angle in standard position with a specific point on its terminal side. We'll cover how to determine the angle's measure, find its reference angle, and understand the significance of the point's coordinates. By the end, you'll have a comprehensive understanding of how to approach these types of problems confidently.

Understanding Angles in Standard Position

What Is an Angle in Standard Position?

An angle in standard position is an angle whose vertex is at the origin of the coordinate plane, with its initial side lying along the positive x-axis. The terminal side is the side where the angle "ends" after rotation from the initial side. The measure of the angle is typically expressed in degrees or radians, and depending on the context, the angle can be acute, right, obtuse, or reflex.

Characteristics of an Acute Angle

An acute angle measures greater than 0° and less than 90°. In the context of standard position, an acute angle's terminal side will be located between the positive x-axis and a line that forms an angle less than 90° with the x-axis. Recognizing whether an angle is acute involves analyzing the point on its terminal side and calculating its measure.

Analyzing the Point B (-3, 4) on the Terminal Side

What Does the Coordinates Tell Us?

Given the point B (-3, 4), we know that the terminal side of the angle passes through this point. The coordinates (-3, 4) are in the Cartesian plane, where:

- • $x = -3$ (the horizontal distance from the origin)
- • $y = 4$ (the vertical distance from the origin)

Since the x-coordinate is negative and the y-coordinate is positive, point B lies in the second quadrant of the coordinate plane.

Implications for the Angle's Location

Because the terminal side passes through $(-3, 4)$, and the point is in the second quadrant, the angle's measure will be between 90° and 180°. However, the problem states that the angle is acute, which suggests that the initial assumption needs to be clarified: perhaps the angle is in the first quadrant, or the problem involves the reference angle.

To reconcile this, consider the following:

- The reference angle is the acute angle between the terminal side and the x-axis.
- Even if the point is in the second quadrant, the reference angle can be acute.

Therefore, to determine whether the angle in question is acute, we need to analyze the reference angle associated with point B.

Calculating the Reference Angle

Step 1: Find the Distance from the Origin to the Point (r)

The distance r from the origin to point B is given by the Pythagorean theorem:

$$
r = \sqrt{x^2 + y^2}
$$

Substituting the coordinates:

$$
r = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5
$$

Step 2: Find the Value of the Reference Angle (θ_{ref})

The reference angle is the acute angle between the terminal side of the angle and the x-axis. It can be found using the ratio of the y-coordinate to r:

$$
\theta_{ref} = \arcsin\left(\frac{|y|}{r}\right)

$$

or equivalently,

$$
\theta_{ref} = \arctan\left(\frac{|y|}{|x|}\right)
$$

Using the tangent ratio:

$$
\theta_{ref} = \arctan\left(\frac{4}{3}\right)
$$

Calculating:

$$
\theta_{ref} \approx \arctan(1.333) \approx 53.13^\circ
$$

Since the point is in the second quadrant, the actual angle θ in standard position is:

$$
\theta = 180^\circ - \theta_{ref} \approx 180^\circ - 53.13^\circ \approx 126.87^\circ
$$

This is an obtuse angle, not acute.

Key insight: The point (−3, 4) lies in the second quadrant, resulting in an angle greater than 90°. Therefore, if the problem specifies that the angle is acute, then the point must lie in the first quadrant, or the problem may be asking about the reference angle.

Conclusion: For the point (−3, 4), the angle in standard position is approximately 126.87°, which is obtuse, not acute. Therefore, if the question asks whether the angle is acute, the answer is no unless further clarification or different point coordinates are provided.

Determining the Actual Angle and Its Trigonometric Values

Using the Coordinates to Find $\sin \theta$, $\cos \theta$, and $\tan \theta$

Given:

$$
x = -3,\quad y = 4,\quad r = 5
$$

The basic trigonometric ratios are:

- $\sin \theta = \frac{y}{r} = \frac{4}{5} = 0.8$
- $\cos \theta = \frac{x}{r} = \frac{-3}{5} = -0.6$
- $\tan \theta = \frac{y}{x} = \frac{4}{-3} = -\frac{4}{3} \approx -1.333$

Since $\sin \theta$ is positive and $\cos \theta$ is negative, the angle is located in the second quadrant, consistent with earlier analysis.

Note: These ratios help in solving various trigonometric problems related to the point and the angle.

Applications and Additional Considerations

Using the Point to Find the Exact Angle Measure

To find the actual measure of the angle in degrees:

$$
\theta = \arctan \left(\frac{y}{x} \right)
$$

Since x is negative and y is positive, the arctangent gives an angle in the fourth quadrant, so we need to adjust for the correct quadrant:

$$
\theta = \arctan \left(\frac{4}{-3} \right) \approx -53.13^\circ
$$

Adding 180° to move to the second quadrant:

$$
\theta \approx 180^\circ - 53.13^\circ = 126.87^\circ
$$

This confirms the earlier calculation.

Understanding the Significance of the Point's Coordinates

Knowing the coordinates of a point on the terminal side of an angle allows you to:

- Calculate the radius (distance from origin)
- Determine the reference angle
- Find the exact measure of the angle in standard position
- Compute the sine, cosine, and tangent values

These are fundamental skills in trigonometry, especially for solving real-world problems involving angles and vectors.

Summary and Key Takeaways

- Angles in standard position have their vertex at the origin, initial side along the positive x-axis, and terminal side passing through a point in the coordinate plane.
- The point (−3, 4) lies in the second quadrant, resulting in an angle of approximately 126.87°, which is obtuse, not acute.
- The reference angle corresponding to point (−3, 4) is approximately 53.13°, which is acute.
- Using the coordinates, you can compute trigonometric ratios and the exact measure of the angle in standard position.
- Understanding the relationship between points and angles in the coordinate plane is essential for solving a variety of trigonometry problems.

Final note: When working with points and angles, always consider the quadrant in which the point lies to correctly identify the angle's measure and sign of trigonometric functions. In cases where the problem specifies an acute angle, ensure that the point's coordinates correspond to the first quadrant or that you're analyzing the reference angle.

If you want to explore more about angles in different quadrants, how to convert between degrees and radians, or applications in physics and engineering, feel free to ask!

Frequently Asked Questions

What is the measure of the angle in standard position with point B (−3, 4) on its terminal side?

The measure of the angle can be found using the tangent function: θ = arctangent (y/x) = arctangent (4/(−3)) ≈ 126.87°. Since the point is in the second quadrant, the angle in standard position is approximately 126.87°, which is an obtuse angle, not acute.

Is the angle formed by point B (−3, 4) in standard position acute, right, or obtuse?

The angle is approximately 126.87°, which makes it an obtuse angle, not acute.

How do you determine if a point lies on the terminal side of an angle in standard position?

A point (x, y) lies on the terminal side of an angle in standard position if the angle's initial side is along the positive x-axis and the point's coordinates satisfy the angle's direction and position in the coordinate plane. The angle can be found using the arctangent of y/x, considering the signs to determine the correct quadrant.

What is the significance of the coordinates (-3, 4) in determining the angle in standard position?

The coordinates (-3, 4) indicate the point's position in the second quadrant, which affects the calculation of the angle's measure and its classification as acute, right, or obtuse.

Can the point (-3, 4) be on the terminal side of an acute angle in standard position?

No, because the calculated angle is approximately 126.87°, which is greater than 90°, so it is not an acute angle.

How do you calculate the reference angle for the point (-3, 4)?

The reference angle is the acute angle between the terminal side and the x-axis. For point (-3, 4), the reference angle is θ = arctangent ($|y|/|x|$) = arctangent (4/3) ≈ 53.13°.

What is the role of the tangent function in determining the angle for point B (-3, 4)?

The tangent function relates the y and x coordinates: $\tan \theta = y/x$. Using this, θ = arctangent (4/(-3)) gives the reference angle in the correct quadrant.

If the point B (-3, 4) is on the terminal side, what is the exact measure of the angle in degrees?

The angle in standard position is approximately 126.87°, since arctangent (4/(-3)) ≈ -53.13°, and adjusting for the second quadrant gives 180° - 53.13° = 126.87°.

Why is the angle with point B (-3, 4) not considered an acute angle?

Because its measure is approximately 126.87°, which is greater than 90°, and less than 180°, classifying it as an obtuse angle.

How does knowing the coordinates of a point help in

understanding the angle in standard position?

Coordinates help determine the quadrant and facilitate calculation of the angle's measure using inverse tangent functions, allowing us to classify the angle as acute, right, or obtuse.

Additional Resources

If Is An Acute Angle In Standard Position And B (-3, 4) Is A Point On The Terminal Side Of The Angle,

Understanding the fundamental concepts of angles and coordinate geometry is essential for students and professionals working in fields such as mathematics, engineering, physics, and computer graphics. When an angle is described as being in standard position, and a specific point lies on its terminal side, it opens up a range of analytical opportunities to explore its properties, measure its size, and understand its orientation in the coordinate plane. This article delves into the scenario where an acute angle in standard position has a point B(-3, 4) on its terminal side, unpacking the geometric and algebraic principles involved.

What Does It Mean for an Angle to Be in Standard Position?

Before analyzing the specifics of the point B(-3, 4), it is vital to clarify what is meant by an angle being in standard position.

Definition of Standard Position:

- An angle is said to be in standard position if:
- Its vertex is located at the origin (0,0) of the coordinate plane.
- Its initial side coincides with the positive x-axis.
- Its terminal side extends from the origin into the plane, forming a certain measure of rotation.

Implications:

- This positioning allows for consistent measurement of angles, typically in degrees or radians, with positive angles measured counterclockwise from the initial side.
- It simplifies the process of defining the angle's measure solely based on the terminal side's position relative to the initial side.

In our scenario, the angle's vertex is at the origin, and its terminal side passes through some point in the plane, including the point B(-3, 4). Since B is on the terminal side, the angle's measure relates directly to the position of B in the coordinate system.

Determining Whether the Angle Is Acute

An acute angle measures less than 90 degrees (or $\pi/2$ radians). To verify whether the angle in question is indeed acute, we need to analyze the position of point B relative to the initial side (positive x-axis).

Key Steps:

1. Identify the Direction of the Terminal Side:
- The terminal side passes through point B(−3, 4).
- Since the initial side is along the positive x-axis, the angle between the initial and terminal sides is determined by the position of B.

2. Calculate the Reference:
- The angle θ between the positive x-axis and the line segment OB (where O is the origin) can be calculated using the vector from the origin to B.

3. Use the Dot Product Method:
- The dot product of vectors can help determine the angle between two vectors.
- The initial side vector: i = (1, 0).
- The vector to B: OB = (−3, 4).

4. Compute the Dot Product:
- $i \cdot OB = (1)(-3) + (0)(4) = -3$.

5. Calculate Magnitudes:
- $|i| = 1$.
- $|OB| = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.

6. Find the Cosine of the Angle:

$$
\cos \theta = \frac{\textbf{i} \cdot \textbf{OB}}{|\textbf{i}| \times |\textbf{OB}|} = \frac{-3}{1 \times 5} = -\frac{3}{5} = -0.6
$$

7. Determine the Angle:

$$
\theta = \arccos(-0.6).
$$

- $\arccos(-0.6) \approx 126.87^\circ$.

Conclusion:

Since approximately $126.87^\circ > 90^\circ$, the angle is obtuse, not acute. Therefore, the initial assumption that the angle might be acute does not align with the calculated value based on the point B(−3, 4).

Re-evaluating the Scenario: Is the Angle Actually Acute?

Given the calculation above, the angle formed by the initial side (positive x-axis) and the terminal side passing through B(−3, 4) is approximately

126.87°, which makes it obtuse. However, the initial question states that the angle is acute. This suggests a need for clarification or a different interpretation.

Possible Clarifications:

- The point B(-3, 4) might be on the other side of the origin relative to the initial side, leading to a different angle measurement.
- The angle could be measured clockwise, resulting in a smaller measure.
- The point B could define a reflex angle (greater than 180°), but the focus is on the acute angle, which is less than 90°.

Alternate Approach:

Suppose the problem refers to the reference angle between the terminal side and the x-axis, which is always between 0° and 90°, regardless of which quadrant the point lies in.

- To find the reference angle, take the absolute value of the coordinate ratios:

$$
\text{Reference angle } \alpha = \arctan \left(\frac{|y|}{|x|} \right).
$$

- For B(-3, 4):

$$
\alpha = \arctan \left(\frac{4}{3} \right) \approx 53.13^\circ.
$$

This indicates that the reference angle is approximately 53.13°, which is acute.

Implication:

- The reference angle is always acute and helps determine the actual angle based on the quadrant.
- Since B(-3, 4) is in the second quadrant (x negative, y positive), the actual angle measured from the positive x-axis is:

$$
180^\circ - 53.13^\circ \approx 126.87^\circ,
$$

confirming the earlier calculation.

Conclusion:

- The reference angle associated with B(-3, 4) is acute.
- The actual angle in standard position is approximately 126.87°, which is obtuse.
- If the question emphasizes the reference angle, then yes, it is acute.

Understanding Coordinates and the Unit Circle Connection

The point B(-3, 4) can also be understood through its relationship to the unit circle, which is fundamental in trigonometry.

Key Concepts:

- The unit circle has a radius of 1, centered at the origin.
- Any point on the terminal side of an angle in standard position can be represented as ($\cos \theta, \sin \theta$), scaled appropriately for points outside the unit circle.

Applying to B(-3, 4):

- The vector from the origin to B has magnitude 5, as previously calculated.
- To find the direction or angle associated with B, normalize the vector:
$$
\left(\frac{-3}{5}, \frac{4}{5} \right).
$$
- These are the cosine and sine values of the reference angle:
$$
\cos \alpha = -\frac{3}{5} \approx -0.6,
$$
$$
\sin \alpha = \frac{4}{5} = 0.8,
$$
which aligns with the earlier calculations.

Quadrant Identification:

- Since x is negative and y is positive, B is in the second quadrant.
- The angle in standard position in the second quadrant with these cosine and sine values is approximately 126.87°, confirming the earlier findings.

Practical Applications of This Analysis

Understanding how to analyze points like B(-3, 4) in the context of angles in standard position has several practical applications:

- Navigation and Robotics: Determining directions based on coordinate points.
- Computer Graphics: Rotating objects and understanding orientations in the plane.
- Physics: Analyzing vectors of forces, velocities, or other vectors in two dimensions.
- Engineering Design: Calculating angles of components relative to axes.

Knowing how to interpret points on the terminal side of an angle helps in designing systems, solving geometric problems, and understanding the behavior of periodic functions like sine and cosine.

Summary and Key Takeaways

- An angle in standard position has its vertex at the origin, initial side along the positive x-axis, and its terminal side passing through a specific point.

- The point B(-3, 4) lies in the second quadrant, leading to an angle of approximately 126.87° with the positive x-axis.
- The reference angle associated with B is about 53.13°, which is acute.
- The calculation of the angle involves vector dot products, magnitude calculations, and inverse trigonometric functions.
- Recognizing the quadrant and the reference angle simplifies understanding the true measure of the angle and its position in the coordinate plane.

In conclusion, while the actual measure of the angle formed

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